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Optimal Path Planning under Priori Information in Stochastic, Time-varying Networks

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Abstract—A novel path planning approach is presented to solve optimal path in stochastic, time-varying networks under priori traffic information. Most existing studies make use of dynamic programming to find optimal path. However, those methods are proved to be unable to obtain global optimal value, moreover, how to design efficient algorithms is also another challenge.

This paper employs a decision theoretic framework for defining optimal path: for a given source S and destination D in urban transit network, we seek an $S-D$ path of lowest expected travel time where its link travel times are discrete random variables. To solve deficiency caused by the methods of dynamic programming, such as curse of dimensionality and violation of optimal principle, an integer programming model is built to realize assignment of discrete travel time variables to arcs. Simultaneously, pruning techniques are also applied to reduce computation complexity in the algorithm. The final experiments show the feasibility of the novel approach.

Keywords—pruning method, stochastic, time-varying networks, optimal path planning.

I. Introduction

RECENTLY, determining optimal route in stochastic, time-varying networks (STV networks) becomes one of most important research topics. In STV networks, travel times are modeled as random variables with time-varying distributions, which often provide a better modeling tool in transportation applications [1, 2, 3].

Hall studies for the first time about STV networks [4]. It is shown that in a stochastic, time-varying network, the standard shortest path algorithms (such as Dijkstra's algorithm) aren't adapted to finding optimal paths in the network. The best route from any given node to goal node depends not only on link travel time, but also on arrival time to the node. Thus, the optimal path planning is not simple path but a policy that describes which node should be visited once the arrival time to a node is realized. Hall suggests dynamic programming for finding optimal policy. Based on the Hall's work, many studies ([5], [6], [7], [8]) are presented on how to compute the optimal routing policy in STV networks.

For the sake of deficient information about the distribution of link trave time, dynamic programming(DP) is proved to be less applicable in describing all states of link travel time. A simple STV network is shown as Fig 1. The expected travel time is computed as follow DP equation

$$E(x_{i+1}) = E(x_i) + E(x_{i+1}/x_i) \tag{1}$$

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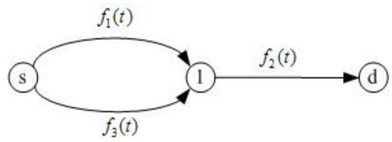

Fig. 1. A simple STV network

where $E(x_{i+1})$ is expected travel time when arriving at node n_{i+1}. $E(x_{i+1}/x_i)$ is expected value of link travel time between node n_i and node n_{i+1}. In the simple network, $f_i(t) = \{(t, T_i, P_{i,t})\}$, and t defines the time when arriving the node n_i, T_i is the arc travel time between two linked nodes, $P_{i,t}$ is the probability of the link travel time. For example, $f_1(t) = \{(1, 11, 0.5), (2, 12, 0.5)\}$, $f_2(t) = \{(12, 19, 0.5), (6, 12, 0.5)\}$, $f_3(t) = \{(4, 11, 1)\}$. According to the optimal principle of DP, the minimum travel time of link $s-1$ is 5.5 (the routing policy is (1,11) $\bigcup$ (node S) $\rightarrow$ node 1), and the same value as link $1-d$ is 5. However, the actual of global optimal path choice is from (2,12,0.5) to (6,12,0.5).

As far as DP is concerned, the decision procedure used in STV networks depends on a lot of state variables for its dynamicity and time dependency, which requires enormous computational times. The problem has been proved an NP problem [9,10].

In this paper we focus on the presence of time-varying traffic conditions in transportation networks, where these conditions can greatly affect the outcomes of the planned schedule. The major goal of this paper is to show the way that each vehicle or traveler make decisions in order to select optimal path through a global mathematical programming model. The numerical results show the feasibility of the approach.

The rest of the paper is organized as follows: Section 2 describes the problem. A mathematical programming model is presented in Section 3. In Section 4, an efficient algorithm is proposed with pruning techniques. The analysis of computation complexity is introduced in Section 5. Section 6 proves its feasibility with numerical examples. The final section concludes the paper.

II. Network description and problem definition

The optimal path planning problem is formulated as a mathematical programming in STV networks. Let $G = (N, A, T, P)$ be a directed network, N is the set of nodes, $|N| = n$, and A is the set of arcs, $|A| = m$. On account of priori traffic information, travel times along the arcs are represented by discrete random variables with distribution functions that are time varying over the period of interest, $t_0 \leq t \leq t_0 + I\delta$, referred to as the congested period in transportation network. The network is considered at a set of T of discrete time $\{t_0 + i\delta\}$, where i is an integer, $i = 1, 2, 3, ..., I$, and δ is smallest increment of time. Beyond time period T, travel times are static and deterministic. P is the probabilistic description of link travel times. It is assumed that travelers know the probabilistic description a priori. $\tau_{i,j}^k(t)$ is defined as possible travel time from node i to node j at time t, $k = 1, ..., K_{i,j}(t)$, where $K_{i,j}(t)$ is the number of possible travel time values on arc (i, j) at time t. For a node $i \in N$, the set of successor and predecessor nodes are given by $\Gamma^{+1}(i) = \{j|(i, j) \in A\}$ and $\Gamma^{-1}(i) = \{j|(i, j) \in A\}$, respectively. To simplify the computation, waiting at the node is not permitted. Travel time $\tau_{i,j}^k(t)$ occurs with probability $p_{i,j}^k(t)$, and

$$\sum_{k=1}^{K_{i,j}(t)} p_{i,j}^k(t) = 1, \forall t \in T \tag{2}$$

For a given source s and destination d, the optimal path planning is equal to finding an optimal path visiting sequence under multiple constraints. In the paper, the optimal object is finding least expected travel time paths in the STV network.

