International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:8, No:6, 2014

Optimal Economic Restructuring Aimed at an Increase in GDP Constrained by a Decrease in Energy Consumption and CO_2 Emissions

Alexander Y. Vaninsky

***Abstract*—**The objective of this paper is finding the way of economic restructuring - that is, change in the shares of sectoral gross outputs - resulting in the maximum possible increase in the gross domestic product (GDP) combined with decreases in energy consumption and CO_2 emissions. It uses an input-output model for the GDP and factorial models for the energy consumption and CO_2 emissions to determine the projection of the gradient of GDP, and the antigradients of the energy consumption and CO_2 emissions, respectively, on a subspace formed by the structure-related variables. Since the gradient (antigradient) provides a direction of the steepest increase (decrease) of the objective function, and their projections retain this property for the functions' limitation to the subspace, each of the three directional vectors solves a particular problem of optimal structural change. In the next step, a type of factor analysis is applied to find a convex combination of the projected gradient and antigradients having maximal possible positive correlation with each of the three. This convex combination provides the desired direction of the structural change. The national economy of the United States is used as an example of applications.

***Keywords*—**Economic restructuring, Input-Output analysis, Divisia index, Factorial decomposition, E3 models.

I. Statement of the Problem

CONVENTIONAL wisdom and literature sources state that the goals of economic growth and preserving the Earth's atmosphere and natural resources are conflicting with each other. Economic development has typically been at odds with energy saving and environmental protection as they pursue different, often mutually exclusive, goals. On the one hand, it is crucial to meet the basic needs of people living in poverty. On the other hand, natural resources depletion, environmental degradation, global warming, and climate change, which economic development partially stimulates, will heavily impact the national well-being. The imbalance between emissions of greenhouse gases and their absorption leads to a continual increase of their atmospheric concentration and contributes to a warming of the planet via the greenhouse effect. The latest figures state that if the rise in global average temperature exceeds some level, estimated between 2^0C and 4^0C, the losses in the world income may reach 0.2% to 2% and catastrophic global consequences may be imminent. With about 4.1 billion metric tons of carbonadded yearly to the atmosphere directly through human activity, CO_2 emissions are the important factor that is within our power to curb according to [2], [4].

Tackling the issue of rising greenhouse gas emissions necessitates a reduction in energy consumption in residential and industrial sectors, development of more environmentally-friendly means of energy production, more efficient technology for industrial and non-industrial use, transportation, and agriculture as well as advocacy for energy-saving habits and behaviors. Special attention should also be paid to environmental policy and legislation. Because CO_2 emissions are man-made to a large extent, environmental policy should focus on doing whatever is necessary to curb them. On the energy side of the problem, we notice that the processes of energy production and consumption are the main sources of CO_2 emissions. Also, some of them, like the hydraulic fracturing, undermine the living conditions and have a strong negative impact on the biosphere.

Approaches are known in the literature (see, for example, Lozano and Gutierez [7]) that suggest partial solutions to the problem by focusing on just one objective: GDP, CO_2 emissions, or energy consumption, respectively. In contravention to them, the objective of this research is to resolve all three problems simultaneously. It will be shown that it is possible to achieve this goal through specially-designed economic restructuring of the national economy.

Firstly, we develop mathematical models that suggest particular directions of optimal economic restructuring for gross domestic product (GDP), energy consumption, and CO_2 emissions by using a projected gradient or antigradients respectively, [1]. Secondly, we use a type of factor analysis to construct a directional vector of economic restructuring having maximal positive correlations with each of them. This vector solves the problem, namely, that if the economy is being restructured in its direction, then GDP is increasing while energy consumption and CO_2 emissions are decreasing.

In this paper, we use the fact that the gradient (antigradient) vector points out the direction of steepest increase (decrease) in the function value and its projection on a subspace spanned over the variables of interest retains this property for the function's limitation to the subspace. The proof of the last statement may be found, for example, in Maital and Vaninsky [8]. To find the gradient vector for GDP, we use Leontief's input –output model [6]. For the energy consumption and CO_2 emissions, factorial models similar to the Kaya-identity, Kaya [5] are developed. Projected gradient (antigradient) is found

A. Y. Vaninsky is with Hostos Community College of The City University of New York, Bronx, NY 10451 USA (phone: +1-718-319-7930; fax: +1-718-518-6607; e-mail: avaninsky@hostos.cuny.edu).

based on the results of Meerovich [10]), Vaninsky and Meerovich [18], and Vaninsky [13]-[17].

