

# On the Numerical Simulation of Flow Past an Oscillating Circular Cylinder in a Circular Path: Oscillation Amplitude Effect

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**Abstract**—This paper presents results obtained from the numerical solution for the flow past an oscillating circular cylinder at Reynolds number of 200. The frequency of oscillation was fixed to the vortex shedding frequency from a fixed cylinder,  $f_0$ , while the amplitudes of oscillations were varied from 0 to  $1.1a$ , where  $a$  represents the radius of the cylinder. The response of the flow through the fluid forces acting on the surface of the cylinder are investigated. The lock-on phenomenon is captured at low oscillation amplitudes.

**Keywords**—Lock-on; streamwise oscillation; transverse oscillation; fluid forces, combined motion

## I. INTRODUCTION

THE Phenomenon of vortex-shedding from an oscillating bluff body has been the subject of study since the last century. In practice, the study of the periodic vortex shedding and wake development behind a bluff body leads to better understanding of the cause of vortex-induced vibration and its subsequent suppression and control. The cases of a cylinder performing one-degree of freedom (1-DoF) forced streamwise or transverse oscillation are studied extensively by many researchers (see the recent works by Al Mdallal [2]; Al Mdallal et al. [1]; Barrero-Gil and Fernandez-Arroyo [4]; Carmo et al. [5]; Konstantinidis and Liang [9]; Marzouk and Nayfeh [10]; Suthon and Dalton [12] and the references therein). On the other hand, only few studies were concerned with the problem of flow past a circular cylinder with combined two-degree of freedom (2-DoF) streamwise and transverse oscillation (we may refer the reader to Baranyi [3]; Didier and Borges [8]; Stansby and Rainey [11]; Williamson et al. [13]). Thus, the primary objective of the present work is to give further numerical and physical investigations on the model of flow past a circular cylinder with combined two-degree of freedom (2-DoF) streamwise and transverse oscillation.

The main objective of this paper is to numerically investigate the response of the fluid forces generated by an oscillating circular cylinder with combined streamwise and transverse oscillations in the presence of uniform stream. The cylinder, whose axis coincides with the  $z$ -axis, is placed horizontally in a cross-stream of an infinite extent where the flow of a viscous incompressible fluid of constant velocity  $U$  past the cylinder in the positive  $x$ -direction.

At dimensional time,  $t^* = 0$ , the cylinder suddenly starts to perform (i) time-dependent streamwise oscillations, and (ii) time-dependent transverse oscillations. The two-dimensional flow configuration of the physical model is shown in Figure 1. Note that the dashed line in Figure 1 represents the circular path of the cylinder. The streamwise and transverse cylinder displacements are, respectively, expressed by

$$X^*(t^*) = A^* \cos(2\pi f^* t^*), \quad Y^*(t^*) = A^* \sin(2\pi f^* t^*)$$

where  $f^*$  and  $A^*$  are, respectively, the dimensional frequency and amplitude of the harmonic motions. Here the quantities are non-dimensionalized by adopting the following relations:  $f = af^*/U$ ,  $A = A^*/a$  and  $t = Ut^*/a$ .

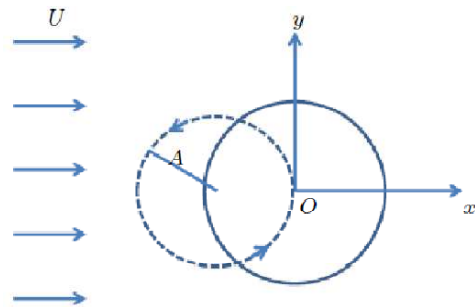


Fig. 1 The physical model and coordinate system

This study is concerned in analysing the surface fluid forces at  $R = 200$ ,  $f = f_0 = 0.0977$  and  $0 \leq A \leq 1.1$ .

