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Numerical Simulation of the Transient Shape Variation of a Rotating Liquid Droplet

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***Abstract*—**Transient shape variation of a rotating liquid dropletis simulated numerically. The three dimensional Navier-Stokes equations were solved by using the level set method. The shape variation from the sphere to the rotating ellipsoid, and to the two-robed shapeare simulated, and the elongation of the two-robed droplet is discussed. The two-robed shape after the initial transient is found to be stable and the elongation is almost the same for the cases with different initial rotation rate. The relationship between the elongation and the rotation rate is obtained by averaging the transient shape variation. It is shown that the elongation of two-robed shape is in good agreement with the existing experimental data. It is found that the transient numerical simulation is necessary for analyzing the largely elongated two-robed shape of rotating droplet.

***Keywords*—**Droplet, rotation, two-robed shape, transient simulation.

I. Introduction

A LEVITATED liquid droplet is used to measure material properties of molten metal at high temperature, since the levitated droplet is not in contact with a container, and the effect of container wall is eliminated for precise measurement [1]. The levitation of liquid droplet, which is also used for container-less processing of material, is controlled by using electromagnetic [2] electrostatic [3], or ultrasonic force [4] in the vertical direction. Additionally, rotation is sometimes imposed on the droplet to stabilize its motion. The rotational motion is controlled by acoustic forces perpendicular to the vertical axis. Viscosity and surface tension are, respectively, obtained from the damping and the frequency of droplet shape oscillations. Viscosity is also obtained from the shape of a rotating droplet [5]. The droplet shape is varied gradually from a sphere to a rotating ellipsoid, and to a two-lobed shape. The measurement of the shape of a two-lobed droplet is not easy, since the two-lobed droplet is elongated in time, and disrupted finally [5].

Oscillating and rotating droplets have been simulated numerically by solving the Navier-Stokes equations, and the nonlinear effects of oscillation amplitude and rotation rate such as frequency shift were discussed [6]-[8]. The equilibrium shape of rotating droplet has been simulated, and the transition between ellipsoidal shape and two-robed shape were discussed [9]. However, numerical simulations for the transient shape variation of rotating droplet from a spherical shape to a two-robed shape have not yet been performed. The transient shape variation of rotating droplet is thus simulated numerically in this study. The droplet is assumed to be levitated in the centre of the simulation region, and the rotational motion is imposed as the initial rigid rotation. The flow field in and around the droplet is calculated using the level set method [10]. In the level set method, the level set function, which is the distance function from the two-phase interface, is calculated by solving the transport equation using the local flow velocity. Incompressible Navier-Stokes equations are solved to obtain the flow field. The simulation program is parallelized by the domain decomposition technique using the Message Passing Interface (MPI) library, and parallel calculations are performed using a massively parallel computer system. Transient of droplet shape is simulated by changing the initial rotational rate, and the variation of two-lobed shape is discussed.

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II. Numerical Method

A. Governing Equations

Governing equations for the droplet motion are the equation of continuity and the incompressible Navier-Stokes equation [10]:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

and

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \cdot (2\mu \mathbf{D}) - \mathbf{F}_s \tag{2}$$

where ρ, $\mathbf{u}$, p and μ, respectively, are the density, the velocity, the pressure and the viscosity, $\mathbf{D}$ is the viscous stress tensor, and $\mathbf{F}_s$ is a body force due to the surface tension. External force fields such as the gravity or the electrostatic force are not simulated in this study. The surface tension force is given by

$$\mathbf{F}_s = \sigma \kappa \delta \nabla \phi \tag{3}$$

where σ, κ, δ and ϕ are the surface tension, the curvature of the interface, the Dirac delta function and the level set function, respectively. The level set function is a distance function defined as the normal distance from the interface: ϕ=0 at the interface, ϕ<0 in the liquid droplet region, and ϕ>0 in the ambient gas region. The curvature of the interface is expressed in terms of ϕ:

$$\kappa = \nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|}\right) \tag{4}$$