III. Basic definition and mathematical model

Definition 1 Assume $x \in N - \{s\}$, then $W_x(t_0)$ is the path's expected travel times from node s to node x when starting at time t_0.

$$W_x(t_0) = E_{s,x_1,..,x}[t_x - t_0] \tag{3}$$

where E is the operator of expected value.

Definition 2 Assume $x \in N - \{s\}$, then F_x^η means the set of selection of possible travel time through the path between s and x.

$$F_x^\eta = \{k_0, k_1, ..., k_x\} \tag{4}$$

where $F_x^\eta \subseteq F_x$, k_i is defined as the k_ith selection at node i.

Definition 3 The indicator variable z_{ijk} is defined as follows.

$$z_{ijk} = \begin{cases} 1 & \text{if arc ij is selected at kth travel time,} \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Through the definitions above, the mathematical model of path planning is formulated as follows:

$$min[W_d(t_0)_{|z_{ijk}}] \tag{6}$$

Its constraints are

$$\sum_j \sum_{k=1}^{K_{i,j}(t)} - \sum_j \sum_{k=1}^{K_{i,j}(t)} z_{jik} = \begin{cases} 1 & \text{if } i = s, \\ -1 & \text{if } i = d, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$
$$\forall i, j \in N, (i, j) \in A$$

$$\sum_j \sum_{k=1}^{K_{i,j}(t)} z_{ijk} \leq 1, \forall i, j \in N, (i, j) \in A \quad k = 1, ..., K_{i,j}(t) \tag{8}$$

$$t_d = \sum_j \sum_{k \in F_d^\eta} z_{ijk} \cdot \tau_{i,j}^k(t) \tag{9}$$

$$\sum_{k=1}^{K_{i,j}(t)} p_{i,j}^k(t) = 1, \forall t \in T \tag{10}$$

$$F_x^\eta \subseteq F_x, x \in N \tag{11}$$

$$s, x_1, ..., d \in N \tag{12}$$

In the mathematical model, constraints (7) denotes relationship between arcs and paths. Equation (9) describes the possible travel times from start to destination. The model's decision variable is z_{ijk}. Through the assignment of possible travel times to arcs, the results of optimal path planning are realized.

IV. Algorithm

To solve the mathematical model efficiently, we derive a heuristic algorithm by using pruning techniques.

For the quantities of intervals and number of possible travel time at intervals, the worst case complexity of the model is $\Theta(n^2KI)$, where $K = max(K_{i,j}(t))$. In order to decrease the solution space, the pruning techniques in the network is needed to be implemented. First, the definition of stochastically consistent is introduced as follows.

Definition 4 The network is stochastically consistent if all i,j,$s \leq t$ [11],

$$Pr\{s + c_{ij}(s) \leq t + c_{ij}(t)\} = 1 \tag{13}$$

where $Pr\{\cdot\}$ denotes probability operator, and c_{ij} is the possible travel time between arc (i, j). The equation defines the consistent of paths as its constituent arcs also comply with the rule of stochastically consistent.

In order to prune the paths in the transportation network, the concept of dominance is needed to be proposed. Path $Pa2$ is dominated by path $Pa1$ iff the latter's expected travel time is less than the former at intervals of t. With the concept of consistent, Miller-Hooks [11] defines the first-order stochastic dominance (FSD) as

$$F_1^t(x) \geq F_2^t(x), \forall x \tag{14}$$

where $F_1^t(x)$ and $F_2^t(x)$ are the distribution function of possible travel time, x is the possible path travel time. Thus, path $Pa2$ is dominated by path $Pa1$ according to above equation, and path $Pa2$ is pruned. The weak FSD condition, $F_1^t(x) > F_2^t(x)$ for at least one value of x, is also applied to reduce the number of paths. Under priori information, the distribution of the path travel time is known in advance. As a result, the pruning techniques can help to reduce the solution space before computation.

After deployed the pruning techniques, the solution space is obviously decreased, then based on the revised solution space, a solution algorithm (Algorithm1) is presented to compute the results of the optimal path planning.

