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Optimal Placement of Piezoelectric Actuators on Plate Structures for Active Vibration Control Using Modified Control Matrix and Singular Value Decomposition Approach

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Abstract**—**The present work deals with the optimal placement of piezoelectric actuators on a thin plate using Modified Control Matrix and Singular Value Decomposition (MCSVD) approach. The problem has been formulated using the finite element method using ten piezoelectric actuators on simply supported plate to suppress first six modes. The sizes of ten actuators are combined to outline one actuator by adding the ten columns of control matrix to form a column matrix. The singular value of column control matrix is considered as the fitness function and optimal positions of the actuators are obtained by maximizing it with GA. Vibration suppression has been studied for simply supported plate with piezoelectric patches in optimal positions using Linear Quadratic regulator) scheme. It is observed that MCSVD approach has given the position of patches adjacent to each-other, symmetric to the centre axis and given greater vibration suppression than other previously published results on SVD.

Keywords**—**Closed loop Average dB gain, Genetic Algorithm (GA), LQR Controller, MCSVD, Optimal positions, Singular Value Decomposition (SVD) Approaches.

I. Introduction

THE development of smart structures for the active vibration control, shape control, active noise control and damage monitoring gained lots of attention due to increased curiosity in space exploration, nano-positioning, advent of fast processors, real time operating systems. A smart structure consists of sensor(s) to make the system observable, a controller to give the suitable gain to the actuator according to sensor's observations, and actuator(s) to make the structure controllable. Piezoelectric materials have been widely used as sensors - actuators for active vibration control because piezoelectric materials provide inexpensive, reliable, fast response, large operating bandwidth, low weight, low power consumption while actuating and sensing the vibrations in flexible structures. The performance active vibration control depends not only on control law but also on sensor/actuator selection and placement [1]. The optimal placement of sensors/actuators has been obtained using various objective functions like maximizing degree of controllability, minimizing control effort, minimizing spillover effects, maximizing modal forces applied by piezoelectric actuators. The genetic algorithm has been used for the optimal placement of piezoelectric sensors and actuators for beams, flat plates and shells. Rao and Pan [2] applied genetic algorithm to solve zero-one optimization problem for finding the optimal location of actuators by taking an upper bound on the dissipation energy. Sadri et al. in 1999 [3] proposed Gray – coded integer genetic algorithm for optimal location of two piezoelectric actuators for isotropic thin plate by taking two objective functions as modal controllability and controllability gramian. Han and Lee [4] investigated optimal placement of piezoelectric sensors and actuators for vibration control of a composite plate using genetic algorithms with consideration of controllability, observability, and spillover prevention as fitness function. Wang et al. [5] studied optimal placement and size of a collocated pair of piezoelectric patch actuators on beams. The product of singular values/controllability index is taken as the objective for the optimal design of a piezoelectric patch actuator. Quek et al. [6] optimized piezoelectric sensor/actuator pairs to suppress the first two modes of vibration based on modal controllability. LQR (Linear Quadratic Regulator) performance is taken as objective function for the optimal placement of ten sensor/actuator pairs by Kumar and Narayanan [7] to suppress the first six modes of vibration. Bruant I. et al. [8] addressed optimization criteria for placing the piezoelectric sensors and actuators for active vibration control ensuring good observability or controllability and considering residual modes to limit the spillover effect. Genetic algorithms have been used to find the optimal location and orientation of piezoelectric patches on simply supported plate. Bruant I. et al. [9] proposed a genetic algorithm to solve a bi-objective problem i.e. minimum numbers of sensors and optimal location needed ensuring good observability. JM Hale and AH Daraji [10] investigated optimal placement of ten sensor/actuator pairs to suppress the first six modes of vibration by taking a new objective function based on modified Hinfinity.