At the next stage, we use the factor analysis approach [11] to find a directional vector of optimal economic restructuring. This vector is constructed as a convex combination of the normalized projected gradient and antigradient vectors obtained at the previous step having maximal positive correlations with each of them. This vector solves the problem: Economic restructuring following its direction results in a maximum possible increase in GDP combined with the decreases in energy consumption and CO_2 emissions. One of the advantages of the suggested approach is the very limited amount of statistical information needed for the calculations. As an example of applications, we consider the economic restructuring of the U.S. economy of 2009.

The paper uses statistical information of the World Input-Output Database (WIOD) [12]. This research paves the way for the subsequent investigation comprising the entirety of leading national and regional economies aimed at finding the way of international cooperation in economic restructuring intended to combine economic growth, energy conservation, and preserving the atmosphere.

The paper is organized as follows. Section II describes the mathematical model, Section III provides an example of applications and discusses the obtained results.This identity is based on the following factorial decomposition:

II. Mathematical Model

In this paper, we measure economic growth as the increase in the gross domestic product (GDP) that is what the people of a given country can use to satisfy their needs and wants. The GDP is calculated as the total of its sectoral components. To find the gradient of the GDP and its projection, we use an input-output model developed by Leontief [6] transformed into a structured form. A conventional input-output model relates gross output, intermediate inputs, and final product in a single matrix equation. This equation allows us to estimate the total requirements in the gross output needed to satisfy a desired level of the final product. A matrix equation of the model is this:

$$\boldsymbol{X} = \boldsymbol{AX} + \boldsymbol{Y}, \tag{1}$$

In this paper, we transform the conventional input-output model into a specially-structured form whereby the only quantitative indicator is total gross output. All other elements of the model are relative indicators such as shares of sectoral gross outputs in total or shares of sectoral final product in the corresponding gross output. The suggested approach is based on a mathematical model developed in Vaninsky [13] that considers GDP as an objective function with the structured input-output model as a set of constraints imposed on its arguments. In that publication, it was shown that the projected gradient of GDP is a multiple of a function of the structural arguments and that the components of the projected gradient, corresponding to the sectoral structure of the gross output, are proportional to the deviations of the corresponding final-product components from the average value. This result is crucial for this paper's objective because it enables us to find the gross-output components of the projected gradient without knowledge of the technological matrix. This is the basis behind the computational simplicity of the suggested approach because the technological matrix may be obviated.

Following Ghosh [3], we transform the model (1) to a structured form. To do that, we divide each row of the matrix (1) by Xi , the gross output of i-sector, correspondingly. We get

$$1 = \sum_{j=1}^{n} C_{ij} + U_i \, , \, i = 1,...,n \, , \tag{2}$$

where $Cij = (AijXj)/Xi$ is a share of gross output of i-sector obtained from the j-sector for technological use, $Ui=Yi /Xi$ is a share of i-final product in i-gross output, and n is the number of sectors in the economy. Each equation in (2) corresponds to a particular sector. The sectors are related through variables $Di,$ the shares of i-gross output in total:

$$Di = Xi /X, \tag{3}$$

where$X$ is total gross output. As follows from the definition,

$$\sum_{i=1}^{n} D_i = 1 \cdot \tag{4}$$

An input-output model, given by (2) through (4), is referred to below as a structured input-output model. In this paper, we follow Vaninsky[13] and append the structured input-output model (2) through (4) with an objective function. The input-output model it becomes a set of constraints imposed on the arguments of the objective function. Since the GDP is a sum of the sectoral components of the vector $\boldsymbol{Y}$ in the (1), it may be written as

$$Y = \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} X_i U_i = \sum_{i=1}^{n} (XD_i) U_i = X \sum_{i=1}^{n} D_i U_i \, , \tag{5}$$

where$Y$ is GDP, equal to the sum of the components of vector $\boldsymbol{Y}$. The components of the gradient $\boldsymbol{\nabla Y}$of the objective function Y are as follows:

$$\frac{\partial Y}{\partial X} = \sum_{i=1}^{n} D_i U_i \, , \quad \frac{\partial Y}{\partial D_i} = XU_i \, , \quad \frac{\partial Y}{\partial U_i} = XD_i \, , \quad \frac{\partial Y}{\partial C_{ij}} = 0 \quad , \tag{6}$$

seeVaninsky [13], [17] for detail.