In the algorithm1, a backward optimal path planning method is deployed in several steps. We first run shortest path algorithm at last interval, and then the successor as to the destination is selected. Finally, algorithm1 run the main loop

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Algorithm 1 Backward optimal path planning algorithm

Initialization
At interval $I-1$, run a shortest path problem algorithm on the deterministic and static network $G'(N,A)$ where arc (i,j) has a deterministic travel time $c_{ij,t}$.
Let $e_\mu(i,t) = \infty$ be the expected travel time of path between node i and d, and $\mu(i,t) = \infty$ be set of selected nodes ,$\forall i \in N - \{d\}$,$\forall t < t_0 + I\delta$, $e_\mu(i, t_0 + (I-1)\delta) = c_{i,d}$, $e_\mu(d,t) = 0, \forall t \in T$.
Main loop
for $t = t_0 + (I-1)\delta$ to 0 **do**
  for $(i,j) \in A$ **do**
    Temp=$\sum_{k=1}^{K_{i,j}(t)}[\tau_{i,j}^k(t) + e_\mu(j, t + c_{ij,t})] \times p_{i,j}^k(t)$;
    If Temp$< e_\mu(i,t)$
    $e_\mu(i,t) = temp$
    $\mu(i,t) = j$
  end for
  Store $\mu(\cdot)$ in list π
end for
Output path planning list π.

to find optimal path planning results from destination to start node backward in time and space.The computation complexity is $\Theta(nm)$ in the fist step while the second step is $\Theta(nK + mK)$. Therefore, the complexity of Algorithm1 is $\Theta(nm + nK + mK)$.

V. Numerical example

A simple STV network is presented as Fig 2. The network has an origin and destination node.

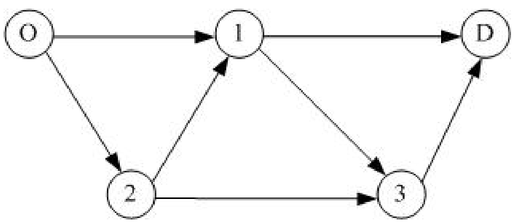

Fig. 2. A simple STV example network

Table 1 lists travel time/probability of each arc with different departure time. The table's content written as 'A/B' describe link travel time (A) and corresponding probability (B).

Through the algorithm, results of optimal path planning are presented as table 2, and optimal paths at each departure time is listed properly.

VI. Conclusions

A decision theoretic framework is developed in resolving problem of optimal path planning under priori information in STV networks. In order to solve problems caused by the methods of DP used in STV networks, such as decrease curse of dimensionality and comply with violation of optimal principle, an integer programming model is built to realize the assignment of discrete travel times to arcs. Simultaneously, pruning techniques are also applied to reduce computation complexity in the algorithm. The final experiments show the feasibility of the novel approach. Future work is to practice the novel approach in real transportation network.

TABLE I
Travel time/probability of each arc with different departure time.

	Departure time								
Link	0	1	2	3	4	5	6	7	8
(O,1)	1/0.5	1/0.5	1/0.5	1/0.5	1/0.5	1/0.5	1/0.5	1/0.5	1/0.5
	2/0.5	2/0.5	2/0.5	2/0.5	2/0.5	2/0.5	2/0.5	2/0.5	2/0.5
(O,2)	1	1	2/0.3	1/0.6	1/0.5	1/0.5	1/0.5	1/0.5	1/0.5
	-	-	1/0.7	2/0.4	2/0.5	2/0.5	2/0.5	2/0.5	2/0.5
(2,1)	3/0.5	2/0.6	2/0.2	3/0.1	1/0.5	5/0.5	4/0.4	1/0.5	4/0.6
	4/0.5	3/0.4	4/0.8	2/0.9	2/0.5	2/0.5	2/0.6	2/0.5	2/0.4
(1,3)	4/0.7	3/0.5	2/0.5	1/0.5	5/0.5	4/0.4	1/0.5	1/0.5	3/0.4
	2/0.3	2/0.5	5/0.5	2/0.5	2/0.5	2/0.6	2/0.5	2/0.5	2/0.6
(2,3)	5	6	6	6	3/0.5	7/0.7	3/0.5	1/0.5	3/0.6
	-	-	-	-	2/0.5	2/0.3	6/0.5	2/0.5	2/0.4
(1,D)	5/0.6	4/0.5	5	3	4	3/0.5	7/0.7	3/0.5	4/0.5
	7/0.4	6/0.5	-	-	-	7/0.5	9/0.3	6/0.5	8/0.5
(3,D)	4/0.7	3/0.5	2/0.2	4/0.6	3/0.5	4/0.5	7/0.3	3/0.4	5/0.6
	2/0.3	2/0.5	5/0.8	2/0.4	6/0.5	3/0.5	5/0.7	5/0.6	4/0.4

TABLE II
The results of optimal path planning.

Departure time	Optimal path planning between OD
0	$(O-1-D)$
1	$(O-1-D)$
2	$(O-1-D), (O-1-3-D)$
3	$(O-2-1-D), (O-1-3-D)$
4	$(O-1-D), (O-1-3-D)$
5	$(O-1-3-D), (O-2-1-D)$
6	$(O-2-1-D), (O-1-3-D)$
7	$(O-2-1-D), (O-1-3-D)$
8	$(O-1-D), (O-1-3-D)$

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