The density and viscosity are, respectively, given by

$$\rho = \rho_l + (\rho_g - \rho_l)H \tag{5}$$

and

$$\mu = \mu_l + (\mu_g - \mu_l)H \tag{6}$$

where the subscripts g and l indicate gas and liquid phases, respectively. In (5) and (6), H is a smeared Heaviside function defined by

$$H = \begin{cases} 0 & (\phi < -\varepsilon) \\ \frac{1}{2}[1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi}\sin(\frac{\pi\phi}{\varepsilon})] & (-\varepsilon \le \phi \le \varepsilon) \\ 1 & (\varepsilon < \phi) \end{cases} \tag{7}$$

where ε is a small positive constant for which $|\nabla\phi| = 1$ for $|\phi| \le \varepsilon$. The evolution of ϕ is given by

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} = 0 \tag{8}$$

In order to maintain the level set function as a distance function, reinitialization of the level set function is proposed by solving the following equation [10]:

$$\frac{\partial\phi}{\partial\tau} = sign(\phi_0)(1 - |\nabla\phi|) \tag{9}$$

where τ is an artificial time, and $sign(\phi_0)$ indicates the sign of the level set function at the beginning of the reinitialization procedure. The level set function becomes a distance function in the steady-state solution of (9).The smoothed sign function proposed for numerical treatment of reinitialization [11] is used for (9);

$$sign(\phi) = \frac{\phi}{\sqrt{\phi^2 + h^2}} \tag{10}$$

where h is the spatial increment in the finite difference method for solving the governing equations. A smoothed version of the sign function was also used in [6].

The following equation is also solved to preserve the total mass in time [12];

$$\frac{\partial\phi}{\partial\tau} = (A_o - A)(P - \kappa)|\nabla\phi| \tag{11}$$

where A_0 denotes the total mass for the initial condition and A denotes the total mass corresponding to the level set function. $P$is a positive constant for stabilization, and 1.0 was used in [12]. The total mass is conserved in the steady-state solution of the above equation. The effects of (9) and (11) have been discussed in detail [13].

B. Numerical Condition

The finite difference method is used to solve the governing equations. The staggered mesh is used for spatial discretization of velocities. The convection terms are discretized using the second order upwind scheme and other terms by the second order central difference scheme. Time integration is performed by the second order Adams-Bashforth method. The SMAC method is used to obtain pressure and velocities [14]. The pressure Poisson equation is solved using the Bi-CGSTAB method. The domain decomposition technique is applied and the MPI library is used for parallel computations, and the block Jacobi preconditioner is used for the parallel Bi-CGSTAB method [15], [16].

The simulation region is a three-dimensional cubic region shown in Fig. 1.A spherical droplet is located in the centre of the simulation region. The size of the simulation region is 12.8 mm x 12.8mm x 12.8mm, and the radius of the droplet is 2.0 mm. Water and air properties are assumed for the inside and outside of the droplet, respectively. The periodic boundary conditions are applied at all sides of the simulation region. Rotation is imposed initially as a rigid rotation around the vertical centre axis. It is reported for simulating rotating-oscillating droplets that the necessary size of the simulation region is 6.0mm x 6.0mm x 6.0mm for the droplet radius of 2.0mm [6]. In this study, the droplet is deformed much, and the elongation of two-robed shape is simulated. The simulation region is thus almost three times larger than the droplet size as shown in Fig. 1.

The number of calculation grid points is 128 x 128 x 128, and the grid size is 0.1mm in all directions. The time step size is set equal to 15.0μs so that the maximum Courant number is smaller than 0.5. External force terms such as the gravitational acceleration and the electrostatic force are not included for simplicity.

