Optimal Production Planning in Aromatic Coconuts Supply Chain Based On Mixed-Integer Linear Programming

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Abstract—This work addresses the problem of production planning that arises in the production of aromatic coconuts from Samudsakhorn province in Thailand. The planning involves the forwarding of aromatic coconuts from the harvest areas to the factory, which is classified into two groups; self-owned areas and contracted areas, the decisions of aromatic coconuts flow in the plant, and addressing a question of which warehouse will be in use. The problem is formulated as a mixed-integer linear programming model within supply chain management framework. The objective function seeks to minimize the total cost including the harvesting, labor and inventory costs. Constraints on the system include the production activities in the company and demand requirements. Numerical results are presented to demonstrate the feasibility of coconuts supply chain model compared with base case.

Keywords—Aromatic coconut, supply chain management, production planning, mixed-integer linear programming.

I. INTRODUCTION

AROMATIC coconuts is an agricultural commodity of Thailand where have a trend for more exportation [1]. However, the aromatic coconut exporting company in Thailand is often facing problem in managing large volume of aromatic coconuts due to an unbalance between supply from harvest areas and demands of customers as a result from an inefficiency of production planning.

There are a few large companies in Thailand that operate along the complete aromatic coconut industry supply chain. Raw material for these companies can be supplied from self-owned and/or contracted areas. At processing plant, after receiving raw material, a decision has to be made whether the coconut is sent to warehouse for later processing or directly sent to the processing line. The processing line involves several steps; trimming into tapered-cylinder form, quality classification, chemical treatment, and packaging in different ways depending on customer preferences.

Supply chain modeling and supply chain management have received a lot of attention among companies in recent years, [2]. It provides a tool for integrated planning of several interrelated planning situations. A driving force to the development of supply chain management systems has been the development of company wide database for data collection

and efficient optimizers to solve the resulting, often large, optimization models. A basic description of supply chain modeling is found in Gunnarsson and Villalobos [3]. A more detailed description of industrial cases can be found in Stadtler and Kilger [4]. Examples include a case of forestry production by Troncoso and Garrido [5] and a case of wood turning company by Pastor [6].

With respect to developing and applying planning model in different instances of the food industry and in particular in the fruit industry, mathematical programming planning models have been proposed. For example as in a case of packaging plant in the fruit industry by Blanco [7] who give an overview of operations management modeling in the Argentina fruit industry and developed a mixed integer linear programming to formulated planning and scheduling models. The model can be applied to estimate the fruit processing capacity of the facility in order to establish future sales policies. Bilgen and Ozkarahan [8] review models for the production and distribution problem in wheat supply chain. The resulting mixed integer linear programming problem was solved to optimality using decomposition methods to reduce the computational effort required, and another case about supply chain network of pea-based novel protein foods was studied by Apaiah and Hendrix [9]. They studied the harvesting and transportation planning problem under uncertainty. The problem was formulated as a mathematical programming model to improve the performance of supply chain networks have mainly focused on finding the lowest cost at which novel protein foods can be manufactured while deciding on location of production and modes of transportation based on minimizing the sum of production and transportation costs.

The purpose of this work is to formulate an optimization model describing the production planning problem, which includes its storage and processing activities. The model is a mixed integer linear programming model and is intended to operate in a minimum cost mode. The model can be used both as an operational planning tool, and a strategic tool to analyze the effects on the current planning in various situations.

The outline of the paper is as follow. In Section II we describe the supply chain problem from harvest areas to the plant. Then, in Section III, we formulate the mathematical model for the problem. In Section IV we describe the solution method and present computational results using data from actual operation of one aromatic coconut company in Thailand, and finally, a conclusion in Section V.

II. PROBLEM DESCRIPTION

The problem is an actual supply chain problem of aromatic coconut from one company in Samudsakhorn province of Thailand which has multiple sources, several processing lines, several demand nodes, and several time periods. The supply chain problem of the company contains decisions concerning which the design of production flow, the timing of shipping and the storage at warehouse. Main decisions are also whether or not a harvest are should be contracted and consider the flow of aromatic coconuts in each process in order to satisfy demand. In addition we have to consider restrictions on capacities of harvested areas, production lines, shipping and storage at warehouse. The flow diagram for a single processing line is shown in Fig. 1.