The problem of determining the optimal locations of actuators/sensors for the active vibration control of flexible structures is of considerable interest in engineering design

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where improved performance and efficiency of response is sought from each controller. Although many authors have been worked in the area of active vibration control using smart structures, there is still scope and need for improvement in better control and actuation. An objective function has been developed to find out optimal placement of piezo-patches on simply supported beam. The optimal positions obtained from the present method have increased the closed loop average dB gain as compared to optimal positions obtained from previous published objective functions. In this study, the optimal locations of ten actuators for simply supported square plate has been carried out to suppress first six modes by considering the new approach i.e. MCSVD using genetic algorithm. LQR (Linear Quadratic Regulator) optimal control scheme has been applied to study the control effectiveness. A comparison is made between the optimal locations of piezo-actuators obtained through various SVD approaches [5], [13].

II. Formulation And Methodology

The representation of the coupled dynamic system has been expressed as [11], [12]:

$$\begin{bmatrix}[M] & [0]\\ [0] & [0]\end{bmatrix}\begin{Bmatrix}\{\ddot{p}\}\\ \{\ddot{\emptyset}_a^e\}\end{Bmatrix}+\begin{bmatrix}[D_p] & [0]\\ [0] & [0]\end{bmatrix}\begin{Bmatrix}\{\dot{p}\}\\ \{\dot{\emptyset}_a^e\}\end{Bmatrix}+\begin{bmatrix}[K_{uu}^e] & [K_{ua}^e]\\ [K_{au}^e] & [K_{aa}^e]\end{bmatrix}\begin{Bmatrix}\{p\}\\ \{\emptyset_a^e\}\end{Bmatrix}=\begin{Bmatrix}\{F_u\}\\ \{F_{\emptyset_a^e}\}\end{Bmatrix} \quad (1)$$

where M is the mass matrix, D_p is the damping co-efficient, K_{uu}^e is the elastic stiffness matrix, K_{aa}^e is the structural dielectric stiffness matrix for actuator, K_{ua}^e is the piezoelectric coupling matrix for actuator. F_u and $F_{\emptyset_a^e}$ are the applied mechanical load vector and electrical charge vector; where,

$$D_p = \Upsilon [M] + \beta [K]$$

Υ and β are the damping constants.

Converting the differential equation to the ordinary equation yields the modal displacement equation of the smart plate as the following equation:

$$\ddot{\eta}_i(t) + 2\zeta_i\omega_i\dot{\eta}_i(t) + \omega_i^2\eta_i(t) = f_c = \sum_{n=1}^{N_a} B\emptyset_{a_n}(t) \quad (2)$$

where B is the modal actuation force which depends on not only the modal shape functions but also the locations and sizes of the actuator elements, $\emptyset_a$ is the control voltage of nth actuator, η_i, $\dot{\eta}_i$ and, $\ddot{\eta}_i$ represent the modal displacement, velocity and acceleration respectively, ω_i and ζ_i are the natural frequency and damping ratio of the ith mode. Equation (2) can be transformed in to state space equations as

$$\dot{x} = Ax + B_c u \quad (3)$$

$$y = Cx \quad (4)$$

where $x = [\dot{\eta}_1\eta_1 \ldots\ldots \dot{\eta}_n\eta_n]^T$ are the generalized modal coordinates

$$A = \begin{bmatrix} 0_{nxn} & I_{nxn} \\ -M^{-1}K & -M^{-1}D_p \end{bmatrix}_{2nx2n}$$

$$u = [\emptyset_{a1}\ \emptyset_{a2} \ldots\ldots\ldots\ldots \emptyset_{an}]^T$$

$$B_C = \begin{bmatrix} 0_{nxp} \\ M^{-1}B(\emptyset_{a)} \end{bmatrix}_{2nxn_a}$$

$$y = [q_1 \ldots\ldots\ldots\ldots q_{ns}]^T, C = \begin{bmatrix} 0 & \ldots & 0 \\ C_{11} & \ldots & C_{n_s1} \\ \vdots & \vdots & \vdots \\ 0 & \vdots & 0 \\ C_{n1} & \ldots & C_{n_sN} \end{bmatrix}$$

A is the system matrix, B is the control matrix which gives control force from actuators, matrix C is with respect to the sensor's output and p is the numbers of piezo-patches as actuators.

