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Numerical study of transient laminar natural convection cooling of high Prandtl number fluids in a cubical cavity: Influence of the Prandtl number

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***Abstract*—** This paper presents and discusses the numerical simulations of transient laminar natural convection cooling of high Prandtl number fluids in cubical cavities, in which the six walls of the cavity are subjected to a step change in temperature. The effect of the fluid Prandtl number on the heat transfer coefficient is studied for three different fluids (Golden Syrup, Glycerin and Glycerin-water solution 50%). The simulations are performed at two different Rayleigh numbers ($5 \cdot 10^6$ and $5 \cdot 10^7$) and six different Prandtl numbers ($3 \cdot 10^5 \geq$Pr≥ 50). Heat conduction through the cavity glass walls is also considered. The propsed correlations of the averaged heat transfer coefficient (Nu) showed that it is dependant on the initial Ra and almost independent on Pr. The instantaneous flow patterns, temperature contours and time evolution of volume averaged temperature and heat transfer coefficient are presented and analyzed.

***Keywords*—** Transient natural convection, High Prandtl number, Variable viscosity.

I. Introduction

NATURAL convection in cubical cavities have attracted numerous researchers due to its implications in a wide set of technological applications and because its geometric simplicity. The problem of natural convection heat transfer has been studied experimentally as well as numerically and analytically during past decades.

George & Capp [1] utilized the classical scaling arguments to analyze the turbulent natural convection boundary layer next to a heated vertical surface. They divided the boundary layer into two parts. An inner region, where the mean convection terms are negligible, that is identified as constant heat flux layer and an outer region which constitutes most of the boundary layer and where the conduction terms are considered negligible. In their work, they suggested universal velocity and temperature profiles for asymptotic values of Rayleigh number when it approaches to infinity.

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Experiments in turbulent natural heat transfer boundary layers of air were conducted by Armfield & Patterson [2],Warner & Arpaci [3] and Cheesewright [4]. Some other experiments were carried out in different liquids by Lock & Trotter [5], Vliet & Liu [6], Fuji et al. [7] and Kutateladze et al. [8]. In all these experiments, the overall characteristics of the natural convection were studied. Turbulent transport in a natural convection along a vertical flat plate was experimentally studied by Kitamura et al. [9] and it was shown that the large eddy motions play an important role on the turbulent transport.

Most of the work done in the thermal storage area considers water as working fluid. Patterson & Imberger [10] numerically investigated the transient natural convection in a rectangular cavity with instantaneous cooling and heating of two opposed vertical sidewalls. They carried out scaling analysis and showed that a number of different initial flow types are possible. These types of flow have a strong dependence on the Prandtl number and the aspect ratio of the rectangular cavity. Nelson et al. [11] and Shyu et al. [12] studied the degradation of the thermal stratification in stratified storage tanks and concluded that highly conductive walls contribute to the degradation of the thermal stratification. Hyun [13] conducted numerical simulations to study the effect of the Prandtl number on the heating of stratified fluid in an enclosure. This author showed that the evolution of the flow and temperature fields are influenced by the Prandtl number.

The consideration of a high Prandtl number fluid with or without variable viscosity as working fluid in the transient cooling natural convection processes is rare. To the authors' knowledge only a few studies have been reported. Lin & Akins [14] , studied experimentally the natural convection in cubical enclosures using different kinds of fluids ($6 \leq Pr \leq 9000$) and different sizes of cubes. These authors found that the inclusion of the time and/or Pr number does not improve the correlation between Nusselt and Rayleigh number and that the use of the conventional correlations is accurate enough for prediction purposes. Ogawa et al. [15] carried out three-dimensional steady calculation of natural convection in a fluid with variable viscosity. They classified the flow patterns into two main regimes depending on the behaviour of the upper boundary layer; the stagnant lid regime and the whole layer regime. They concluded that the Nusselt number of the top surface is highly dependent on the viscosity in the whole layer regime. Davaille & Jaupart [16] reported experimental results of transient natural convection at high Ra numbers with large viscosity variation in a tank with insulated bottom wall and cold top wall. They studied the effect of the viscosity in the unstable boundary layer on the onset of instabilities. They introduced a new viscous temperature scale to compute the heat transfer rate at the cold top wall of the tank and claimed that this scale is sufficient to account for the characteristics of convection. Cotter & Michael [17] numerically studied the influence of the external heat transfer coefficient and aspect ratio of the enclosure on the transient natural convection of a warm crude oil in a vertical cylindrical storage tank located in a cold environment. Recently Oliveski et al. [18] investigated numerically and experimentally the two dimensional transient natural convection in a tank of oil with constant viscosity. The thermal boundary conditions used in the simulations were determined experimentally.