1) Fresh coconut from harvest areas both forwarding from self-owned areas and contracted areas, a decision has to be made whether the coconuts is stored in warehouse for further processing (X_1) or is processed (X_2). In general, a storage non-processed coconut is undesirable due to economic reasons but it could be necessary if an income of coconuts exceed the processing capacity of the line.

2) The coconuts from harvest areas and warehouse (X_3) are fed to the processing line. These coconuts have been trimmed out the shell, in a module for first process coconut trim called PI.

3) After first trim stage, the coconuts (X_4) enter to the second trim (PII) stage, where they are trimmed on the top part into cone-cover shape and part of green peel is cut. The coconuts enter to the pre-classification (depending on size and weight) where non-tradable coconut is separated for juice production (W1).

4) Once the coconuts have been classified by size, these coconuts (X_5) enter the third trim module (PIII), where they are trimmed on the top and bottom to tapered cylinder form with cone-cover top (the stem end or spikelet end of the fruit).

5) The coconuts receive a treatment with water containing special chemical products (DIP) that prevent brown skin on coconut. The coconut is then treated and dried before further processing.

6) Each coconut enters to quality classification sector where it is classified by quality, according to the degree of defects or damage. Some waste is also produced at this stage (W2).

7) After the quality classification sector, the coconut (X_6) enters to the packaging section (PAK) where it is packed according to the characteristics of the container specified by the client.

8) Finally, processed coconut is sent directly to customers (X_7) or there is a possibility to store in cold storage facilities (X_8) for further delivery (X_9).

In the following a brief description of several important issues are discussed to present a more complete picture of the activities.
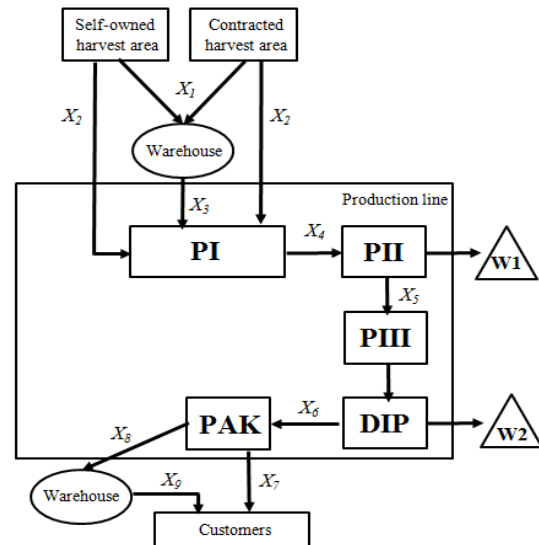

Fig. 1 An illustration of the possible flows in the model

A. Supply of Coconuts

The supplying company obtains coconuts from several sources. The harvested areas can be classified into two groups; self-owned harvest areas and contracted harvest areas. At the areas that are owned by the company, the coconuts have to be removed during the planning period. The coconut in contracted harvest areas can be made available by entering into a contract with a supplier.

B. Warehouse

Warehouse is used to balance seasonal variation of supply and demand and also to offer more shipping possibilities. It receives coconut for storage from two sources during the whole season: fresh aromatic coconuts that exceed the processing capacity of the line which must are storage in warehouse for later processing and containers of coconut produced in the packaging section which need are storage in cold storage. Moreover, there is a separate capacity warehouse for fresh aromatic coconuts from harvested areas and coconut products that must be stored in cold storage. There are different costs of storage between cold and general storage.

C. Waste

A fraction of non-tradable coconut due to aesthetic issues (damage, imperfections, size, etc.) are eliminated from the processing system in the different classification modules and sold for coconut juice production.

D. Labor Policy

The company has a permanent labor staff, which covers a single eight-hour working shift along the whole season.

III. MATHEMATICAL MODEL

In this section we present the mathematical model of aromatic coconut supply chain problem. We first describe the sets of variables, and then follow the constraints and the objective function.

Let I be the set of harvest areas, C the set of customer areas and T the set of time periods. The set of harvesting areas contains subsets for self-owned areas (I_S) and harvest areas with a potential to be contracted (I_{SC}). We will use index i for harvest areas, c for customer areas and t for time periods.

