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On the Accuracy of Basic Modal Displacement Method Considering Various Earthquakes

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Abstract**—**Time history seismic analysis is supposed to be the most accurate method to predict the seismic demand of structures. On the other hand, the required computational time of this method toward achieving the result is its main deficiency. While being applied in optimization process, in which the structure must be analyzed thousands of time, reducing the required computational time of seismic analysis of structures makes the optimization algorithms more practical. Apparently, the invented approximate methods produce some amount of errors in comparison with exact time history analysis but the recently proposed method namely, Complete Quadratic Combination (CQC) and Sum Root of the Sum of Squares (SRSS) drastically reduces the computational time by combination of peak responses in each mode. In the present research, the Basic Modal Displacement (BMD) method is introduced and applied towards estimation of seismic demand of main structure. Seismic demand of sampled structure is estimated by calculation of modal displacement of basic structure (in which the modal displacement has been calculated). Shear steel sampled structures are selected as case studies. The error applying the introduced method is calculated by comparison of the estimated seismic demands with exact time history dynamic analysis. The efficiency of the proposed method is demonstrated by application of three types of earthquakes (in view of time of peak ground acceleration).

Keywords Time history dynamic analysis, basic modal displacement, earthquake induced demands, shear steel structures.

I. Introduction

THE time history analysis of large scale structures requires much computational effort. This drawback more resonates optimization problems in which the dynamic analyses are performed for many times for a structure [1]. Consequently, approximating the time history responses of structures may effectively reduce the computational burden.

Maybe the first outstanding work for avoiding time history analysis is performed by E. Rosenblueth [2] in his PhD thesis. He introduced the SRSS rule for modal combination by using response spectrum. The SRSS rule predicts the peak of responses without any need to derive complete response of the structure. This modal combination rule provides good response estimates for structures with well-separated natural frequencies only. This drawback has been later recognized by Wilson et al. [3] and that is why they produced a replacement for SRSS calling CQC rule for modal combination. These probabilistically methods are based on the assumption that the structure behaves linearly and more over the input earthquake excitation is a stationary wave. Therefore, although estimations of the methods are acceptable for engineering problems, for scientific problem researcher seek for better replacements with more accuracy.

Several researches have been implemented towards structural optimization against earthquake effects. Enhancement of genetic algorithm in optimization process through application of neural networks was implemented by Gholizadeh and Salajegheh [4]. Wavelet transforms has been applied by Salajegheh and Heidari [5], towards reduction of analysis run time of structures being excited by earthquake strong ground motions. In the mentioned research, the earthquake data points have been halved through transformation of acceleration time histories by Wavelet Transforms. Wavelet transformation divided the acceleration time history function into two main parts, namely low and high frequencies. High frequency part of strong ground motion data had a minor effects on seismic induced demands, hence has been omitted, while low frequency part of strong ground motion (with lower data points compared with original time function) has been applied to achieve seismic induced demands of the structure. Liang Su et al. [6] proposed mean response spectrum to achieve linear response of structures against strong ground motions of several earthquakes.

Lagaros et al. [7] examined the influence of various design procedures on the dynamic performance of real-scale steel buildings. In addition, Zou and Chan [8] and Kocer and Arora [9] have used traditional and evolutionary search techniques to optimize the seismic design of structures by using the response spectrum or time history analysis.

Prendes Gero et al. [10]–[11] employed a modified elitist Genetic Algorithm (GA) for the design optimization of 3D steel structures. Also, they compared their proposed optimization algorithm with the common commercial solutions for structural optimization. Cheng et al. [12] used a multi-objective GA-based formulation incorporating game and fuzzy set theory for the optimum design of dynamically loaded 2D frames.

In the present research, a logical methodology is introduced towards prediction of earthquake induced response of structures. Modal response of a hypothetical structure is calculated though time history analysis and through scaling of coordinate axes the modal response of real structure is predicted.

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II. Basic Modal Displacement

For a structure with n degrees of freedom, the dynamic equation of motion under the ground acceleration $\ddot{U}_g$ can be written as

$$M\ddot{U} + C\dot{U} + KU = -Ml\ddot{U}_g \quad (1)$$

where M, C and K are mass, damping and stiffness matrices of the structure and U and l are vector of displacements and ones, respectively.