The objective of the current work is to study the effect of the fluid Prandtl number on the instantaneous flow field topology in terms of instability near the top wall, and on the averaged heat transfer coefficient (Nu).

II. Physical and Mathematical Models

The case under consideration is the three dimensional unsteady natural convection of high Prandtl number fluids (Golden Syrup, Glycerin and Glycerin-water solution 50 %) in cubical cavities. In reality, the viscosities of these fluids have dependence on temperature. However, according to Younis et al. [19] no significant effects are observed in the flow field and heat transfer coefficient for viscosity contrast less than 10. Therefore, variable viscosity is only adopted for Golden syrup for temperature increments considered. The physical properties of the working fluids are summarized in table I.

The cavities are made of glass with different dimensions (see table II). The thickness of the walls is 10% of the cavity dimension, and the heat conduction through these walls is considered. The cavities are full of hot fluid, and the walls of the cavities are rigid and immobile. Initially, the fluid in the cavity is considered to be at rest and at constant temperature and the temperature of the six walls is set to constant value through the cooling process.

TABLE I
Physical properties of the working fluids

Working fluid	ρ $\frac{kg}{m^3}$	$\beta \cdot 10^5$ K^{-1}	$\alpha \cdot 10^7$ $\frac{m}{s^2}$	ν $\frac{m}{s^2}$
Golden syrup	1.438	43	1.2	see eq.(1)
Glycerin	1.258	61	0.94	$1.27 \cdot 10^{-3}$
Glycerin-water 50%	1.211	51	1	$1 \cdot 10^{-6}$

According to Davaille and Jaupart [16], the viscosity variation with temperature for the Golden syrup is assumed to be of the form:

$$\mu = \mu_o \, exp\left(\frac{1}{AT^2+BT+C}\right) \tag{1}$$

Where: $\mu_o = 4.485 \cdot 10^{-8} \; Pa \; s$,$A = -7.5907 \cdot 10^{-7}$,$B = 3.8968 \cdot 10^{-4}$ and $C = 4.0130 \cdot 10^{-2}$, and T is in Celsius. Figure 1 shows the dependence of the dynamic viscosity on the temperature.

The system of natural convection is governed by the three-dimensional unsteady Navier - Stokes equations and the energy equation along with the Boussinesq approximation. The governing equations in non-dimensional form in Cartesian coordinates can be written as:

$$\frac{\partial u_i^*}{\partial x_i^*} = 0 \tag{2}$$

$$\frac{\partial u_i^*}{\partial t^*} + \frac{\partial (u_i^* u_j^*)}{\partial x_j^*} = -\frac{\partial P^*}{\partial x_i^*} + \frac{\partial}{\partial x_j^*}\left[Pr(T^*)\left(\frac{\partial u_i^*}{\partial x_j^*} + \frac{\partial u_j^*}{\partial x_i^*}\right)\right] + \delta_{i1}\, Ra_o\, Pr_o\, T^* \tag{3}$$

$$\frac{\partial T^*}{\partial t^*} + \frac{\partial (u_j^* T^*)}{\partial x_j^*} = \frac{\partial^2 T^*}{\partial x_j^* \partial x_j^*} \tag{4}$$

The non-dimensional form the governing equations are obtained by using the following scalings:

$$x_i^* = \frac{x_i}{L} \;,\; u_i^* = \frac{u_i L}{\alpha} \;,\; t^* = \frac{t\alpha}{L^2}$$
$$P^* = \frac{P}{\rho(\frac{\alpha}{L})^2} \;,\; T^* = \frac{T - T_i}{T_i - T_w}$$

The Prandtl number that appears in the diffusion term in equation (3) is calculated using equation (1) in the case of the simulations correspond to the Golden syrup. The Prandtl and Rayleigh numbers (Pr_o and Ra_o) in the buoyancy term are based on kinematic viscosity evaluated at the reference temperature $T_o = (T_w + T_i)/2$.

The studied cases are summarized in table II.