## II. COMPUTATIONAL FLOW MODEL

A frame of reference is used in which the axes translate and oscillate with the cylinder. Modified polar coordinates  $(\xi, \theta)$ , with the origin at the centre of the cylinder, are used. Here  $\xi = \ln(r) = \ln(r^*/a)$ , where  $r$  is the dimensionless radial coordinate. The equations of motion can be written in terms of the vorticity  $(\zeta)$  and the stream function  $(\psi)$  in dimensionless form as

$$e^{2\xi} \frac{\partial \zeta}{\partial \xi} = \frac{2}{R} \left( \frac{\partial^2 \zeta}{\partial \xi^2} + \frac{\partial^2 \zeta}{\partial \theta^2} \right) + \frac{\partial \psi}{\partial \xi} \frac{\partial \zeta}{\partial \theta} - \frac{\partial \psi}{\partial \theta} \frac{\partial \zeta}{\partial \xi} \quad (1)$$

$$\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \theta^2} = e^{2\xi} \zeta \quad (2)$$

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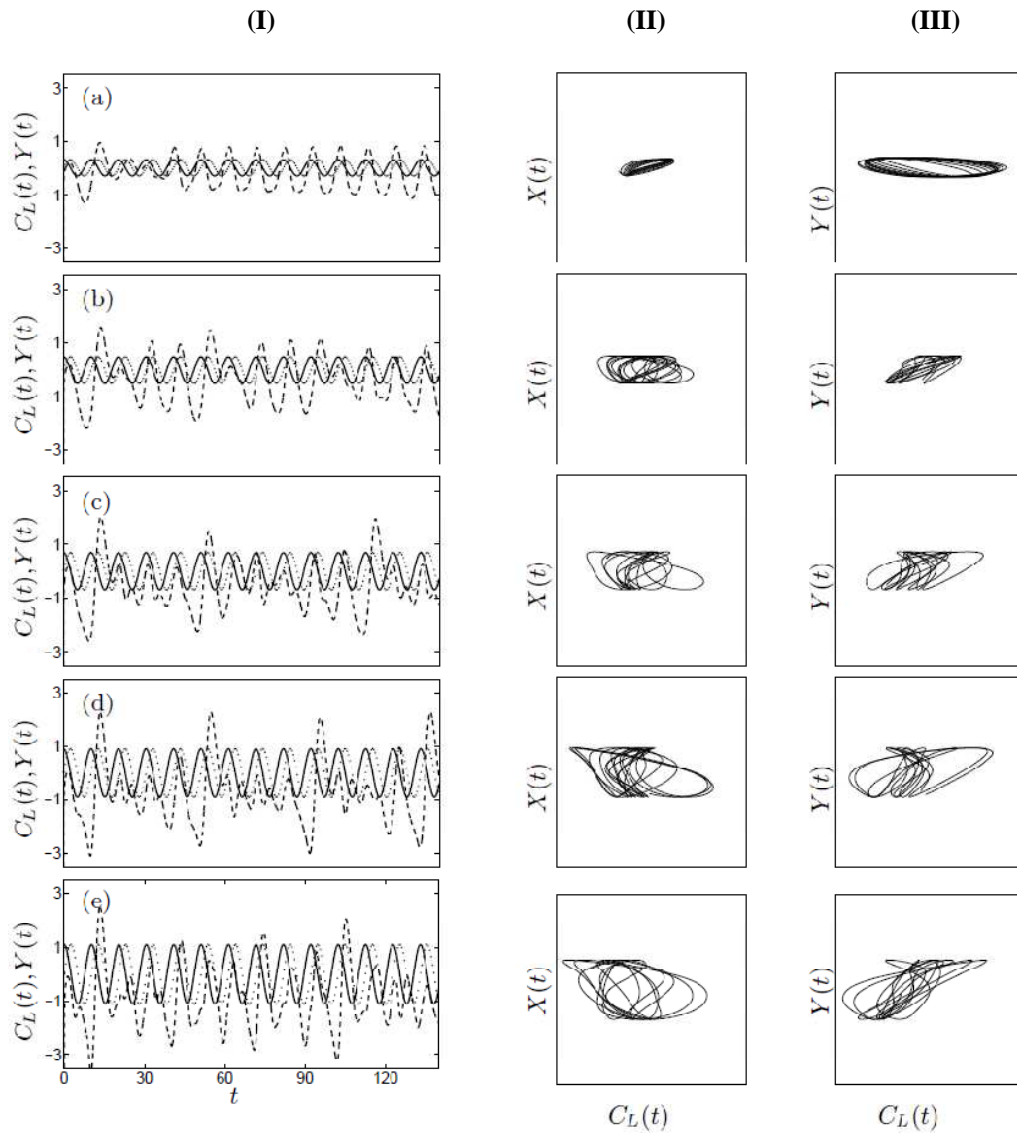