It is known that the gradient of the limitation of a function to a surface is a projection of the gradient of the function on a tangent subspace, see Maital and Vaninsky [9] for detail. In the problem defined by (2) through (6) we project gradient (5) on the hyperplane defined by (2) and (4) as was proposed in Vaninsky and Meerovich [18] and Vaninsky [14], [15]; details may be found in Maital and Vaninsky[9].

As shown in Vaninsky[17], the projected gradient of the objective function (5) is

$$\begin{aligned}\mathrm{Proj}_H \nabla \boldsymbol{Y} = X(&U_1 - \bar{U}, U_2 - \bar{U}, \ldots, U_n - \bar{U},\\ &-\frac{D_1}{n+1}, \ldots, -\frac{D_1}{n+1}, -\frac{D_2}{n+1}, \ldots -\frac{D_2}{n+1}, \ldots, -\frac{D_n}{n+1},\\ &\ldots -\frac{D_n}{n+1}, \frac{nD_1}{n+1}, \ldots, \frac{nD_n}{n+1}, \ 0)\end{aligned} \tag{7}$$

where $\nabla \boldsymbol{Y}$ is a gradient of function Y, H is a hyperplane defined by (2) and (4), $Proj_H \nabla Y$ is a projection of $\nabla \boldsymbol{Y}$ on H, and $\bar{U} = (U_1 + \ldots + U_n)/n$ stands for the average value of the variables U_i. The first n components of the projected gradient correspond to the variables D_i - the shares of i-gross output in total. We compute a projection of the gradient (7) on the subspace spanned over the variables $D_i$and get

$$\frac{\partial Y}{\partial D_i} = X_0(U_i - \bar{U}) \tag{8}$$

with all other components equal to zero. For a directional vector, the factor X_0 may be ignored, and we get

$$\boldsymbol{d}_{GDP,i} = (U_i - \bar{U}), \tag{9}$$

where$\boldsymbol{d}_{GDP}$ is the directional vector of the GDP-maximizing restructuring. This formula shows that in the direction of the projected gradient, the impact of structural variable D_i on the final product is proportional to the deviation of the corresponding share of the i-final product in the i-gross output from the average value.

In the next step, we find vectors of economic restructuring optimal for the decrease in the CO_2 emissions and energy consumption, respectively. Consider the CO_2 emissions first. A factorial model of the CO_2 emissions is

$$C = \sum C_i = XC_x = \sum X_i C_{xi} = XC_x \sum \left(\frac{X_i}{X} \frac{C_{xi}}{C_x} \right) = XC_x \sum D_i R_{ci} \tag{10}$$

where$C$ and C_i are total and sectoral CO_2 emissions, X and X_i are total and sectoral gross products, C_x and C_{xi} are the carbon intensity of the total and sectoral gross products, respectively, and R_{ci} stands for the relative carbon intensity of the gross output (the ratio of the C_{xi} to C_x.)

Since we are interested in the optimization of the economic structure, we consider the (10) at the base value of $X = X_0$ and divide both its sides by $C_0 = X_0 \bullet C_x$. We get

$$\hat{C} = C/C_0 = \Sigma D_i R_{ci}, \tag{11}$$

where$\hat{C}$ is CO_2 emissions in terms of the base year. Equation (11) is a factorial model of the CO_2 emissions with the factors Di interconnected by (4). A gradient of the function $\hat{C}$ is

$$\nabla \hat{C} = < R_{C1}, R_{C2}, \ldots, R_{Cn} >, \tag{12}$$

and, as shown in Meerovoch [10], Vaninsky and Meerovich [18], and Vaninsky [15], the projected gradient on the hyperplane defined by (4) is

$$Pr\,\mathrm{oj}_D \nabla \hat{C} = < R_{C1} - \bar{R}_C, R_{C2} - \bar{R}_C, \ldots, R_{Cn} - \bar{R}_C >, \tag{13}$$

where $\bar{R}_c = (R_{c1} + R_{c2} + \ldots + R_{cn})/n$ stands for the average value of the relative carbon intensities. Since our objective is to decrease the CO_2 emissions, the optimal structural change corresponds to the projected antigradient, that is

$$d_C = -Proj_D \nabla \hat{C} = -< R_{C1} - \bar{R}_C, R_{C2} - \bar{R}_C, \ldots, R_{Cn} - \bar{R}_C > . \tag{14}$$

For the same reasons, the projected antigradient of the energy consumption is

$$d_E = -Proj_D \nabla E = -< R_{E1} - \bar{R}_E, R_{E2} - \bar{R}_E, \ldots, R_{En} - \bar{R}_E >, \tag{15}$$

whereR_{Ei} and $\bar{R}_E$ stand for the sectoral relative and averagerelative energy intensity of the gross output respectively.