LQR optimal control theory is used to determine the active control gain. The following quadratic cost function is minimized

$$\mathrm{j} = \frac{1}{2}\int_0^\infty (\{x\}^T [Q]\{x\} + \{u\}^T[R]\{u\})dt \quad (5)$$

$[Q]$, of dimension (2n×2n), and $[R]$ of dimension $(n_a x n_a)$ are controlled the value of the performance index, where n, n_a represent the number of modes and actuators respectively. They are the main design parameters. J represents the weighted sum of energy of the state and control.

Assuming full state feedback, the control law is given by

$$\{u\} = -[K]\{x\} \quad (6)$$

with constant control gain

$$[K] = [R]^{-1}[B]^T[S] \quad (7)$$

Matrix S can be obtained by the solution of the Riccati equation, given by

$$[A]^T[S] + [S][A] + [Q] - [S][B][R]^{-1}[B]^T[S] = 0 \quad (8)$$

Solution of the Reduced Riccati equation (8) gives the value of matrix [S]; if matrix [S] is positive definite then the system is stable and the closed loop matrix $[A]-[B][K]$ is stable. The feedback control gain matrix can be obtained after substitution of [S] in (8).

A. SVD Approaches

[U, σ, V] = svd[B] produces a diagonal matrix σ of the same dimension as [B], with non-negative diagonal elements in decreasing order, and unitary matrices [U] and [V] so that $[B] = [U]*[\sigma]*[V]^T$.

1. Product of SVD

In order to find the optimal location of actuators, Wang and Wang [5] proposed a controllability index, which was obtained by maximizing the global control force, and to achieve maximum control forces, product of singular values can be taken as controllability index. The magnitude of σ_i is the function of location and size of piezoelectric actuators, Wang and Wang proposed that the controllability index is defined by

$$\Omega = maximize_{j=1,N}\left\{\prod_{i}^{n_a} \sigma_i\right\} \quad (9)$$

where N is the number of the available positions and n_a are the number of the actuators.

2. Norm of SVD

The norm of SVD has been maximized to find the optimal locations of the actuators. The objective function is represented as

$$\Omega = maximize_{j=1,N}\left\{\max_{i=1,n_a} \sigma_i\right\} \quad (10)$$

3. Minimum of SVD

The minimum value of the SVD has been maximized to find the optimal locations of the actuators [13].

$$\Omega = maximize_{j=1,N}\left\{\min_{i=1,n_a} \sigma_i\right\} \quad (11)$$

4. Summation of SVD

The objective function has been represented by summation of SVD.

$$\Omega = maximize_{j=1,N}\left\{\sum_{i=1}^{n_a}(\sigma_i)\right\} \quad (12)$$

5. MCSVD Approach

The rows of the control matrix [B] represent the states of the system and columns represent the number of inputs i.e actuators. The sizes of ten actuators are combined to outline one actuator by adding the ten columns of control matrix to form a column matrix. The singular value of column control matrix is considered as the fitness function and optimal positions of the sensors and actuators are obtained by maximizing it with GA (Genetic algorithm).

$$B = \begin{bmatrix} B_{11} & \dots & B_{1a} \\ 0 & \dots & 0 \\ \vdots & \vdots & \vdots \\ B_{n1} & \dots & B_{na} \\ 0 & \dots & 0 \end{bmatrix}_{2nxn_a} \quad (13)$$

$$B = \begin{bmatrix} B_{11} + B_{12} + \dots + B_{1a} \\ 0 \\ \vdots \\ B_{n1} + B_{n2} + \dots + B_{na} \\ 0 \end{bmatrix}_{2nx1} \quad (14)$$

$$\Omega = maximize_{j=1,N}(\sqrt{(\mathrm{eig}([\mathrm{B}]^{\mathrm{T}} * [\mathrm{B}]))} \quad (15)$$

III. Optimal Placement of Sensors-Actuators Using Genetic Algorithm

A Simple Genetic Algorithm uses probabilistic selection as a basis for evolving a population of problem solutions. An initial population is generated and subsequent generations are created according to a pre-specified breeding and mutation methods inspired by nature. These algorithms are highly parallel, guided, random adaptive search techniques. The generation can be terminated by stopping condition such as particular number of generation, time limit, fitness limit, stall generation, stall time limit, function tolerance, nonlinear constraint tolerance [14].