A. Variables

First we define variables representing the supply of coconut in each time period $t \in T$. The volumes of forwarding at a harvest area can be defined as

Sup_i^t — The volume of coconuts that is harvested at area i in time period $t, i \in I_S \cup I_{SC}$,

X_{1i}^t — The volume of coconuts that is forwarded from area i to warehouse in time period $t, i \in I_S \cup I_{SC}$,

X_{2i}^t — The volume of coconuts that is directly forwarded from area i to processing line in time period t, $i \in I_S \cup I_{SC}$.

We also need variables representing the flows of coconuts in production lines. We define

X_3^t — The volume of coconuts that is shipped at warehouse in time period t,

X_4^t — The total volume of coconuts that is forwarded to processing line in time period t,

X_5^t — The total volume remaining after screening at trim process in time period t,

X_6^t — The total volume remaining after screening at dipping process in time period t.

Variables related to transporting of product from company to customers can be defined as:

X_7^t — The volume of product that is directly transported to customer without storing at the end of time period t,

X_8^t — The volume of product that is stored at cold storage at the end of time period t,

X_9^t — The volume of product that is shipped from cold storage to customer at the end of time period t.

We also need variables that are related to storing at warehouse, and we define as follows

inI^t — The volume of coconuts that is stored at warehouse at the end of time period t,

$inII^t$ — The volume of product that is stored at cold storage at the end of time period t,

Where the volume for $t = 0$ define the initial conditions.

All variables defined so far are continuous variables, and they can be interpreted as network flow variables in a multi-commodity network describing the possible flows of coconuts from harvest areas to production lines.

We also need some set of binary variable in the model formulation. For the harvest areas we define

$$Plt_i^t = \begin{cases} 1, \text{ if area } i \text{ is harvested in period } t, i \in I_S \cup I_{SC}, \\ \\ 0, \text{ otherwise.} \end{cases}$$

B. Input Parameters

CaA_i — Maximum harvesting capacity at source i, $i \in I_S \cup I_{SC}$,

$CapPL^t$ — The maximum processing capacity,

$CaWI$ — Draft capacity of warehouse for coconuts storing,

$CaWII$ — Draft capacity of cold storage for coconut product,

Dem_c^t — The demand of customer c in time period t,

RoI — The maximum proportion of waste in pre-classification sector,

$RoII$ — The maximum proportion of waste in quality classification sector,

SoH_i — The harvesting cost at self-owned area,

$SoHC_i$ — The corresponding harvesting cost at contracted area,

$SoLD$ — The labor cost per volume at trim stage,

$SoLP$ — The corresponding labor cost at packaging stage,

$SoIN$ — The inventory cost per volume at warehouse,

$SoIC$ — The corresponding inventory cost at cold storage.

C. Constraints

To describe the need for harvesting at different areas, we have the following constraints

$$\sum_{t \in T} Plt_i^t = 1 \;, \forall i \in I_S, \tag{1}$$

and

$$\sum_{t \in T} Plt_i^t \leq 1 \;, \forall i \in I_{SC}. \tag{2}$$

Constraints (1) specify that each self-owned harvest area has to be harvested exactly once (in exactly one time period) during the planning period, and constraints (2) specify that each potential harvest area to be contracted has to be harvested at most once during the planning period. If a constraint in (2) is satisfied with strict inequality, no harvesting takes place in any of the time periods in that harvest area, which is interpreted as no contract of the harvest area. In such a case, there is no supply from the harvest area.

The constraints

$$Sup_i^t \leq CaA_i Plt_i^t \;, \forall i \in I_S \cup I_{SC}, \forall t \in T, \tag{3}$$

where CaA_i is the volume of coconut available at source i to ensure that the harvested volume of a coconut in a period never exceeds the harvesting capacities.

To assure that the coconuts are harvested at all harvest area i are forwarded to the processing line and warehouse in the same time period, we have the constraints.

