

# Optimization of Partially Filled Column Subjected to Oblique Loading

M. S. Salwani, B. B. Sahari, Aidy Ali, and A. A. Nuraini

**Abstract**—In this study, optimization is carried out to find the optimized design of a foam-filled column for the best Specific Energy Absorption (SEA) and Crush Force Efficiency (CFE). In order to maximize SEA, the optimization gives the value of 2.3 for column thickness and 151.7 for foam length. On the other hand to maximize CFE, the optimization gives the value of 1.1 for column thickness and 200 for foam length. Finite Element simulation is run by using this value and the SEA and CFE obtained 1237.76 J/kg and 0.92.

**Keywords**—Crash; foam; oblique loading.

## I. INTRODUCTION

CURRENT goals for the automotive industry are to improve vehicle crashworthiness while decreasing its weight. Typical actions needed in order to reach these goals are the application of new materials and redesigned structural components. Physical testing can be costly, so simulation and optimization techniques come in hand to ease the issue. Column geometry and shape was known to affect the energy absorption capability. Thus, many researches have been done to optimize the design with regards to crashworthiness.

The optimization process with the target of maximizing the specific energy absorption has been successfully carried out by several researchers. To improve crushing energy absorption, foam has been adopted as one of new filler materials in impact engineering. Introduction of the foam material alters the crash behavior of structural component. With respect to foam-filling structures, [1] performed an optimization on single and triple-cell hexagonal columns filled with aluminum foams. Optimization aimed for

maximum specific energy with simultaneous consideration of section geometry, tube thickness, and foam density. [2] optimized the combination of foam density, column wall thickness, column width, column material strength and total component length to give the minimum mass to the component. It was found that optimum foam filled columns compared to the traditionally designed non-filled columns showed smaller cross section dimensions in addition to less weight. As a consequence, mass-, length- and volume reductions are possible by utilizing foam filler. The work by [3] has been implemented to find an optimum empty tube can absorb. [4, 5] also [4] investigate the strengthening effect of aluminum foam in filled column under axial and bending load. [6] had employed the sequential quadratic programming (SQP) to find the optimum design variables. Optimization procedure has been applied to maximize the SEA and to determine the optimum geometry of foam-filled tubes. The optimization process with the target of minimum weight design of a foam-filled has three design parameters, which are the width of the column wall, gauge thickness, and the relative foam density. [7] had solved the optimization problem of filled sections under combined compression/bending loading.

In regards to energy absorbing structure, exploration of the design optimization methodology is increasingly important. Polynomial response surfaces, radial basis functions, and Kriging are the different surrogate models used in the study of [8]. In the work of [9] metamodels by RSM and radial basis function (RBF) are compared for use in multi-objective optimization. In addition, multi-objective optimization had been performed by maximization of a composite objective function that provides a compromise between CFE and SEA. [10] had proposed an optimization methodology for single and multiple objectives of crashworthy structures. Deterministic and evolutionary algorithms linked with simplified models based on multibody dynamics formulations are presented. The difference between the single-objective and multiple-objective optimizations was brought forward by [11] in a Pareto sense. [12] shown that some useful optimal design principles involved in the performance of crashworthy structure can be discovered by the reliability-based robust Pareto optimization. In this study, polynomial functions are used to optimize the design of partially filled column subjected to oblique loading with the respect of specific energy absorption and crush force efficiency.

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II. METHODOLOGY

A. Finite Element simulation

In this study, a factorial design is used for its uniformity of sampling. In a full factorial design, when k levels are used for n variables, the total number of experiments is k<sup>n</sup>. The present analysis dealt with 2 variables that is the column thickness and the foam length. TABLE I lists the variables and levels used in the simulation.

TABLE I  
DESIGN LEVELS IN THE SIMULATION

Level	Column thickness, <i>t</i> (mm)	Foam length, <i>L</i> (mm)
1	1.1	40
2	1.4	80
3	1.7	120
4	2.0	160
5	2.3	200

A few crashworthiness parameters are used to assess the performance of filled square aluminum column. Parameters involve in this study are the specific energy absorption, SEA and crush force efficiency, CFE. SEA is introduced as:

$$SEA = \frac{E_a}{m}$$

where energy is defined per unit mass, *m*. The CFE is defined as the ratio of the mean crush load to the peak crush load:

$$CFE = \frac{P_m}{P_{max}}$$

A model of square aluminum column is developed using the Belytschko-Tsay shell element. Material properties of the aluminum column are shown in TABLE II.