TABLE II
Studied cases

Case	Working fluid	Cavity dimension (m)	Ra_i	Pr_i
1	Glycerin-water 50%	0.066	$5 \cdot 10^7$	53
2	Glycerin	0.36	$5 \cdot 10^7$	$1.3 \cdot 10^4$
3	Golden syrup	0.75	$5 \cdot 10^7$	$1 \cdot 10^5$
4	Glycerin-water 50%	0.057	$5 \cdot 10^6$	50
5	Glycerin	0.305	$5 \cdot 10^6$	$1.14 \cdot 10^4$
6	Golden syrup	0.58	$5 \cdot 10^6$	$2.87 \cdot 10^5$

The initial and wall temperatures corresponding to the cavity dimensions indicated in table II are summarized in table III.

For cases 3 and 6 the viscosity contrast $(\mu(T_w)/\mu(T_i))$ is 230 and 25 respectively.

III. Numerical Methods

The in-house three-dimensional finite volume code 3DINAMICS is employed to solve the discretized equations along with the associated boundary conditions. The code utilizes the staggered variable arrangement. The spatial discretization of both the diffusive and convective terms is carried out by the central differencing scheme. The time discretization for all

TABLE III
INITIAL AND BOUNDARY CONDITIONS

Case	Initial temperature ^oC	Wall temperature ^oC
1	30	10
2	30	10
3	50	10
4	23	20
5	23	20
6	35	10

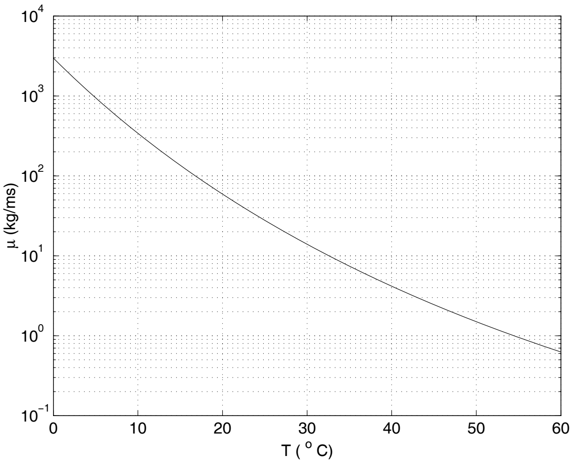

Fig. 1. Variation of Golden syrup viscosity with temperature .

terms is performed with the second order Crank-Nicholson scheme, except for the buoyancy term which is discretized in time using the second order Adams-Bashforth scheme. The Tri-Diagonal Matrix solver (TDMA) is employed to solve the resulting system of algebraic equations. The coupling between the velocity and pressure field is handled with the Fractional Step Method, and the resulting Poisson equation is solved with the Conjugate Gradient Method. This code has been validated and tested in numerical simulation of turbulent and laminar natural convection flows by Pallares et al. [20] and Valencia et al. [21]. The complete mathematical formulation and associated numerical methods of the code can be found in Cuesta [22].

In order to have reliable results, the thermal and velocity boundary layers developed near the walls of the cavity should be well resolved in terms of space and time. The flow contains different length scales ranging from the small scales of the thermal boundary layer to the big scales of the large rolling motions in the interior of the cavity. To be able to capture the whole range of scales it is necessary to use a fine mesh in the boundary layers and cavity center. Taking this consideration into account, a non-uniform mesh of 60^3 control volumes is employed. At the Ra and Pr numbers considered, the results obtained with a mesh of 70^3 control volumes do not show a significant difference in comparison with the present results. The mesh is symmetrically distributed with respect to the symmetry elements of the cavity.

IV. RESULTS AND DISCUSSION

The instantaneous flow field and temperature contours shown in figure 2 are plotted in the vertical midplane of the cavity and correspond to case5.

In general, the flow near the lateral wall is descending due to the cooling mechanism and ascending in the center. The flow in the lower half of the cavity is almost stagnant because of the stable stratification, while a number of secondary flow are presented in upper half of the cavity due to the unstable stratification in this region. Figure 2(a) clearly shows the generation of a central descending plume induced by the flow instability near the top wall. The front contour of the plume descends as time evolves until it is attached to the bottom region of the cavity where the stratification is stable. This feature is repeated in figures 2(b) to 1(h), the plumes are thin due to the high Pr. Figures 2(b), 2(c), 2(d) and 1(e) show two small plumes located midway between the main descending plume in the cavity center and the lateral walls.