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$$\sum_{i \in I_S \cup I_{SC}} Sup_i^t = \sum_{i \in I_S \cup I_{SC}} \left(X_{1i}^t + X_{2i}^t \right), \forall t \in T \qquad (4)$$

These constraints also assure that the forwarded quantity is equal to the available quantity. Part of the incoming coconuts (Sup_i^t) feeds the processing line (X_2) and if the production capacity is exceed the rest is derived to storage (X_1).

The warehouse has a limited storage capacity of fresh coconuts. Let $CaWI$ denote the storing capacity at warehouse. The capacity constraints can then be formulated as

$$\sum_{i \in I_S \cup I_{SC}} X_{1i}^t \leq CaWI \ , \forall t \in T \qquad (5)$$

In constraints (5) the capacity is defined as the volume of fresh coconuts stored at the end of the period. To ensure that the volumes stored at a warehouse never exceeds the stated storage capacity.

The balancing constraints for fresh coconuts at warehouse are expressed as:

$$inI^t = inI^{t+1} + \sum_{i \in I_S \cup I_{SC}} X_{1i}^t - X_3^t \ , \forall t \in T \qquad (6)$$

The maximum processing capacity of the processing line has to do with the volume of coconuts that can be handled at the entrance of the PI module, which is in turn dependent on the processing capacity. Then, the constraints

$$\sum_{i \in I_S \cup I_{SC}} X_{2i}^t + X_3^t \leq CapPL^t \ , \forall t \in T \qquad (7)$$

make sure that is capacity is not exceed.

The network structure indicates that we need some sets of flow balancing constraints. For coconuts entering PI module (X_4) if confirmed by fresh coconuts entering the system (X_2) and non-processed cold stored coconuts (X_3) if required, the balancing constraints can be formulated as

$$\sum_{i \in I_S \cup I_{SC}} X_{2i}^t + X_3^t = X_4^t \ , \forall t \in T \qquad (8)$$

The coconuts leaving PII module is the fraction of non-wasted pieces entering the module are expressed as

$$X_4^t = (1 - RoI) X_5^t \ , \forall t \in T \qquad (9)$$

where RoI is the fraction of wasted coconuts which non-size in pre-classification sector. And after DIP module, in a similar way for quality classification sector the constraints become

$$X_5^t = (1 - RoII) X_6^t \ , \forall t \in T \qquad (10)$$

Since coconuts from DIP process have to be packed immediately in the same time period, either directly to a customer (X_7) or to a cold storage (X_8) for later transportation, the flow balancing constraint become

$$X_6^t = X_7^t + X_8^t \ , \forall t \in T \qquad (11)$$

We also have to consider a number of capacity restrictions regarding storing of product at cold storage. Let the total storing capacity be denoted by $CaWII$. Then, the constraints

$$X_8^t \leq CaWII \ , \forall t \in T \qquad (12)$$

and the balancing constraints for stored products at cold storage are expressed as

$$inII^t = inII^{t+1} + X_8^t - X_9^t \ , \forall t \in T \qquad (13)$$

Finally, we have to express constraints ensuring that the demand at customer is satisfied. The demand at customer c in time period t is denoted by Dem_c^t. The demand constraints can now be expressed as

$$X_7^t + X_9^t \geq \sum_{c \in Cus} Dem_c^t \ , \forall t \in T \qquad (14)$$

and

$$\sum_{t \in T} (X_7^t + X_9^t) \geq \sum_{c \in Cus} \sum_{t \in T} Dem_c^t \qquad (15)$$

In constraints (14) and (15) to assure that the total demand at the end of period t is satisfied.

D. Objective Function

The objective for the supplying and planning company is to minimize the total cost for satisfying the contracted demand. The total cost can be expressed as

$$z = C^{har} + C^{lob} + C^{inv}$$

where C^{har} = harvesting cost, C^{lob} = labor cost, and C^{inv} = inventory cost.

Let SoH_i be the harvesting cost of self-owned area, and let $SoHC_i$ be the corresponding harvesting cost for contracted area. The total harvesting cost can now be expressed as:

$$C^{har} = \sum_{i \in I_S} \sum_{t \in T} SoH_i Plt_i^t + \sum_{i \in I_{SC}} \sum_{t \in T} SoHC_i Plt_i^t \qquad (16)$$

where the first term expresses the fixed harvesting cost at self-owned harvest areas and the second term expresses the fixed harvesting cost at contracted harvest areas.