The eigenvalue problem written as

$$K\Phi_i = \lambda_i M\Phi_i \quad (2)$$

The eigenvalue problem leads to computation of the mode shapes also known as eigenvectors, Φ, and eigenvalues also known as frequencies λ_i.

The eigenvalues are numbered in the descending order ($\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots$).

Squares of circular frequencies are equal to eigenvalues,

$$\lambda_i = \omega_i^2 \quad (3)$$

where ω_i is circular frequency of the ith mode.

By a transformation, the equation of motion is written in the modal coordinates.

$$U = \Phi q \quad (4)$$

After pre-multiplication of Φ^T to the transformed equation, the system of differential equations is decoupled into n separate differential equations as:

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i = -\Gamma_i\ddot{u}_g(t);\ i = 1, 2, \ldots n \quad (5)$$

where q_i is the ith modal coordinate (the ith coordinate of vector q) and Γ_i is the modal participation factor and written as

$$\Gamma_i = \Phi_i^T Ml \quad (6)$$

Let us consider a specific design solution for the structure. This specific determined structure is hereafter called the basic structure or the basic model. All parameters regarding the basic model are denoted by the over bar sign. Hence, for the basic structure the dynamic equation of motion in its modal coordinates is according to "(5)".

If the design parameters of the basic structure change, the modal information of the structure alters. The n decoupled differential equations for the changed structure are written as calculation "(5)". However, solving "(5)", n times for n modes by employing both Duhamel's integral or direct integration methods (e.g. Wilson θ and β Newmark methods) is computationally expensive. Therefore, in this paper, we propose to modify the modal displacements of the basic structure in a way to obtain the modal displacements of the new structure with the new design parameters written as

$$q_i(t) = \lambda_i\bar{q}_i(\mu_i t);\ i = 1, 2, \ldots n \quad (7)$$

The parameters λ_i and μ_i scale the basic modal coordinates along time (horizontal axis) with the amount of λ_i times and displacements (vertical axis) with the amount of $1/\mu_i$ times, respectively (see Fig. 1). In fact, it is assumed that by changing design variables, the pattern of modal displacements do slightly change, and the location and amount of maximums and minimums modifies only.

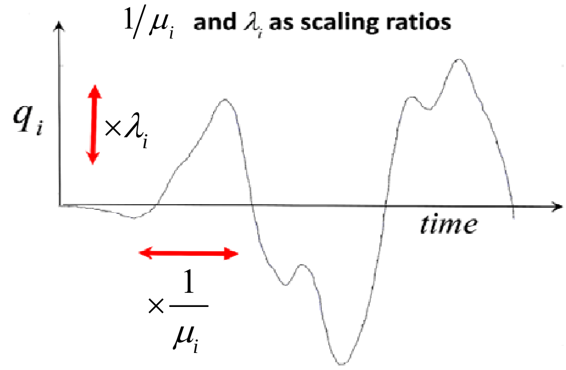

Fig. 1 $1/\mu_i$ and λ_i as scaling factors along the corresponding axes for the ith modal displacement

Here, we explain that how the almost optimum amounts for λ_i and μ_i; $i = 1, 2, \ldots n$ are derived. It is known that the time required for a single degree of freedom system to complete a cycle of vibration when subjected to an earthquake ground motion is very close to the natural period of the system. This interesting result, valid for typical ground motions containing a wide range of frequencies, and can be proven using random vibration theory [13].

Result implies that μ_i can written as

$$\mu_i = \omega_i(x)/\bar{\omega}_i \quad (8)$$

Also, the modal displacements should be scaled in a way to the maximum amount of modal displacements to be equal to the values suggested by displacement spectrum of the earthquake for a single degree of freedom system.

Remind that the plot of the peak values for displacement as a function of circular frequency, ω, is called the displacement spectrum.

The value λ_i as written as

$$\lambda_i = D(\omega_i(x))/D(\overline{\omega}_i) \quad (9)$$

where $D(.)$ presents the displacement spectrum of the input earthquake.

III. Applied Strong Ground Motions

To demonstrate the efficiency of the proposed method towards prediction of seismic induced demands, time history analysis of sampled shear structures against three horizontal strong ground motions, Northridge, Artificial earthquake and Superstition hill is implemented. Applied acceleration time histories are shown in Figs. 2-4.