In what follows, we use formulas of the projected gradient (9) and antigradients (14) and (15) to find a directional vector that has maximum positive correlations with each of them. To find this vector, we use the technique of factor analysis, see[11]. The optimization problem is this:

$$\begin{aligned}&\mathit{Maximize}\\ &\mathit{Corr}(\boldsymbol{d}, \boldsymbol{d}_{GDP})^2 + \mathit{Corr}(\boldsymbol{d}, \boldsymbol{d}_E)^2 + \mathit{Corr}(\boldsymbol{d}, \boldsymbol{d}_{CO2})^2\\ &\mathit{Subject\ to}\\ &\boldsymbol{d} = \alpha_1 \cdot \boldsymbol{d}_{GDP} + \alpha_2 \cdot \boldsymbol{d}_E + \alpha_3 \cdot \boldsymbol{d}_C\\ &\alpha_1, \alpha_2, \alpha_3 \geq 0\\ &\alpha_1 + \alpha_1 + \alpha_1 = 1\\ &\mathit{by\ changing}\ \alpha_1, \alpha_2, \alpha_3,\end{aligned} \tag{16}$$

where$Corr(\,)$stands for the correlation coefficient.

Solution to this problem is a convex combination of the vectors $\boldsymbol{d}_{GDP}$, $\boldsymbol{d}_E$, and $\boldsymbol{d}_C$that maximizes a sum of squares of its correlation coefficients with the vectors of projected gradient or antigradients respectively (referred to in the statistical literature as the coefficients of determination).To solve the problem, we developed a computational procedure using the Excel's Solver.

The obtained directional vector of the structural change serves the three goals simultaneously. It may be noted that in some special cases such vector does not exist, for example, if the three vectors are collinear or coplanar with an angle of 120^0 between each two of them. However, such situations are very unlikely in practice where typical dimensions of the vectors are in the range from 15 to 500 and may be ignored.

III. Example of Applications and Discussion

In this section we use data provided by the World Input-Output Database (WIOD) to find the optimal structural change in the U.S. economy of 2009which affords the latest data available at the time of this paper's preparation. The input-output methodology underlying the WIOD statistical information may be found in Timmer[12].

The U.S. economy is considered as a collection of 34 sectors shown in Table I. Table II presents the shares of sectoral gross outputs, energy, CO_2 emissions in total (columns 2, 4, and 5, respectively), shares of final product in the sectoral gross output (column 3), and the relative energy and CO_2 gross output intensities (columns 6 and 7, respectively). The sector Private Households with Employed Persons, the second smallest by the gross output, was excluded because it had zero energy consumption and CO_2 emissions. Its gross output was added to the sector Public Administration and Defense; Compulsory Social Security which is the largest one. Sectoral amounts of energy consumption and CO_2 emissions were calculated by using data of the intermediate output because the data related the sectoral gross outputs were not available to the author. Columns 2 and 3 of the Table II are referred to as D_i and U_i in formulas (5) through (8), (10) to (11) etc. respectively. Columns 4 and 5 of the Table II are the sectoral shares in energy consumption and CO_2 emissions respectively. They were calculated as E_i/E and C_i/C, respectively, as shown by (10) for CO_2 emissions. Columns 6 and 7 of the Table II are energy and CO_2 intensities of the sectoral gross output with respect to the total gross output intensities. As follows from (10) they are equal to $R_{ei}=E_{xi}/E_x$ and $R_{ci}=C_{xi}/C_x$, respectively, and may be calculated as $R_{Ei}=(E_i/E)/(X_i /X)$ and $R_{ci}= (C_i /C)/(X_i /X)$, correspondingly, as shown in Table II.