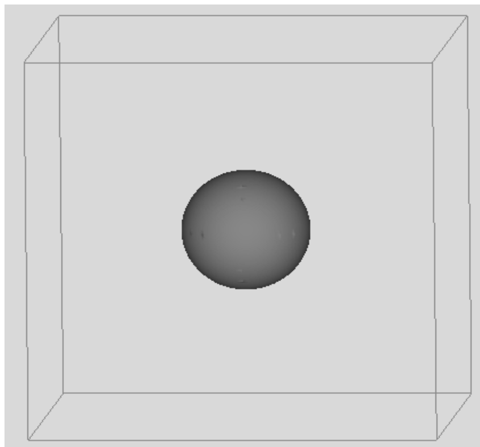

Fig. 1 Simulation region and initial droplet shape

III. Results and Discussion

The variation of droplet shape is shown in Fig. 2. The initial rotation rate is 30.0rps or 188.5rad/s. Top views in every 10,000 time steps or 0.15s from the initial shape shown in Fig. 1 are depicted in Fig. 2. In every 10,000 time steps, the droplet rotates about 4.5 times. It is seen that the droplet deforms in the beginning stage of rotation within several times of rotations. The droplet then becomes a non-axisymmetric shape and elongated at about 30,000 time steps or after 13.5 times of rotations. At 40,000 time steps or after 18 times of rotations, the droplet becomes the two-lobed shape. The two-lobed shape is stable, and the droplet continues rotating. The two-lobed shape is, however, elongated gradually, and the rotation rate becomes smaller. It is noted that the location of the droplet in the simulation region is shifted gradually in time, since no external force is applied for controlling the droplet position.

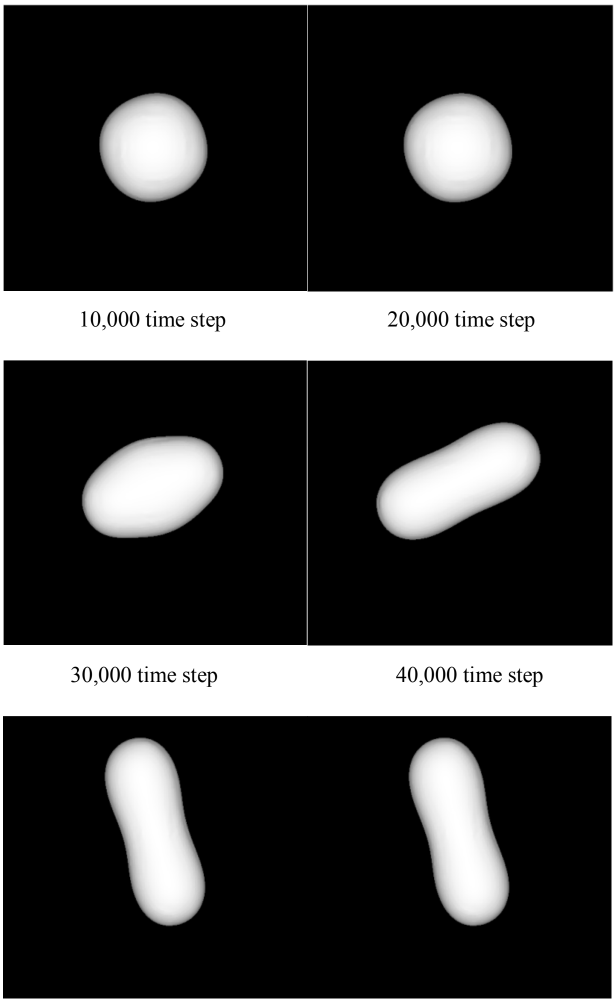

Fig. 2 Variation of droplet shape for 30.0rps

Another example is shown in Fig. 3, where the initial rotation rate is 50.0rps or 314.2rad/s. The droplet rotates about 7.5 times in every 10,000 time steps in this case. The droplet shape is shown to be unstable in the beginning stage of rotation due to the large rotation rate. The two-lobed shape, however, appears at about 20,000 time steps after the initial unstable shape. The two-lobed shape is stable afterwards, as is the case for the small rotation rate shown in Fig. 2. The two-lobed shape seems to be elongated at about 30,000 time steps, and shortened at about 50,000 time steps. The shape oscillation of two-robed droplet between an elongated shape and a shortened shape is indicated in Fig. 3.