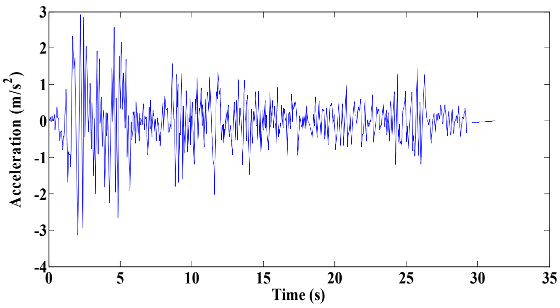

Fig. 2 Acceleration history of Northridge

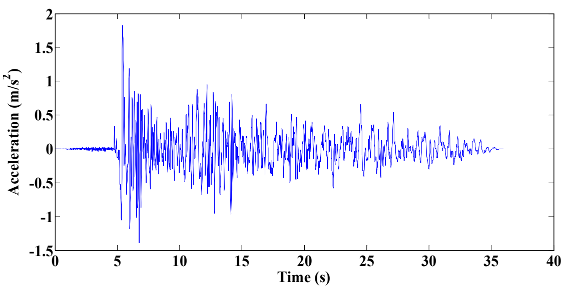

Fig. 3 Acceleration history of Artificial earthquake

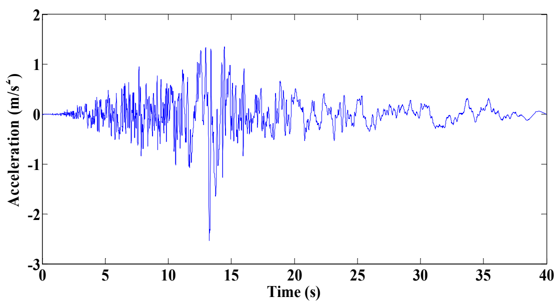

Fig. 4 Acceleration history of Superstition hill

IV. Shear Steel Frames

5-story 1-bay steel shear frames are considered as case studies (see Fig. 5). The mentioned frames have 5 degrees of freedom. Strong ground motions of earthquakes are applied horizontally and weight of each story is considered to be 21 tons. To account for different story stiffness, column sections are selected from the cross sections shown in Table I.

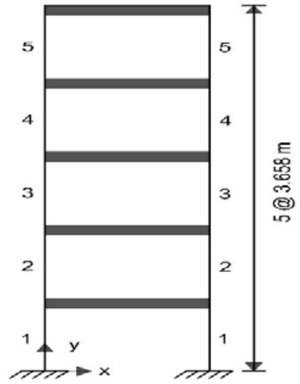

Fig. 5 A Five-story shear frame

TABLE I
AVAILABLE PROFILES FOR SHEAR FRAMES

No.	Profile
1	Box 180*180*16
2	Box 220*220*17.5
3	Box 240*240*20
4	Box 260*260*20
5	Box 280*280*20
6	Box 300*300*20
7	Box 320*320*20
8	Box 340*340*20

Five types of shear frames, which are analyzed in this study, are shown in Table II.

TABLE II
SHEAR FRAME STRUCTURES AND SECTIONS USED IN THEM

Structure no.	Element groups no.				
	1	2	3	4	5
1	3	3	2	2	1
2	4	3	3	1	1
3	5	4	3	2	1
4	7	4	3	3	3
5	8	7	6	6	4

Hypothetical and base structures according to BMD method and for the considered case studies are shown in Table III.

TABLE III
NEW MODEL AND BASE MODEL FOR BMD IN SHEAR FRAMES

New structure no.	Base structure no.
1	3
2	5
3	1
4	1
5	3

V. Numerical Results

To demonstrate the accuracy of the proposed method towards prediction of seismic induced demand values, achieved results applying BMD method are compared with two commonly used methods, namely SRSS and CQC. Approximation errors in prediction of roof displacement applying each method are shown in Tables IV-VI, in which results of time history analysis of the sampled structures are considered as exact solution. Furthermore, to show the required computational effort for each method, analysis run time is shown below the approximation errors in the same tables.

