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Numerical Modeling of Gas Turbine Engines

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***Abstract*—**In contrast to existing methods which do not take into account multiconnectivity in a broad sense of this term, we develop mathematical models and highly effective combination (BIEM and FDM) numerical methods of calculation of stationary and quasi-stationary temperature field of a profile part of a blade with convective cooling (from the point of view of realization on PC). The theoretical substantiation of these methods is proved by appropriate theorems. For it, converging quadrature processes have been developed and the estimations of errors in the terms of A.Ziqmound continuity modules have been received.

For visualization of profiles are used: the method of the least squares with automatic conjecture, device spline, smooth replenishment and neural nets. Boundary conditions of heat exchange are determined from the solution of the corresponding integral equations and empirical relationships. The reliability of designed methods is proved by calculation and experimental investigations heat and hydraulic characteristics of the gas turbine first stage nozzle blade.

***Keywords*—**Multiconnected systems, method of the boundary integrated equations, splines, neural networks.

I. Introduction

THE development of aviation gas turbine engines (AGTE) at the present stage is mainly reached by assimilation of high values of gas temperature in front of the turbine (T_Γ). The activities on gas temperature increase are conducted in several directions. Assimilation of high (T_Γ) in AGTE is however reached by refinement of cooling systems of turbine blades. It is especially necessary to note, that with T_Γ increase the requirement to accuracy of results will increase. In other words, at allowed values of AGTE metal temperature $T_{lim} = (1100...1300K)$, the absolute error of temperature calculation should be in limits ($20-30K$), that is no more than 2-3%.

This is difficult to achieve (multiconnected fields with various cooling channels, variables in time and coordinates boundary conditions). Such problem solving requires application of modern and perfect mathematical device.

II. Problem Formulation

In classical statement a heat conduction differential equation in common case for non-stationary process with distribution of heat in multi–dimensional area (Fourier-Kirchhoff equation) has a kind [1]:

$$\frac{\partial(\rho C_v T)}{\partial t} = div(\lambda\, grad T) + q_v, \qquad (1)$$

where ρ, c_v and λ - accordingly material density, thermal capacity, and heat conduction; q_v - internal source or drain of heat, and T - is required temperature.

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Research has established that the temperature condition of the blade profile part with radial cooling channels can be determined as two-dimensional [2]. Besides, if to suppose constancy of physical properties and absence of internal sources (drains) of heat, then the temperature field under fixed conditions will depend only on the skew shape and on the temperature distribution on the skew boundaries. In this case, equation (1) will look like:

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad (2)$$

When determining particular temperature fields in gas turbine elements are used boundary conditions of the third kind, describing heat exchange between the skew field and the environment (on the basis of a hypothesis of a Newton-Riemann). In that case, these boundary conditions will be recorded as follows:

$$\alpha_0 (T_0 - T_{\gamma_0}) = \lambda \frac{\partial T_{\gamma_0}}{\partial n} \qquad (3)$$

This following equation characterizes the quantity of heat transmitted by convection from gas to unit of a surface of a blade and assigned by heat conduction in a skew field of a blade.

$$-\lambda \frac{\partial T_{\gamma_i}}{\partial n} = \alpha_i (T_{\gamma_i} - T_i) \qquad (4)$$

Equation (4) characterizes the heat quantity assigned by convection of the cooler, which is transmitted by heat conduction of the blade material to the surface of cooling channels: where T_0 - temperature of environment at $i = 0$; T_i - temperature of the environment at $i = \overline{1,M}$ (temperature of the cooler), where M - quantity of outlines; T_{γ_0} - temperature on an outline γ_i at $i = 0$ (outside outline of blade); T_{γ_i} - temperature on an γ_i at $i = \overline{1,M}$ (outline of cooling channels); α_0 - heat transfer factor from gas to a surface of a blade (at $i = 0$); α_i - heat transfer factor from a blade to the cooling air at $i = \overline{1,M}$; λ - thermal conductivity of the material of a blade; n - external normal on an outline of researched area.

III. Problem Solution

At present for the solution of this boundary problem (2)-(4) four numerical methods are used: Methods of Finite Differences (MFD), Finite Element Method (FEM), probabilistic method (Monte-Carlo method), and Boundary Integral Equations Method (BIEM) (or its discrete analog — Boundary Element Method (BEM)).

Let us consider BIEM application for the solution of problem (2)-(4).