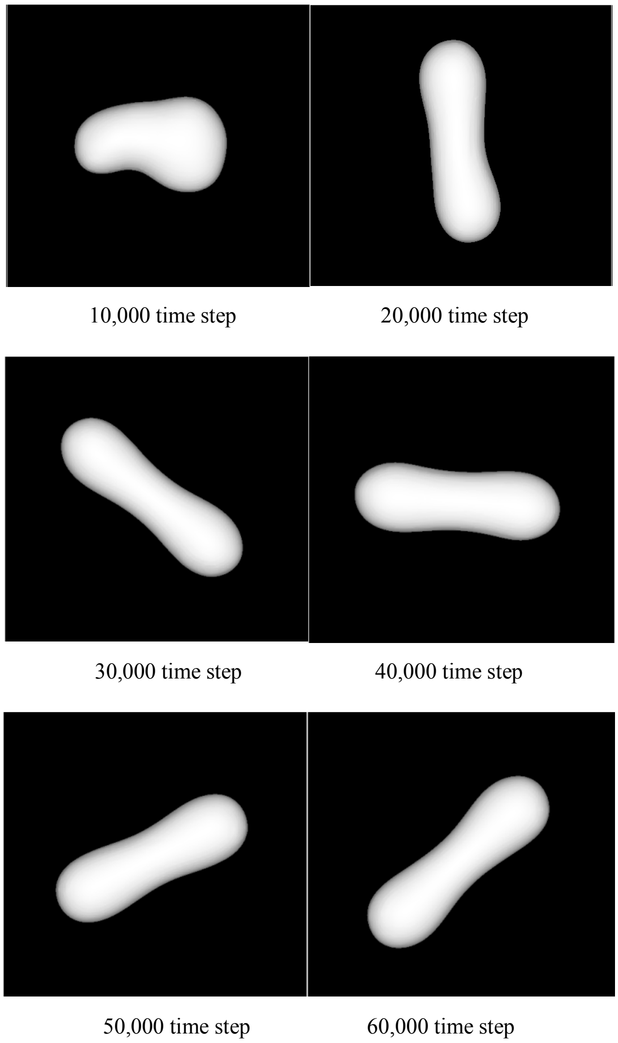

Fig. 3 Variation of droplet shape for 50.0rps

The elongation of the droplet is shown in Fig. 4 as a function of time. The elongation in Fig. 4 is defined as the maximum length of the droplet normalized by the initial diameter. For the case with the initial rotation rate of 30rps, the droplet shape is oscillated in the beginning stage, and the maximum length increases gradually up to about 30,000 time steps. The droplet shape is almost symmetrical during this period as shown in Fig. 2. The droplet becomes asymmetric after about 30,000 time steps, and the two-robed shape appears. The maximum length increases almost monotonically up to 50,000 time steps as shown in Fig. 4. The elongation continues after 50,000 time steps with slight oscillations.

The initial transient for the case with the rotation rate of 50rps is much different from that for 30rps. The droplet shape becomes very unstable immediately after the initiation of rotation, and the maximum length increases with large oscillations as shown in Fig. 4. After about 20,000 time steps; however, the droplet shape becomes almost two-robed shape as shown in Fig. 3. The maximum length increases afterwards with large oscillations. The two-robed shape is stable as shown in Fig. 3, and the shape oscillation between an elongated shape and a shortened shape becomes small, though the elongation continues after 20,000 time steps. It is noted that the increase in the maximum length of two-robed shape is almost the same after about 80,000 time steps for both the cases with different initial rotation rate. The rotation rate is actually almost the same for both the cases shown in Fig. 4 after 80,000 time steps. This indicates that the rotation of two-robed shape becomes almost the same even with the different initial rotation rate. Although the initial rotation energy is larger for the case with the rotation rate of 50rps than that for the case with 30rps, the rotation energy is almost the same after about 80,000 time steps. The rotation energy for the case with 50rps is thus found to be consumed for the large shape variation.