TABLE IV
DISPLACEMENT ERROR PERCENT FOR THE ROOF OF SHEAR FRAMES IN VARIOUS ANALYSIS SUBJECTED NORTHRIDGE GROUND MOTION

Structure no. and time (s)	Analysis method		
	SRSS	CQC	BMD
1	3.20	3.21	2.98
Time	0.006	0.012	0.015
2	7.84	7.86	6.66
Time	0.006	0.014	0.011
3	7.45	7.47	3.24
Time	0.004	0.010	0.014
4	3.96	3.95	2.24
Time	0.004	0.009	0.010
5	2.57	2.57	1.72
Time	0.004	0.009	0.013

TABLE V
DISPLACEMENT ERROR PERCENT FOR THE ROOF OF SHEAR FRAMES IN VARIOUS ANALYSIS SUBJECTED ARTIFICIAL EARTHQUAKE

Structure number and time (s)	Analysis method		
	SRSS	CQC	BMD
1	10.33	10.35	2.39
Time	0.004	0.009	0.016
2	9.34	9.36	0.14
Time	0.004	0.010	0.016
3	11.04	11.06	4.48
Time	0.004	0.010	0.017
4	3.20	3.21	3.44
Time	0.004	0.009	0.014
5	4.71	4.72	3.85
Time	0.004	0.010	0.021

TABLE VI
DISPLACEMENT ERROR PERCENT FOR THE ROOF OF SHEAR FRAMES IN VARIOUS ANALYSIS SUBJECTED SUPERSTITION HILL GROUND MOTION

Structure number and time (s)	Analysis method		
	SRSS	CQC	BMD
1	0.78	0.75	13.34
Time	0.004	0.010	0.021
2	5.15	5.11	4.69
Time	0.004	0.010	0.020
3	0.75	0.79	8.12
Time	0.006	0.014	0.022
4	2.63	2.64	0.54
Time	0.004	0.010	0.020
5	3.63	3.62	0.72
Time	0.004	0.010	0.024

To illustrate the efficiency of the proposed method in prediction of seismic demand time function, roof displacement time history of third structure, being excited by three assumed earthquakes and calculated based on time history analysis, is compared with BMD results in Figs. 6-8.

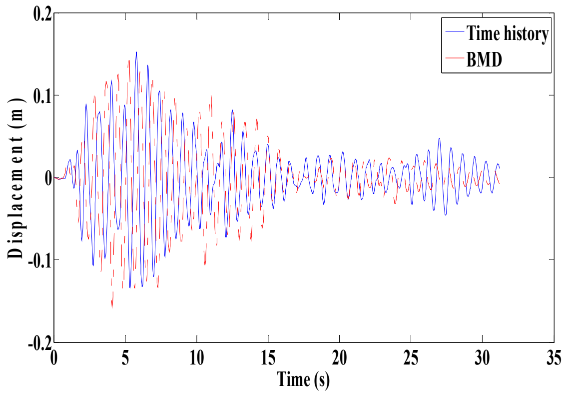

Fig. 6 Roof displacement derived by BMD compared with exact ones for third shear frame, under Northridge quake

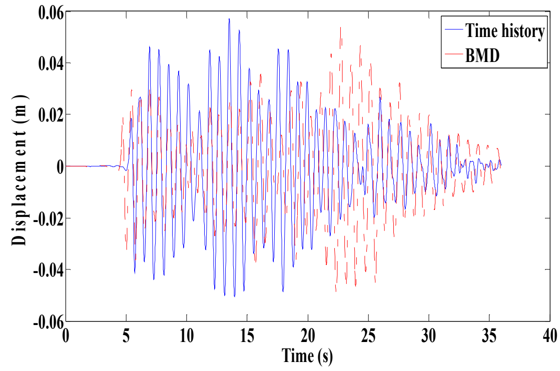

Fig. 7 Roof displacement derived by BMD compared with exact ones for third shear frame, under Artificial quake

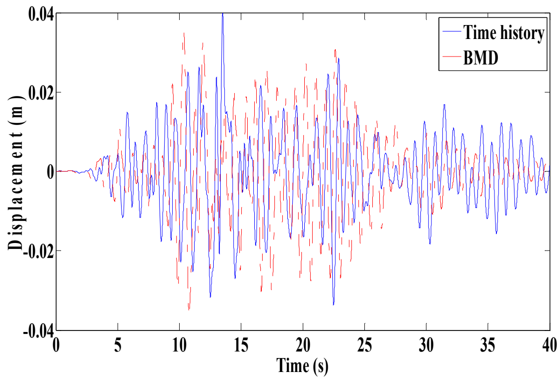

Fig. 8 Roof displacement derived by BMD compared with exact ones for third shear frame, under superstition hill quake

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