Table III shows the GDP projected gradient and the energy consumption and CO_2 emissions projected antigradients. They were calculated by using (9), (15), and (14) respectively by subtracting the corresponding average value from the sectoral values.The gradient and antigradient vectors were normalized as shown in Table IV: The square roots of the sum of squares of the coordinates of each vector were made equal to one, as suitable for the factor analysis, see [11]. The result is shown in columns 2 through 4 of the Table IV. Column 5 of Table IV presents the vector of optimal structure change obtained as a convex combination of the normalized gradient and antigradient vectors that maximizes the sum of squares of correlation coefficients. Recall that the square of the correlationcoefficient is usually referred to as the determination coefficient; it represents a part of the variance that may be explained by the correlate. A normalized vector of optimal structural change is shown in column 6.

The optimization procedure was similar to that used in the Factor Analysis, [11]. The computation was performed by using the Excel's Solver.

TABLE I
SECTORS INCLUDED IN THE MODEL

Sector	Abbreviation
(1)	(2)
Agriculture, Hunting, Forestry and Fishing	AHFF
Mining and Quarrying	MQ
Food, Beverages and Tobacco	FBT
Textiles and Textile Products	TTP
Leather, Leather and Footwear	LLF
Wood and Products of Wood and Cork	WPWC
Pulp, Paper, Paper , Printing and Publishing	PPPPP
Coke, Refined Petroleum and Nuclear Fuel	CRPNF
Chemicals and Chemical Products	CCP
Rubber and Plastics	RP
Other Non-Metallic Mineral	ONMM
Basic Metals and Fabricated Metal	BMFM
Machinery, Nec	MN
Electrical and Optical Equipment	EOE
Transport Equipment	TE
Manufacturing, Nec; Recycling	MNR
Electricity, Gas and Water Supply	EGWS
Construction	C
Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel	SMRMRF
Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles	WTCT
Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods	RT
Hotels and Restaurants	HR
Inland Transport	IT
Water Transport	WT
Air Transport	AT
Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies	OSAT
Post and Telecommunications	PT
Financial Intermediation	FI
Real Estate Activities	REA
Renting of M&Eq and Other Business Activities	ROBA
Public Admin and Defense; Compulsory Social Security	PAD
Education	ED
Health and Social Work	HSW
Other Community, Social and Personal Services	OCSPS

Notes.
[a]Nec stands for Not elsewhere classified.

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:8, No:6, 2014

TABLE II
SHARES IN TOTAL AND RELATIVE INTENSITIES

Sector	Gross Output	Final Product[a]	Energy[b]	CO_2[b]	Relative energy intensity	Relative CO_2 intensity
(1)	(2)	(3)	(4)	(5)	(6)= (4)/(2)	(7)= (5)/(2)
AHFF	0.0138	0.1948	0.0087	0.0120	0.6324	0.8683
MQ	0.0141	0.2882	0.0153	0.0265	1.0851	1.8820
FBT	0.0313	0.5654	0.0110	0.0145	0.3525	0.4642
TTP	0.0024	0.2318	0.0021	0.0021	0.8757	0.8770
LLF	0.0001	0.1204	0.0000	0.0000	0.2562	0.3307
WPWC	0.0032	0.0494	0.0046	0.0035	1.4595	1.0930
PPPPP	0.0172	0.3027	0.0185	0.0146	1.0764	0.8520
CRPNF	0.0191	0.3211	0.3170	0.0445	16.6005	2.3315
CCP	0.0242	0.2638	0.0545	0.0319	2.2498	1.3169
RP	0.0068	0.1095	0.0009	0.0012	0.1368	0.1746
ONMM	0.0037	0.0655	0.0084	0.0262	2.2641	7.0978
BMFM	0.0193	0.0013	0.0170	0.0241	0.8792	1.2489
MN	0.0112	0.3628	0.0032	0.0039	0.2820	0.3495
EOE	0.0195	0.2874	0.0022	0.0026	0.1149	0.1318
TE	0.0238	0.3856	0.0042	0.0048	0.1765	0.2014
MNR	0.0056	0.4917	0.0006	0.0009	0.1002	0.1588
EGWS	0.0156	0.5117	0.3170	0.4855	20.2874	31.0718
C	0.0466	0.8308	0.0159	0.0100	0.3420	0.2143
SMRMRF	0.0085	0.8131	0.0013	0.0014	0.1488	0.1657
WTCT	0.0407	0.4836	0.0051	0.0075	0.1250	0.1834
RT	0.0477	0.8638	0.0135	0.0187	0.2829	0.3920
HR	0.0307	0.7830	0.0139	0.0148	0.4521	0.4832
IT	0.0137	0.3198	0.0242	0.0459	1.7602	3.3395
WT	0.0014	0.6473	0.0065	0.0134	4.7953	9.8579
AT	0.0054	0.5657	0.0186	0.0372	3.4684	6.9381
OSAT	0.0079	0.0926	0.0071	0.0134	0.8957	1.6881
PT	0.0249	0.4173	0.0058	0.0075	0.2324	0.3006
FI	0.0979	0.3386	0.0061	0.0073	0.0624	0.0744
REA	0.0928	0.7063	0.0056	0.0023	0.0605	0.0248
ROBA	0.1157	0.2019	0.0171	0.0247	0.1479	0.2139
PAD	0.1195	0.9494	0.0455	0.0608	0.3811	0.5084
ED	0.0090	0.9015	0.0049	0.0037	0.5482	0.4049
HSW	0.0680	0.9737	0.0151	0.0207	0.2213	0.3049
OCSPS	0.0386	0.5714	0.0086	0.0119	0.2225	0.3089
Average		0.4416			1.8522	2.2310