Some changes have been made to the genetic algorithm for finding the optimal locations of the actuators on the plate structure efficiently.

(i). The initial population is generated randomly with constraints. The length of chromosome will be equal to the total number (10) of piezo-patches considered. No two individuals/genes of generated population allowed having same value and individuals are of absolute value. A constraint is placed on the chromosome value and individuals are varied from 1 to total number of finite elements of structures considered i.e. 100 in the present case.

(ii). The minimization problem is converted into a maximization problem with fitness $f(x) = -\Omega$

(iii). The algorithm creates a sequence of new populations. At each step, the algorithm uses the individuals in the current generation to create the next population. To create the new population, the algorithm performs the following steps:

a) Scores each member of the current population by computing fitness i.e. controllability index.
b) Selects members, called parents, based on their fitness.
c) Some of the individuals in the current population that have greater fitness are chosen as elite. These elite individuals are conceded to the next population.
d) Produces offspring from the parents. Offspring's are produced either by combining the vector entries of a pair of parents—*crossover* or by making random changes to a single parent—*mutation*. The aim of the crossover is to exchange information between two individuals while ensuring that no two individuals of generated offspring allowed are having same values in given domain for present case. Thus, a constraint has been placed to get unique individuals with single point crossover. The random changes made to single parent should be absolute value. Therefore, a constraint has been placed to get absolute value towards positive infinity of individuals while ensuring that no two individuals of generated offspring allowed are having same value with uniform mutation.
e) Replaces the current population with the children to form the next generation.

(iv). Computation is terminated after the convergence of fitness function and the chromosome based on the best value gives the optimal locations of actuators.

(v). To achieve the global optimum, put the previous result i.e. the locations of actuators achieved, in the initial random population and repeat the process.

TABLE I
MATERIAL PROPERTIES AND DIMENSIONS OF SMART PLATE

Physical Parameters	Plate	Piezoelectric sensor/actuator (PZT-5H)
Length(m)	0.16	0.02
Breath(m)	0.16	0.02
Thickness(m)	0.6e-3	1.06e-3
Elastic Modulus(Pa)	2.07e11	6.3e10
Density(Kg/m^3)	7800	7500
Poisson's Ratio	0.3	0.3
Piezodielectric Constant (F/m)		2.84e-8
Piezodielectric Constant (F/m)		2.84e-8
Piezoelectric Constant (Vm/N)		-24.48

TABLE II
PARAMETERS FOR GENETIC ALGORITHM

Parameter	**Simply Supported Plate**
Population Size	500
Elite Count	1
Crossover Fraction	0.9
Mutation Fraction	0.01
Stopping Condition	20generations

IV. RESULTS AND DISCUSSIONS

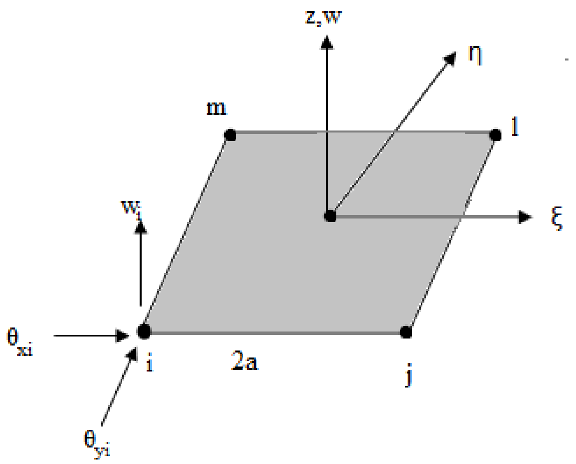

Fig. 1 Square Element with nodes (i, j, l, m) and DOF ($w_i \theta_{xi}\ \theta_{yi}$)