Fig. 2 (I): —, the time variation of  $C_L(t)$ ; - - -, streamwise cylinder displacement,  $X(t)$ ; ....., transverse cylinder displacement,  $Y(t)$ , (II): the Lissajous patterns ( $C_L$  versus  $X$ ), and (III): the Lissajous patterns ( $C_L$  versus  $Y$ ) for combined (2-DoF) streamwise and transverse oscillation case when  $R = 200 : f / f_0 = 1.0$  : (a)  $A = 0.3$ , (b)  $A = 0.5$ , (c)  $A = 0.7$ , (d)  $A = 0.9$ , (e)  $A = 1.1$

The boundary conditions for  $\psi$  and  $\zeta$  are based on the no-slip and impermeability conditions on the cylinder and the free stream condition away from it. These conditions are utilized to deduce sets of global conditions, termed integral conditions, on  $\zeta$  by applying one of the Green's identities to the domain of the field of flow, for more details see Dennis and Chang [6,7]. Further, all flow variables must be periodic functions of the angular coordinate  $\theta$  with period  $2\pi$ , i.e.

$$\zeta(\xi, \theta, t) = \zeta(\xi, \theta + 2\pi, t), \quad \psi(\xi, \theta, t) = \psi(\xi, \theta + 2\pi, t).$$

The method of analysis is based on the solutions of the unsteady Navier-Stokes equations (2) and (3) by means of Fourier analysis and finite difference method. Note that Since the computational domain along the spatial,  $\xi$ , direction is unbounded, we choose a large enough artificial outer boundary,  $\xi_\infty$ , for the numerical treatment. The solutions for both the vorticity and stream function components are combined together by incorporating the Gauss-Seidel iterative method to obtain a consistent solution for the system. For further details about the numerical algorithm the reader is referred to the recent work of Al Mdallal et al. [1] and the references therein.

The simulations are carried out by using the time step  $\Delta t_{j+1} = 10^{-4}$  for the first 100 steps, then was increased to  $\Delta t_{j+1} = 10^{-3}$  for the next 100 steps and finally  $\Delta t_{j+1} = 10^{-2}$  for the rest of the calculations. The number of points in the  $\xi$  direction is taken as 319 with a grid size of  $\Delta z = 0.025$ . The maximum number of terms in the Fourier series is taken as  $N = 60$  for all cases considered in this paper.

### III. NUMERICAL SIMULATION RESULTS

The full set of results for the cases of  $R = 200$ :  $f / f_0 = 0.5 - 4.0$  when  $0 \leq A \leq 1.1$  will be reported elsewhere, but here we concentrate and analyze only for the cases when  $f / f_0 = 1.0$ . It is well-known that the lock-on phenomenon in the case of a cylinder performing a streamwise-only (1-DoF) oscillation occurs when the oscillation frequency is near twice the natural shedding,  $f \approx 2f_0$ . However, it occurs when the oscillation frequency

is near the natural shedding  $f \approx f_0$ , in the case of a cylinder performing a transverse-only (1-DoF) oscillation. In this discussion, we analyzed the lock-on phenomenon via Lissajous patterns of the lift coefficient.

The time history of the fluctuating lift coefficients up to non-dimensional time  $t = 140$  and the corresponding Lissajous patterns (i.e.  $C_L$  versus  $X$  and  $C_L$  versus  $Y$ ) are shown in Figure 2 for the case of combined (2-DoF) streamwise and transverse oscillation at  $R = 200$ :  $f / f_0 = 1.0$ , and  $0.3 \leq A \leq 1.1$ . It is clear that the cylinder excitation in all cases except  $A = 0.3$  produces a non-repetitive signature of the lift coefficient which is also suggested by the corresponding Lissajous patterns. These observations suggest that the vortex shedding

in the near-wake region is almost "but not full" periodic behavior flow values of oscillation amplitudes, i.e.  $A \leq 0.3$ .

Table I shows the predicted values of the maximum lift coefficient,  $C_{L,max}$ , the RMS lift coefficient,  $C_{L,rms}$ , the maximum drag coefficient,  $C_{D,max}$ , the minimum drag coefficient,  $C_{D,min}$ , the RMS drag coefficient,  $C_{D,rms}$ , and the mean drag coefficient,  $\overline{C_D}$ , for the case of combined (2-DoF) streamwise and transverse oscillation at  $R = 200$ ,  $f / f_0 = 1.0$ , and  $0.3 \leq A \leq 1.1$ . These quantities are calculated over four periods of cylinder oscillation for  $6T \leq t \leq 13T$ , where  $T = 1 / f \approx 10.24$ . Obviously, the quantities  $C_{L,max}$ ,  $C_{L,rms}$  and  $C_{D,max}$ , in general, are increasing as the amplitude of oscillation,  $A$ , increases. However, we notice that the values of  $C_{D,max}$  and  $C_{D,rms}$  are only increasing in the interval  $A \in [0.3, 0.9]$ . Finally,  $C_{D,min}$  is decreasing as  $A$  increases.