Notes
[a] Share in the sectoral gross output.
[b] Calculated by using intermediate output.

TABLE III
GDP GRADIENT, AND ENERGY AND CO_2 ANTIGRADIENTS

Sector	GDP[a]	Energy[b]	CO_2[b]
(1)	(2)	(3)	(4)
AHFF	-0.2467	1.2198	1.3627
MQ	-0.1534	0.7671	0.3490
FBT	0.1238	1.4997	1.7668
TTP	-0.2098	0.9766	1.3540
LLF	-0.3211	1.5960	1.9002
WPWC	-0.3921	0.3928	1.1380
PPPPP	-0.1389	0.7758	1.3790
CRPNF	-0.1204	-14.7482	-0.1005
CCP	-0.1777	-0.3976	0.9141
RP	-0.3321	1.7154	2.0563
ONMM	-0.3761	-0.4119	-4.8669
BMFM	-0.4402	0.9730	0.9821
MN	-0.0787	1.5702	1.8815
EOE	-0.1541	1.7373	2.0992
TE	-0.0560	1.6758	2.0295
MNR	0.0501	1.7521	2.0722
EGWS	0.0701	-18.4351	-28.8408
C	0.3892	1.5102	2.0167
SMRMRF	0.3716	1.7035	2.0652
WTCT	0.0420	1.7273	2.0476
RT	0.4222	1.5694	1.8390
HR	0.3415	1.4001	1.7478
IT	-0.1217	0.0920	-1.1085
WT	0.2058	-2.9431	-7.6269
AT	0.1241	-1.6162	-4.7071
OSAT	-0.3490	0.9566	0.5428
PT	-0.0242	1.6198	1.9304
FI	-0.1030	1.7898	2.1565
REA	0.2648	1.7917	2.2061
ROBA	-0.2397	1.7044	2.0171
PAD	0.5078	1.4711	1.7226
ED	0.4599	1.3040	1.8260
HSW	0.5321	1.6309	1.9261
OCSPS	0.1298	1.6297	1.9221
Norm [c]	1.6251	25.0487	31.9914

Notes
[a] Share in the sectoral gross output minus average value.
[b] Opposite of the sectoral energy (CO_2) intensity minus average value.
[c] Square root of the sum of squares of the components.

TABLE IV
NORMALIZED GDP GRADIENT, ENERGY AND CO_2 ANTIGRADIENTS, AND OPTIMAL RESTRUCTURING VECTOR[a]