The simply supported square plate has been discretized in 10x10 quadrangular elements; each element with 4 nodes and each node is having 3 degree of freedom i.e. $w_{i,}\theta_{xi}, \theta_{yi}$ as shown in Fig. 1. The total number of structural degree of freedom (DOF) used is 363. Of the 363 DOF, 18 are zero due to simply supported boundary conditions. The material properties and the dimensions of the smart plate are given in Table I. The number of actuators is assumed to be fixed as ten. The size of the one piezo actuator has been taken equal to the size of one element of the plate. The optimal placement of ten actuators on simply supported plate has been found out by SVD approaches (1)-(5) as an objective function using genetic algorithm and compared with developed MCSVD approach to suppress first six modes. The ten piezo actuators moved on to the 100 positions available on the simply supported plate using genetic algorithm. The parameters taken for the genetic algorithm are given in Table II. Product of Singular values [5] has been taken as fitness function to obtain the optimal placement of actuators (Fig. 2 (a)). It is observed that the closed loop average db gain using LQR controller at Q=10^7, 10^6, 10^5, are 21.48db, 12.33db, 5.19db respectively. Fig. 2 (b) shows the optimal location of the actuators, when the maximum value of SVD of control matrix [B] has been maximized and closed loop average db gain achieved using LQR controller at Q=10^7, 10^6, 10^5, are 21.82db, 15.9db, 8.73db respectively. When the minimum value of the SVD of the control matrix has been taken as objective function and maximize it, the closed loop average db gains obtained using LQR controller at Q=10^7, 10^6, 10^5, are 18.84db, 9.57db, 3.05db respectively with the optimal locations shown in Fig. 2 (c). The optimal placement shown in Fig. 2 (d) has been obtained by maximizing the summation of SVD's of control matrix. It has been found that the closed loop average db gain using LQR controller at Q=10^7, 10^6, 10^5, are 26.91db, 18.05db, 9.27db respectively. The sizes of ten actuators are combined to outline one actuator by adding the ten columns of control matrix to form a column matrix. The singular value of column control matrix is considered as the fitness function and optimal positions of actuators obtained by maximizing it with GA are shown in Fig. 2 (e). In these optimal positions, the closed loop average db gain using LQR controller at Q=10^7, 10^6, 10^5, are 31.83db, 22db, 12db respectively. The optimal positions obtained from the present method having the percentage improvement reduction in closed loop db gain of 29.44%, 21.88%, 18.28% using LQR controller at Q=10^5, 10^6, 10^7 respectively as compared to maximum db gain in other optimal positions.

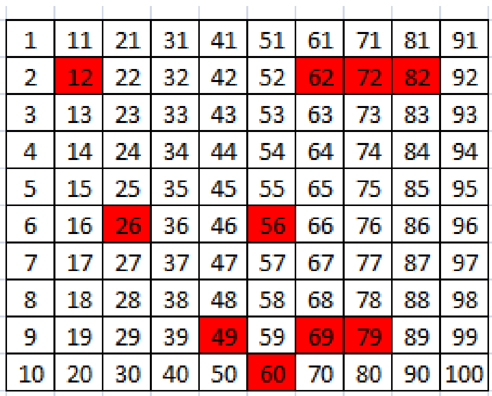

Fig. 2 (a) Optimal locations of actuators obtained by objective functions (1) on simply supported plate

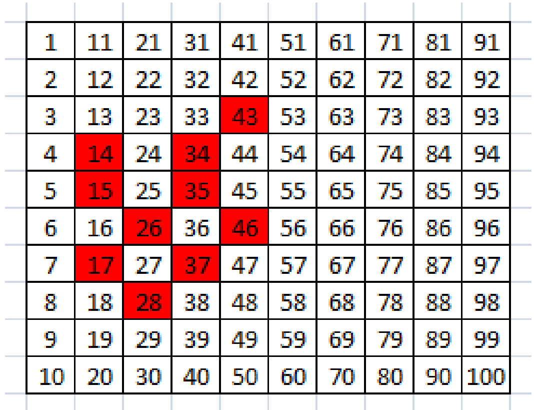

Fig. 2 (b) Optimal locations of actuators obtained by objective functions (2) on simply supported plate