The instantaneous flow fields of cases 1 to 3 are not symmetric with respect to the vertical symmetry planes of the cavity. In these cases the plumes are very thin and non organized. A lot of confined secondary motions are present in the upper half of the cavity, mainly due to the higher Ra. Regarding cases 4 and 6, they follow a similar plume generation mechanism as in case5 with some differences in the time needed for the plume to deform and in plume size. For higher Pr fluid, the plumes are deformed earlier and they are reduced in size.

Figure 3 shows the time evolution of the volume averaged temperature of the cavity. It was expected that the higher Pr the faster the cooling. This is valid for the cases that do not consider the viscosity variation (case1, case2, case4 and case5). However, case3 and case6 were expected to cool faster than other cases due to the larger Pr. As shown in figure 3 it is clear that the viscosity variation affects the flow and temperature fields in such a way that the fluid can not cool as faster as if a constant viscosity was considered. To have better understanding of the viscosity variation influence, the simulation of case3 and case6 assuming constant viscosity is being carried out.

The time evolution of the averaged heat transfer coefficient Nu is presented in figure 4. Nu is plotted against Ra which is based on the temperature difference between the volume averaged temperature of the cavity and the averaged temperature of the inner walls. Although a high contrast in Pr presents - for example the contrast of Pr between case1 and case2 is of order of 10^3 -, It is clear that Nu has no dependence on Pr and only depends on the initial Ra. For both regimes, the higher and the lower Ra regimes, the Nu for the cases that do not consider viscosity variation almost collapse into two groups having very similar slope. However, Nu of case6 also follows the trend of Nu of its group (case4 and case5), while the Nu of case3 does not probably because of the viscosity variation. The data can be correlated as:
First group, $Ra = 5 \cdot 10^7$, excluding case3:

$$Nu = 9.69 \cdot 10^{-7} Ra - 0.72$$

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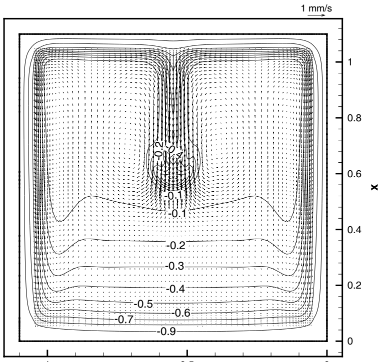
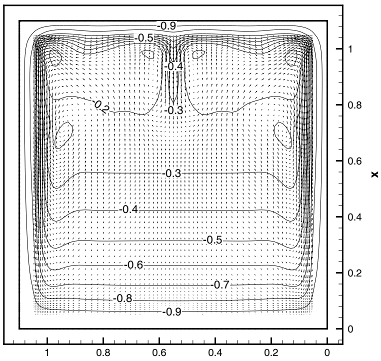
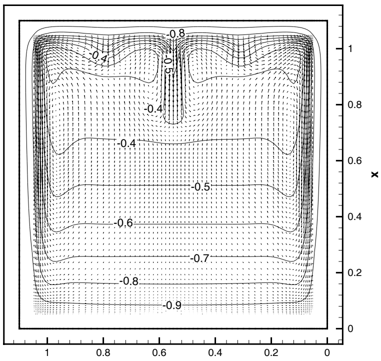