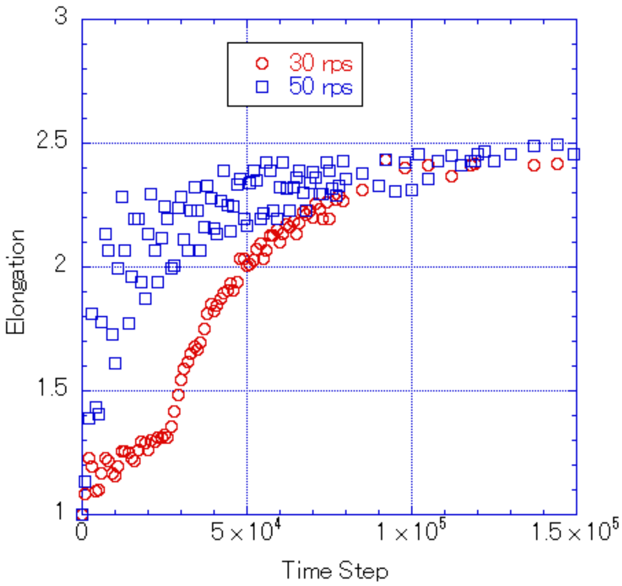

Fig. 4 Elongation of droplet shape

The relationship between the elongation of two-robed droplet and the rotation rate is shown in Fig. 5. In Fig. 5, the horizontal axis is the rotation rate normalized by the resonant frequency given by

$$f = \frac{1}{2\pi}\sqrt{\frac{8\sigma}{\rho_l r^3}} \tag{12}$$

where r is the initial droplet radius. Some experimental data and numerical results are also shown in Fig. 5 not only for non-axisymmetric two-robed shape but also for axisymmetric rotating ellipsoidal shape [5]. The present numerical results are shown by thick squares and circles for 50.0rps case and 30.0rps case, respectively. In Fig. 5, the numerical results for two-robed shape shown by the broken line are not in good agreement with the experimental data for the rotation rate smaller than 0.3 [5]. The numerical results in Fig. 5 are obtained based on the finite element method for the equilibrium shape [9]. The experimental data for largely elongated shape are, however, obtained under transient conditions [5]. The present numerical results are also obtained under transient conditions as shown in Figs. 2-4. For the case with the initial rotation rate of 50.0rps, three data points shown in Fig. 5 are obtained by averaging the results from 45,000 to 48,000, from 85,000 to 95,000, and from 122,000 to 128,000 time steps. For the case with 30.0rps, two data points are from 49,000 to 52,000 and from 73,000 to 80,000 time steps. It is shown in Fig. 5 that the agreement between the experimental data and the present numerical results is good in comparison with the equilibrium calculations. It is thus indicated that the transient numerical simulations are necessary for analyzing largely elongated two-robed droplet.

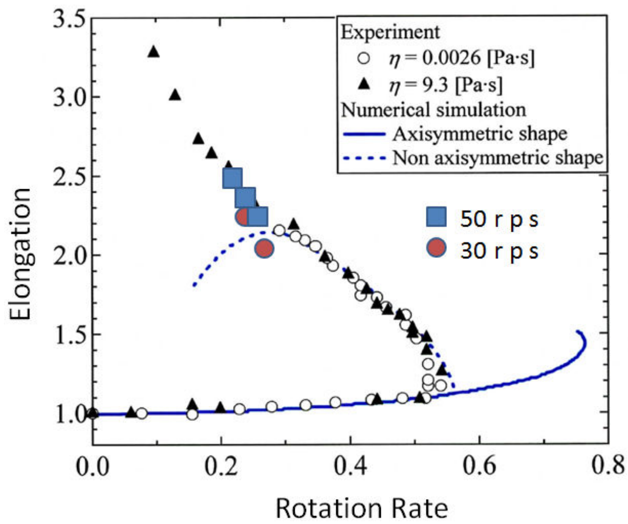

Fig. 5 Relation between elongation and rotation rate

IV. Conclusion

The transient shape variation of the rotating liquid droplet has been simulated numerically in this study. The three-dimensional Navier-Stokes equations were solved by using the level set method. The simulation program was parallelized using the MPI library and the parallel computer system was used. The shape variations from the sphere to the rotating ellipsoid, and to the two-robed shape were simulated, and the elongation of the droplet was discussed. It was shown that the two-robed shape was stable and the elongation continued with slight shape oscillation. The elongation of two-robed shape after the initial transient was found to be almost the same for the cases with different initial rotation rate. The initial transient was from the spherical shape to the two-robed shape for the case with smaller rotation rate, while it was from the unstable irregular shape to the two-robed shape for the case with larger rotation rate. The relationship between the elongation and the rotation rate was obtained by averaging the transient shape. It was shown that the elongation was in good agreement with the existing experimental data, which were not reproduced by the equilibrium calculation. It was thus found that the transient numerical simulation was necessary for analyzing the two-robed shape of rotating droplet.

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