Sector	GDP	Energy	CO_2	Optimal	Optimal, normalized
(1)	(2)	(3)	(4)	(5)	(6)
AHFF	-0.1518	0.0487	0.0426	0.0408	0.0441
MQ	-0.0944	0.0306	0.0109	0.0179	0.0194
FBT	0.0762	0.0599	0.0552	0.0580	0.0628
TTP	-0.1291	0.0390	0.0423	0.0364	0.0394
LLF	-0.1976	0.0637	0.0594	0.0551	0.0597
WPWC	-0.2413	0.0157	0.0356	0.0190	0.0206
PPPPP	-0.0854	0.0310	0.0431	0.0340	0.0368
CRPNF	-0.0741	-0.5888	-0.0031	-0.2907	-0.3145
CCP	-0.1094	-0.0159	0.0286	0.0035	0.0038
RP	-0.2044	0.0685	0.0643	0.0597	0.0646
ONMM	-0.2314	-0.0164	-0.1521	-0.0879	-0.0951
BMFM	-0.2709	0.0388	0.0307	0.0272	0.0294
MN	-0.0485	0.0627	0.0588	0.0580	0.0628
EOE	-0.0948	0.0694	0.0656	0.0635	0.0687
TE	-0.0345	0.0669	0.0634	0.0627	0.0679
MNR	0.0308	0.0699	0.0648	0.0665	0.0719
EGWS	0.0431	-0.7360	-0.9015	-0.7973	-0.8629
C	0.2395	0.0603	0.0630	0.0661	0.0715
SMRMRF	0.2286	0.0680	0.0646	0.0703	0.0761
WTCT	0.0259	0.0690	0.0640	0.0655	0.0709
RT	0.2598	0.0627	0.0575	0.0650	0.0704
HR	0.2101	0.0559	0.0546	0.0591	0.0640
IT	-0.0749	0.0037	-0.0347	-0.0169	-0.0183
WT	0.1266	-0.1175	-0.2384	-0.1704	-0.1844
AT	0.0764	-0.0645	-0.1471	-0.1013	-0.1096
OSAT	-0.2147	0.0382	0.0170	0.0216	0.0234
PT	-0.0149	0.0647	0.0603	0.0606	0.0656
FI	-0.0634	0.0715	0.0674	0.0661	0.0716
REA	0.1629	0.0715	0.0690	0.0725	0.0785
ROBA	-0.1475	0.0680	0.0631	0.0603	0.0652
PAD	0.3125	0.0587	0.0538	0.0626	0.0678
ED	0.2830	0.0521	0.0571	0.0602	0.0652
HSW	0.3275	0.0651	0.0602	0.0692	0.0749
OCSPS	0.0799	0.0651	0.0601	0.0630	0.0682

Notes
[a] Divided by the vector's norm.

In our computations, the weight coefficients were as follows: α_{GDP} = 0.0248, α_{Energy} = 0.4879, and α_{CO2} = 0.4873. As required by the convex linear combination, all of them are non-negative and sum up to one. The values of the weight coefficients reveal that the vector of optimal change tends to serve more the problem of energy conservation and atmosphere protection than the objective of economic growth. This might not be the case if a different economy was analyzed. The key sectors of economic restructuring, as shown by the components of the normalized vector of optimal structural change, were as follows. The main sectors to be expanded (those having the component's value above 0.07 in our case) were REA (0.0785), SMRMRF (0.0761), HSW (0.0749), MNR (0.0719), FI (0.0716), C (0.0715), WTCT (0.0709), and RT (0.0704). Since the differences among the components do not exceed 10%, they may be considered as the candidates for expansion in any appropriate combination. Only 6 sectors had negative components: EGWS (-0.8629), CRPNF (-0.3145), WT(-0.1844), AT (-0.1096), ONMM (-0.0951), and IT (-0.0183). These sectors should be shrunken and their shares in the gross output should be transferred to the expanded sectors. As follows from the numerical values of the components, the impacts of these sectors are quite different. The Electricity, Gas and Water Supply sector is the most desirable candidate for contraction.

Since that sector's contraction is usually undesirable, more detailed analysis may be required for finding the subsectors having the least negative impact on the economy. It may be done, for example, by repeating the above computations with key sectors subject to expansion, and all other sectors aggregated to one sector. The new vector of optimal structural change will provide higher resolution of the previously obtained big picture of economic restructuring.

IV. CONCLUSIONS

This paper suggests an approach to finding structural change in a national or regional economy, leading to increase in the GDP combined with a decrease in energy consumption and CO_2 emissions. The input-output model in the structured form is used to find the projected gradient of the GDP and factorial models are developed for finding the projected antigradients of the energy consumption and CO_2 emissions respectively. A kind of factor analysis was used to compute a resulting vector of structural change satisfying all three goals. The U.S. economy of 2009 was used as an example. The key sectors to be subject to expansion or contraction were determined. The next step of the application of the suggested approach is the detailing of the proposed recommendations on economic restructuring by expansion of the key sectors into subsectors, while contracting the remaining sectors to one aggregated sector. The obtained results are aimed to serve as a basis for economic policy decision-making aimed at serving the goals of economic growth, energy saving, and atmospheric preservation.