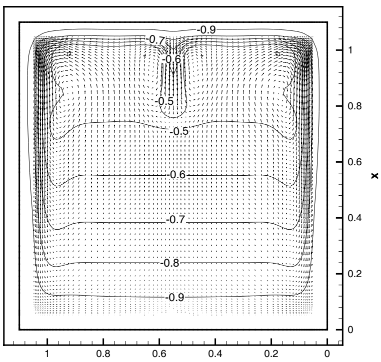
(a) $t^* = 0.0025$



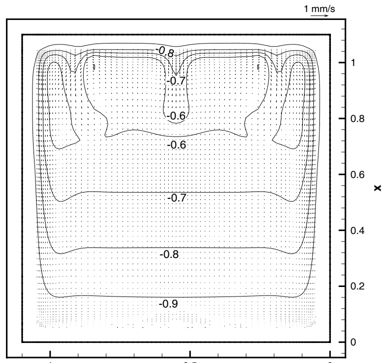
(b) $t^* = 0.005$



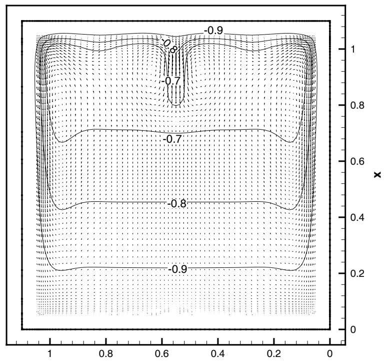
(c) $t^* = 0.0075$



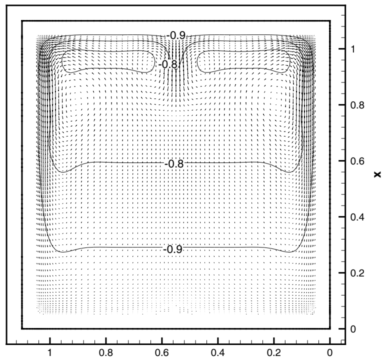
(d) $t^* = 0.01$



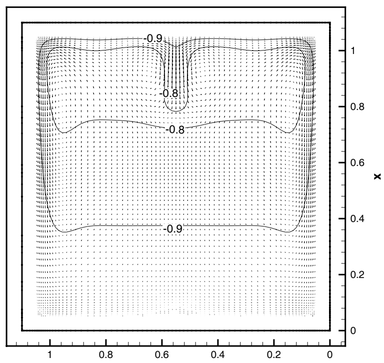
(e) $t^* = 0.0125$



(f) $t^* = 0.015$



(g) $t^* = 0.0175$



(h) $t^* = 0.02$

Fig. 2. Velocity vectors and temperature contours for Glycerin.

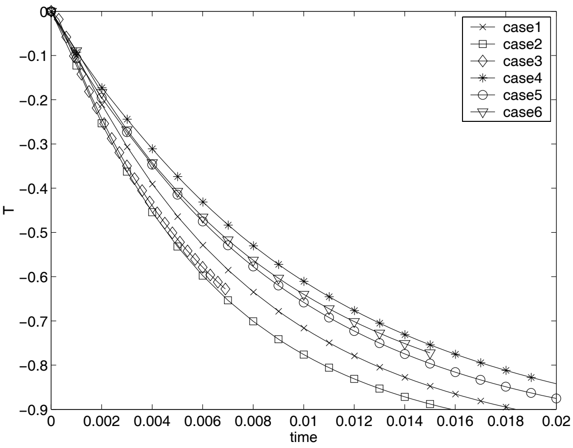


Fig. 3. Time evolution of volume averaged temperature.

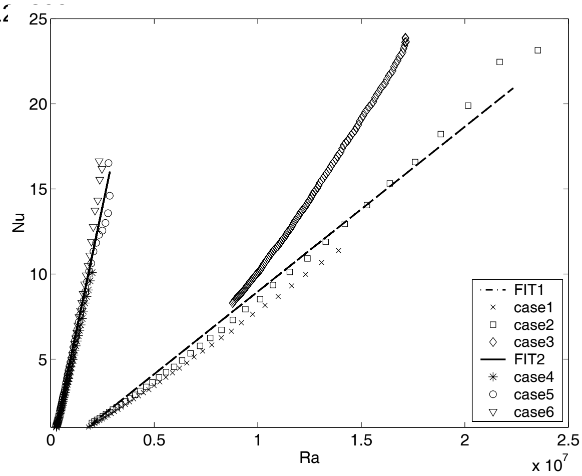


Fig. 4. Averaged heat transfer coefficient Nu.

Second group, $Ra = 5 \cdot 10^6$, including case6:

$$Nu = 5.97 \cdot 10^{-6} Ra - 0.524$$

V. CONCLUSION

The transient laminar natural convection cooling of high Prandtl number fluid in cavity is studied numerically to investigate the influence of the fluid Prandtl number on the heat transfer coefficient. Three different working fluids are considered at two different Rayleigh numbers ($5 \cdot 10^6$ and $5 \cdot 10^7$) and different initial and wall temperatures. The effect of the fluid viscosity variation is also considered in some cases. The numerical study illustrates that for the lower Ra the flow and temperature fields are symmetric, and organized laminar thermal plumes rises in different times with different size and intensity. The plume size is decreasing with Pr. Regarding the higher Ra, non organized small plumes are evident in the upper half of the cavity. The consideration of viscosity variation modifies the flow in terms of the volume averaged temperature and the averaged heat transfer coefficient Nu. These effects need more investigations by considering the same cases without viscosity variation. Also it is found that, Nu is dependent on the initial Ra and mostly independent on Pr, and it is correlated as a function of Ra which is based on the temperature difference between the averaged volume temperature and the averaged temperature of the inner walls.

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