TABLE II  
MECHANICAL PROPERTIES OF AA6060 T4

E (N/mm <sup>2</sup> )	ν	ρ (g/cm <sup>3</sup> )	σ <sub>y</sub> (N/mm <sup>2</sup> )
66820	0.33	2.7	175

The stress-strain definition for the aluminum column is extracted from the stress-strain curve of Fig. 1.

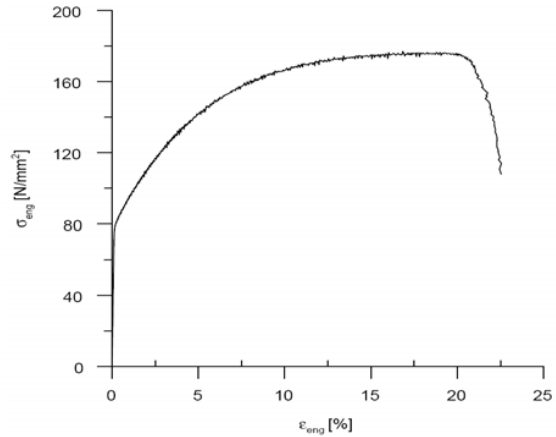


Fig. 1 Engineering stress-strain curve for AA6060[13]

Aluminum foam with a density of 0.365 g.cm<sup>3</sup> is modeled by using solid element. The power-law relationship between foam density, ρ<sub>f</sub> and foam plateau stress, σ<sub>p</sub> used for the purpose of simulation is taken from the equation produced by Gibson and Ashby.

$$\sigma_p = C_{pow} \left( \frac{\rho_f}{\rho_{f0}} \right)^n$$

*C<sub>pow</sub>* and *n* are material constants equals to 800 and 2.38, respectively [14], while ρ<sub>f0</sub> is the density of the foam base material, which is 2.7 g/cm<sup>3</sup> for aluminum. The stress-strain relationship [11] defined for the foam is shown in TABLE III.

TABLE III  
STRESS-STRAIN CURVE OF ALUMINUM FOAM

Stress	0	σ <sub>p</sub> / E	0.6	0.7	0.75	0.8
Strain	0	σ <sub>p</sub>	σ <sub>p</sub>	1.35σ <sub>p</sub>	5.00σ <sub>p</sub>	0.05E

Impact layout of the filled column is shown in Fig. 2. The load was applied at an angle θ, that is 30 degree, through a rigid body. The bottom end of the column is fixed in all degrees of freedom and the upper end is constrained to the rigid body. A velocity of 48 km/hr is prescribed to the rigid body.

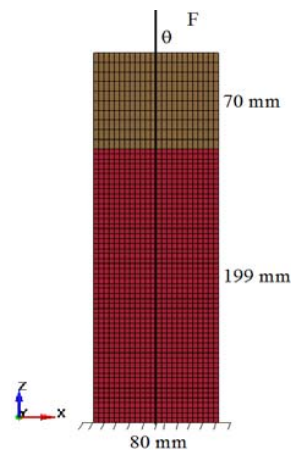


Fig. 2 Impact layout

### B. Polynomial Models Development

Polynomial models are constructed from sampled data gathered from Finite Element simulation. A linear and quadratic polynomial models are developed in the following form:

Model 1: Linear

$$\hat{y} = a_{00} + a_{10}t + a_{01}L$$

Model 2: Quadratic with interaction

$$\hat{y} = a_{00} + a_{10}t + a_{01}L + a_{11}tL + a_{20}t^2 + a_{02}L^2$$

Model 3: Pure quadratic

$$\hat{y} = a_{00} + a_{10}t + a_{01}L + a_{20}t^2 + a_{02}L^2$$

As it can be realized, an optimum approximate model is the one with minimum error. This can be achieved using the least squares technique. The coefficient of determination known as  $R^2$  is typically used to check the model's ability to identify the variation within the output response [15] and is defined as:

$$R^2 = 1 - \frac{SSE}{SST}$$

where

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

and

$$SST = \sum_{i=1}^n (y_i - \bar{y})^2$$

where  $y_i$  is the true output response, here calculated from nonlinear FE analysis,  $\hat{y}_i$  is the approximate response calculated from polynomial models,  $\bar{y}$  is the average of the true response, and  $n$  is the number of design points used to generate the model.  $R^2$  varies between 0 and 1, where values close to 1 mean that the approximate model has high ability to explain the variations within the output response. Generally speaking, the larger the values of  $R^2$  and  $R^2_{adj}$ , and the smaller the value of RMSE, the better the fit. In situations where the number of design variables is large, it is more appropriate to look at  $R^2_{adj}$ .

$$adjR^2 = 1 - \frac{\frac{SSE}{(n-p)}}{\frac{SST}{(n-1)}}$$

### C. Optimization

Optimization has been carried out to maximize the SEA and CFE. Numerical optimization has been done by using Sequential Quadratic Programming (SQP) method. In general SQP expands the nonlinear objective function quadratically

and linear constraints linearly about the current design by using the Taylor's Series.

$$f(\mathbf{X}_p + \Delta\mathbf{X}) = f(\mathbf{X}_p) + \nabla f(\mathbf{X}_p)^T \Delta\mathbf{X} + \frac{1}{2} \Delta\mathbf{X}^T \mathbf{H}(\mathbf{X}_p) \Delta\mathbf{X}$$

The first-order expansion is expressed in terms of gradient and the second-order expansion is expressed in terms of the Hessian matrix,  $\mathbf{H}$ .

The Hessian is updated based on the Lagrangian of the problem. The Lagrangian of the functions is

$$F(t, L) = f(t, L) + \lambda h(t, L) + \beta g(t, L)$$

where  $\lambda$  and  $\beta$  are Lagrange multiplier for the equality,  $h$  and inequality,  $g$  constraint.

When the first order conditions are satisfied, then the design is converged. Otherwise, the iteration will stop once the design is not changing or the number of maximum iterations reached.

## III. RESULTS

### A. Finite Element Simulation

TABLE IV  
SIMULATION RESULTS

L (mm)	t (mm)	m (kg)	SEA (J/kg)	CFE
40	1.1	0.27	385.47	0.27
	1.4	0.32	345.76	0.20
	1.7	0.38	785.63	0.44
	2.0	0.43	893.47	0.47
	2.3	0.48	919.79	0.59
80	1.1	0.35	336.08	0.30
	1.4	0.40	741.44	0.51
	1.7	0.45	869.52	0.56
	2.0	0.51	1023.06	0.61
	2.3	0.56	1161.96	0.75
120	1.1	0.42	658.30	0.71
	1.4	0.48	838.08	0.77
	1.7	0.53	985.42	0.72
	2.0	0.58	1031.66	0.68
	2.3	0.64	1255.62	0.87
160	1.1	0.50	757.42	0.91
	1.4	0.55	975.90	0.84
	1.7	0.61	1171.68	0.92
	2.0	0.66	1131.39	0.82
	2.3	0.71	1300.16	0.92
200	1.1	0.58	789.28	0.92
	1.4	0.63	844.59	0.82
	1.7	0.68	895.66	0.76
	2.0	0.74	967.88	0.73
	2.3	0.79	1030.32	0.72

Result of the simulations run on Finite Element software, Ls-dyna is shown in TABLE IV. Generally, it can be seen that SEA and CFE increases as the column thickness,  $t$  increases. The simulation results also suggest that, the increase in foam length contributes to stable deformation of the column through an improvement of CFE values.

*B. Polynomial models*

Based on the data obtained from finite element simulation, curve fitting of the data is carried out by using the polynomial model. The accuracy of the model mentioned in equation is calculated and shown in TABLE V. It can be concluded that the second model gives the best approximation of the simulated results for both SEA and CFE.