REFERENCES

[1] P. Calamai, and J. More, "Projected gradient methods for linearly constrained problems", *Mathematical Programming,*,vol.39, 1987, pp. 93-116.
[2] *Energy Explained*. U.S. Energy Information Administration, http://www.eia.gov/energyexplained, 2013.
[3] A. Ghosh,*Experiments with Input - Output Models*. Cambridge: Cambridge University Press, 1964.
[4] *IPCC 2013: Climate Change 2013. The Physical Science Basis*. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press. Cambridge, UK and New York, NY, US., www.ipcc.ch/report/ar5/wg1/ , 2013.
[5] Y. Kaya, "Impact of Carbon Dioxide Emission Control on GNP Growth: Interpretation of Proposed Scenarios,"*Paper presented to the IPCC Energy and Industry Subgroup, Response Strategies Working Group*, Paris. (mimeo), 1990.
[6] W. Leontief, *Input - Output Economics*. 2nd. ed. New York: Oxford University Press, 1986.
[7] S. Lozano, and E.Gutierez, "Non-parametric frontier approach to modelling the relationships among population,GDP, energy consumption and CO2 emissions,"*Ecological Economics*, vol. 66, 2008, pp. 687-699.

[8] S. Maital, and A. Vaninsky, "Data Envelopment Analysis with a Single DMU: A Graphic Projected - Gradient Approach,"*European J of Operational Research,* vol.115, No 3,1999, pp. 518-528.

[9] S. Maital, and A. Vaninsky, "Productivity Paradoxes and their Resolution,"*J of Productivity Analysis*, vol. 14, No 3, 2000, pp. 191-207.

[10] V. Meerovich, *Turnover of Financial Funds and Efficiency of Production*. (*OborotSredstviEffektivnost' Proizvodstva*), Moskva: Financy, 1974. (In Russian.)

[11] B. Thompson, *Exploratory and confirmatory factor analysis: Understanding concepts and applications*. Washington DC: American Psychological Association, 2004.

[12] M. Timmer, Ed."The World Input-Output Database (WIOD): Contents, Sources and Methods,"*WIOD Working Paper Number 10*, http://www.wiod.org/publications/papers/wiod10.pdf, 2012.

[13] A. Vaninsky, "Computational Method of Finding Optimal Structural Change in Economic Systems: An Input-output Projected-gradient Approach,"*J of Interdisciplinary Mathematics*, vol. 9, No 1, 2006, pp. 61 - 76.

[14] A. Vaninsky, *Factorial Analysis of Economic Activity.* (*Factornyi Analiz Khozyaistvennoi Deyatel'nosti*), Moskva: FinancyiStatistika, 1987. (In Russian)

[15] A. Vaninsky, "Generalization of the integral method of economic analysis to interconnected and derived factors,"*Automation and Remote Control*, vol 44, No 8, Iss. 2, 1984, pp. 1074 - 1083.

[16] A. Vaninsky, "Key sectors of Optimal Economic Restructuring Constrained by Environmental Protection: A Structured Input - Output Analysis Approach,"*Council For Innovative Research. J of Social Science Research,vol.* 1, No 3, 2013, pp. 71-78. http://www.cirworld.com/index.php/ijssr/article/view/1896/pdf_9, 2013.

[17] A. Vaninsky, "Structural Change Optimization in Input - Output Models,"*J of Interdisciplinary Mathematics*, vol. 12, No 6, 2009, pp. 839 – 861.

[18] A. Vaninsky, and V.Meerovich, "Problems of the methodology of analysis of the impact of structural change on the indicators of production efficiency," (Voprosymetodologiiianalizavliyaniyastrukturnykhsdvigovnapokazatelieffectivnostiproizvodstva.) *Proceedings of the National Scientific Conference "Economic Leverages of the Efficiency of Using Material, Labor, Finance, and Natural Resources"*, vol. 2, 1978, pp. 105 - 107, Moskva: Central Economic - Mathematical Institute. (In Russian).