TABLE I  
PREDICTED VALUES OF  $C_{L,max}$ ,  $C_{L,rms}$ ,  $C_{D,max}$ ,  $C_{D,min}$ ,  $C_{D,rms}$   
AND  $\overline{C_D}$  FOR THE CASE OF COMBINED (2-DOF) STREAMWISE AND  
TRANSVERSE OSCILLATION IN THE PRESENCE OF UNIFORM FLOW AT  
 $R = 200$ ,  $f / f_0 = 1.0$  AND  $0.3 \leq A \leq 1.1$

A	$C_{L,max}$	$C_{L,rms}$	$C_{D,max}$	$C_{D,min}$	$C_{D,rms}$	$\overline{C_D}$
0.3	0.8211	0.6857	2.0522	0.9689	1.5643	1.5298
0.5	1.1054	0.8848	2.3474	0.4396	1.5503	1.4691
0.7	1.9298	1.0086	2.8432	0.1111	1.8381	1.6702
0.9	2.0825	1.2994	3.2472	-0.1607	2.0582	1.8290
1.1	2.1020	1.4006	3.5577	-0.9784	2.0011	1.6465

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### REFERENCES

- [1] Q. M. Al-Mdallal, K. P. Lawrence, and S. Kocabiyik. Forced streamwise oscillations of a circular cylinder: lock-on modes and resulting forces. *Journal of Fluids and Structures*, 23(5):681–701, 2007.
- [2] Q.M. Al-Mdallal. *Analysis and computation of the cross-flow past an oscillating cylinder with two degrees of freedom*. PhD thesis, Memorial University of Newfoundland, St. John's, Canada, 2004.
- [3] L. Baranyi. Numerical simulation of flow around an orbiting cylinder at different ellipticity values. *Journal of Fluids and Structures*, 24:883–906, 2007.
- [4] A. Barrero-Gil and P. Fernandez-Arroyo. Fluid excitation of an oscillating circular cylinder in cross-flow. *European Journal of Mechanics - B/Fluids*, 29(5):364–368, 2010.
- [5] B.S. Carmo, S.J. Sherwin, P.W. Bearman, and R.H.J. Willden. Flow-induced vibration of a circular cylinder subjected to wake interference at low Reynolds number. *Journal of Fluids and Structures*, 27(4):503–522, 2011.

- [6] S. C. R. Dennis and G. Z. Chang. Numerical integration of the Navier-Stokes equations, Technical Summary Report No. 859. *Mathematical Research Center, University of Wisconsin*, 1969.
- [7] S. C. R. Dennis and G. Z. Chang. Numerical solutions for steady flow past a cylinder at Reynolds numbers up to 1000. *Journal of Fluid Mechanics*, 42:471–489, 1970.
- [8] E. Didier and A. R. J. Borges. Numerical predictions of low Reynolds number flow over an oscillating circular cylinder. *Journal of Computational and Applied Mechanics*, 8(1):39–55, 2007.
- [9] E. Konstantinidis and C. Liang. Dynamic response of a turbulent cylinder wake to sinusoidal inflow perturbations across the vortex lock-on range. *Physics of Fluids*, 23(7):In Press, 2011.
- [10] O. A. Marzouk and A. H. Nayfeh. Characterization of the flow over a cylinder moving harmonically in the cross-flow direction. *International Journal of Non-Linear Mechanics*, 45(8):821–833, 2010.
- [11] P. K. Stansby and R. C. T. Rainey. On the orbital response of a rotating cylinder in a current. *Journal of Fluid Mechanics*, 439:87–108, 2001.
- [12] P. Suthon and C. Dalton. Streakline visualization of the structures in the near wake of a circular cylinder in sinusoidally oscillating flow. *Journal of Fluids and Structures*, 27(7):885–902, 2011.
- [13] C. H. K. Williamson, P. Hess, M. Peter, and R. Govardhan. Fluid loading and vortex dynamics for a body in elliptic orbits. *the Conference on Bluff Body Wakes and Vortex-Induced Vibration, Washington, DC, USA, paper number 18*, 1998.