TABLE V  
POLYNOMIAL MODELS ACCURACY

Model	SEA accuracy			CFE accuracy		
	R <sup>2</sup>	Adj R <sup>2</sup>	RMSE	R <sup>2</sup>	Adj R <sup>2</sup>	RMSE
1	0.7941	0.7789	120.8	0.7597	0.7419	0.1172
2	0.8472	0.8154	110.4	0.8661	0.8381	0.09282
3	0.8321	0.8052	113.4	0.7921	0.7589	0.1133

The second order polynomial equations to approximate the data is as follows:

$$SEA(t, L) = -667.6 + 972.3t + 5.929L - 124.4t^2 - 1.074tL - 0.0114L^2$$

$$CFE(t, L) = 0.002481 - 0.02591t + 0.008282L + 0.1073t^2 - 0.002128tL - 9.394e-6L^2$$

*C. Optimization*

The maximization problem is formulated as the following. The first problems is aimed at maximizing the SEA and the second problem is to maximize the CFE. Both of the problems are restricted by the side constraints.

$$\left\{ \begin{array}{l} \text{Maximize} \quad SEA(t, L) = -667.6 + 972.3t + 5.929L - 124.4t^2 - 1.074tL - 0.0114L^2 \\ \text{s.t.} \quad t^l \leq t \leq t^u \\ \quad \quad L^l \leq L \leq L^u \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{Maximize} \quad CFE(t, L) = 0.002481 - 0.02591t + 0.008282L + 0.1073t^2 - 0.002128tL - 9.394e-6L^2 \\ \text{s.t.} \quad t^l \leq t \leq t^u \\ \quad \quad L^l \leq L \leq L^u \end{array} \right.$$

From the polynomial equation, a contour plot for SEA is built as in Fig. 3. It can be shown that SEA is at peak when the column thickness is about 2.0 to 2.3 mm and foam length is about 70 to 200 mm.

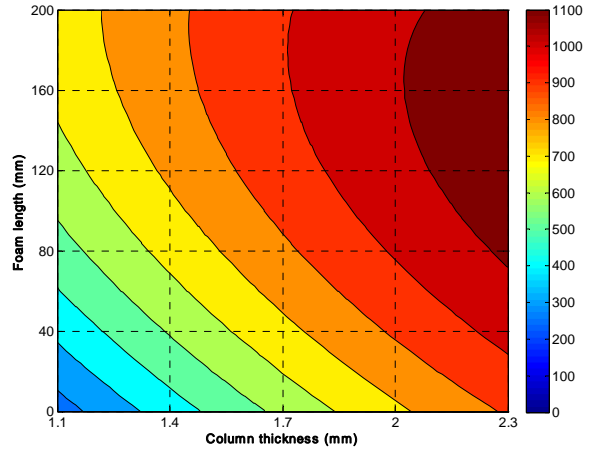


Fig. 3 Contour plot for SEA

From Fig. 4, it can be shown that CFE is at peak when the column thickness is about 1.1 to 1.2 mm and foam length is about 190 to 200 mm.

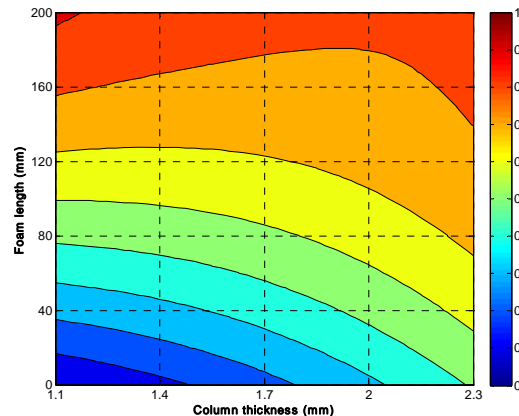


Fig. 4 Contour plot for SEA

From the contour plots, it gives several designs for better crash performance. In order to get a design that can achieves maximum SEA and CFE, Sequential Quadratic Programming (SQP) is used. SQP solves the optimization problem in two iterations. Any initial design selected is converged to the same value. In order to maximize SEA, the optimization gives the value of 2.3 for column thickness and 151.7 for foam length. On the other hand to maximize CFE, the optimization gives the value of 1.1 for column thickness and 200 for foam length. Finite Element simulation is run by using this value and the SEA and CFE obtained 1237.76 J/kg and 0.92.

## IV. CONCLUSION

For optimization purpose, a higher degree polynomial model can be used for better approximation. In the future research, more variables can also be added. SQP method will be much appreciated in handling more than two variables problem, where the polynomial model is not able to be illustrated graphically. Optimization of these crush performance can also be done simultaneously.

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