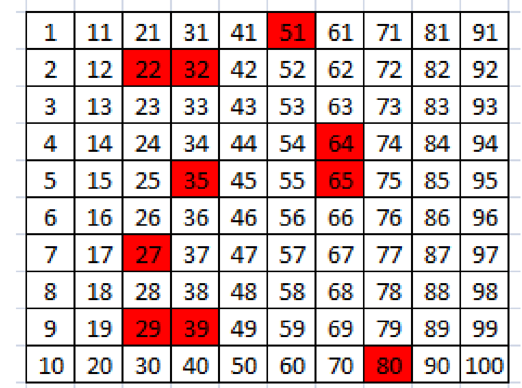

Fig. 2 (c) Optimal locations of actuators obtained by objective functions (3) on simply supported plate

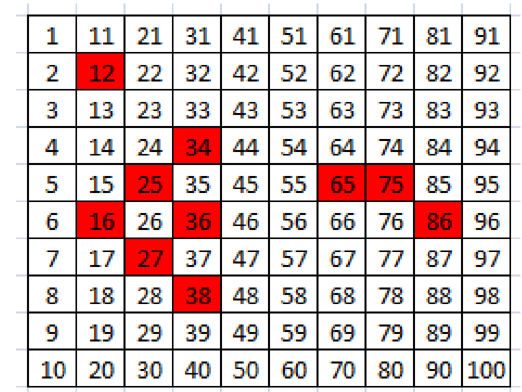

Fig. 2 (d) Optimal locations of actuators obtained by objective functions (4) on simply supported plate

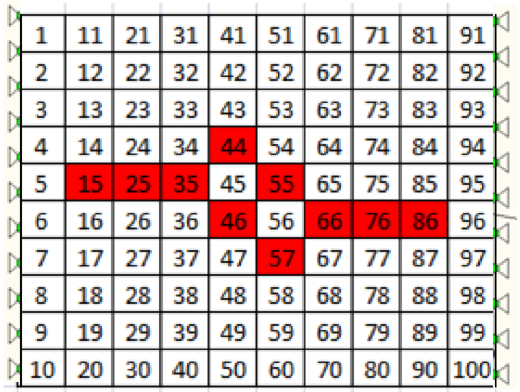

Fig. 2 (e) Optimal locations of actuators obtained by objective functions (5) on simply supported plate

Table III shows the closed loop average dB reduction for the objective functions (1-5). It is observed from Table III that the closed loop average dB reduction is maximum using MCSVD approach.

TABLE III
CLOSED LOOP AVERAGE DB GAIN REDUCTION FOR THE VARIOUS OBJECTIVE FUNCTIONS

Objective Function	**LQR (10^7)**	**LQR (10^6)**	**LQR (10^5)**
(1) $\Omega = maximize_{j=1,N}\{\prod_i^{n_a}\sigma_i\}$	21.48db	12.33db	5.19db
(2) $\Omega = maximize_{j=1,N}\{\max_{i=1,n_a}\sigma_i\}$	21.82db	15.9db	8.73db
(3) $\Omega = maximize_{j=1,N}\{\min_{i=1,n_a}\sigma_i\}$	18.84db	9.57db	3.05db
(4) $\Omega = maximize_{j=1,N}\{\sum_{i=1}^{n_a}(\sigma_i)\}$	26.91db	18.05db	9.27db
(5) MCSVD Approach	**31.83db**	**22db**	**12db**

V. CONCLUSIONS

An objective function has been developed by modifying control matrix and SVD (MCSVD) to optimize ten actuator positions on a simply supported plate using a genetic algorithm to suppress first six modes. The effectiveness of the developed objective function is compared with objective functions in published previous papers. It is observed that the optimal configuration obtained from developed objective function of ten actuators is symmetric about the plate centre axes. The column of the control matrix has been added to form a column matrix. It has also been observed that the positions of ten patches obtained from the developed method are in adjoining to each other. The vibration suppression has been obtained using LQR controller for the present work and compared with various SVD approaches. It has been found that the optimal locations obtained from present approach gives improvement in closed loop average dB gain reduction of 29.44%, 21.88%, 18.28% using LQR controller at Q=10^5, 10^6, 10^7 respectively.

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