Let $SoLP$ be the labor cost per volume at trim stage, and let $SoLD$ be the corresponding labor cost at packaging section. The total labor cost can then be expressed as:

$$C^{lab} = \sum_{t \in T} (SoLP)(X_4^t + X_5^t) + \sum_{t \in T} (SoLD)(X_6^t) \qquad (17)$$

where the first term expresses the labor cost of trim sector (PI, PII, and PIII) and the second term expresses the labor cost of packaging sector.

Finally, we have to express the inventory cost at warehouse and cold storage. Let $SoIN$ be the inventory cost per volume of storing at warehouse, and let $SoIC$ be the corresponding cost at cold storage. We can then express the total inventory costs as:

$$C^{inv} = \sum_{i \in I_S \cup I_{SC}} \sum_{t \in T} (SoIN)X_{1i}^t + \sum_{t \in T} (SoIC)X_8^t \qquad (18)$$

where the first term express the cost of storing at warehouse, and where the second terms express the cost of storing product at cold storage.

IV. COMPUTATIONAL STUDY AND RESULTS

In this section, the computational experiments that were carried out on real application are presented for the coconut supply chain model.

The test problem is given from one of the largest company which exports aromatic coconut in Samudsakhorn province. The company has therefore a large number of harvest areas which can be considered as self-owned. Information regarding to the size of the test problem is given in Table I.

A. Solution Methods

The mixed integer linear programming formulation described in the previous section was implemented using the modeling language AMPL [10], and the problem was solved with standard mathematical programming software with the branch-and-bound algorithm called ILOG CPLEX 8.0 [11]. The computational tests were performed on a Intel CORE^{TM} i5 with 3.30 GHz processor and 4.00 GB of RAM.

TABLE I
THE SIZE OF THE TEST PROBLEM

Number of self-owned harvest areas	128
Number of contracts harvest areas	70
Number of customers	3
Number of time periods	20

The modeling Language AMPL has been used to model the problem. As LP-solver we have used CPLEX. To solve the mixed integer linear programming problem directly using CPLEX, we used the default setting. The tolerance from the optimal integer solution was set to 0.05%.

B. Computational Results

The result of solving is given in Table II. The objective function value is given in cost/container, not to reveal the actual values.

TABLE II
COMPUTATIONAL OF THE MODEL BASED ON THE INPUT DATA STRUCTURES

Objective function (THB)	50 261
Total number of variables	62 840
Number of binary variables	15 480
Number of integer variables	46 940
Number of linear variables	420
Number of constraints	504
Solver memory used (MB)	4 564
Solution time in CPU (second)	251
Gap tolerance	0.05

Table II shows the computational results that the solution time is acceptable and within practical time limits. The quality of solution is very high as we get very small gaps to the optimal integer solution.

The optimal total cost of operation for this case is THB 50 261. This total cost was reduced by 10.12%.when compare to the total cost configuration of the base case.

The result obtained demonstrates clearly the savings of optimization that dictate part of the aromatic coconut production network structure. Such results are a valuable tool for the manager to estimate the processing capacity in order to establish sales commitments for the next business year.

V. CONCLUSION

The main purpose of this paper is to present a model and solution approach that can be used as a decision support tool for production planning of the supply chain of aromatic coconut. The mathematical model developed gives a detailed description of the supply chain problem considered. It has been applied in a minimum cost mode in order to estimate the production capacity of the facility. Results were presented corresponding to a processing plant operating one processing line.

The result of problem was solved by a commercial IP solver, i.e., CPLEX. The quality of the solutions found is very high, the objective function values is within 0.05% of the optimal value. The model is tested on a real industrial case and it has been possible to evaluate a number of strategic analyses. The present model can produce better and more flexible solutions compared to manual planning. In conclusion, we believe that the suggested model and solution approach can be used as an important tool in the decision making by the planning staff at any entrepreneur in the considered industry.

Several improvements to address more realistic versions of the system are possible and will also motivate future work. For example, the consideration of several parallel processes lines, transportation issues, etc. A further extension of the present work is to include the explicit consideration of the stochastic nature of the system into the study.

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