3.1. The function $T = T(x, y)$, continuous with the derivatives up to the second order, satisfying the Laplace equation in considered area, including and its outline $\Gamma = \bigcup_{i=0}^{M}\gamma_i$, is harmonic. Consequence of the Grin integral formula for the researched harmonic function $T = T(x, y)$ is the ratio:

$$T(x,y) = \frac{1}{2\pi}\int_{\Gamma}[T_{\Gamma}\frac{\partial(\ell nR)}{\partial n} - \ell nR\frac{\partial T_{\Gamma}}{\partial n}]ds \tag{5}$$

where R - variable at an integration of the distance between point $K(x, y)$ and "running" on the outline k - point; T_{Γ} - temperature on the outline Γ. The temperature value in some point k lying on the boundary is determined (as limiting at approach of point $K(x, y)$ to the boundary)

$$T_k = \frac{1}{2\pi}\left[\int_{\Gamma}T_{\Gamma}\frac{\partial(\ell nR_k)}{\partial n}ds - \int_{\Gamma}\frac{\partial T_{\Gamma}}{\partial n}\ell nR_k ds\right] \tag{6}$$

With allowance of the boundary conditions (2)-(3), after collecting terms of terms and input of new factors, the ratio (6) can be presented as a linear algebraic equation, computed for the point R :

$$\begin{gathered}\varphi_{k1}T_{\gamma 01} + \varphi_{k2}T_{\gamma 02} + \ldots + \varphi_{kn}T_{\gamma 0m} - \\ - \varphi_{k\gamma_0}T_0 - \varphi_{k\gamma_i}T_i - 2\pi T_k = 0\end{gathered} \tag{7}$$

where n is the quantity of sites of a partition of an outside outline of a blade ℓ_{γ_0} (ℓ_{γ_i} $on\ i = 0$) on small sections ΔS_0 (ΔS_i $at\ i = 0$), m is the quantity of sites of a partition of outside outlines of all cooling channels ℓ_{γ_i} $(i = \overline{1, M})$ on small sections ΔS_i.

Let us note, that unknowns in the equation (7) except the unknown of true value T_k in the k point are also mean on sections of the outlines partition ΔS_0 and ΔS_i temperatures $T_{\gamma 01}, T_{\gamma 02}, \ldots, T_{\gamma 0m}$ and $T_{\gamma i1}, T_{\gamma i2}, \ldots, T_{\gamma im}$ (total number $n + m$).

From a ratio (7), we shall receive the required temperature for any point, using the formula (5):

$$\begin{gathered}T(x,y) = \frac{1}{2\pi}[\varphi_{k1}T_{\gamma 01} + \varphi_{k2}T_{\gamma 02} + \ldots + \varphi_{kn}T_{\gamma 0n} + \\ + \ldots + \varphi_{km}T_{\gamma im} - \varphi_{k\gamma_0}T_{cp_0} - \varphi_{k\gamma_i}T_{cp_i}]\end{gathered} \tag{8}$$

where

$$\varphi_{k1} = \int_{\Delta S_{01}}\frac{\partial(\ell nR_k)}{\partial n}ds - \frac{\alpha_{01}}{\lambda_1}\int_{\Delta S_{01}}\ell nR_k ds$$

$$. \quad . \quad . \quad . \quad .$$

$$\varphi_{kn} = \int_{\Delta S_{0m}}\frac{\partial(\ell nR_k)}{\partial n}ds - \frac{\alpha_{0m}}{\lambda_m}\int_{\Delta S_{0m}}\ell nR_k ds$$

$$\varphi_{k\gamma_0} = \frac{\alpha_{01}}{\lambda_1}\int_{\Delta S_{01}}\ell nR_k ds + \ldots + \frac{\alpha_{0n}}{\lambda_n}\int_{\Delta Sn}\ell nR_k ds$$

$$\varphi_{k\gamma_{ii}} = \frac{\alpha_{01}}{\lambda_1}\int_{\Delta S_{i1}}\ell nR_k ds + \ldots + \frac{\alpha_{im}}{\lambda_m}\int_{\Delta S_{im}}\ell nR_k ds$$

In activities [2] the discretization of aniline $\Gamma = \bigcup_{i=0}^{M}\gamma_i$ by a many discrete point and integrals that are included in the equations as logarithmic potentials, was calculated approximately with the following ratios:

$$\int_{\Delta S_{\gamma_i}}\frac{\partial(\ell nR_k)}{\partial n}ds \approx \frac{\partial(\ell nR_k)}{\partial n}\Delta S_{\gamma_i} \quad , \tag{9}$$

$$\int_{\Delta S_{\gamma_i}}\ell nR_k ds \approx \ell nR_k \Delta S_{\gamma_i} \quad , \tag{10}$$

(where $\Delta S_{\gamma_i} \in L = \bigcup_{i=0}^{M}l_i;\ \ l_i = \int_{\gamma_i}ds$) In contrast to [4], we offer to decide the given boundary value problem (2)-(4) as follows. We locate the distribution of temperature $T = T(x, y)$ as follows:

$$T(x,y) = \int_{\Gamma}\rho\,\ell nR^{-1}ds\ , \tag{11}$$

where $\Gamma = \bigcup_{i=0}^{M}\gamma_i$ -smooth closed Jordan curve; M -quantity of cooled channels; $\rho = \bigcup_{i=0}^{M}\rho_i$ - density of a logarithmic potential uniformly distributed on γ_i $S = \bigcup_{i=0}^{M}s_i$.

Thus curve $\Gamma = \bigcup_{i=0}^{M}\gamma_i$ are positively oriented and are given in a parametric kind: $x = x(s)$; $y = y(s)$; $s \in [0, L]$; $L = \int_{\Gamma}ds$.

Using BIEM and expression (11) we shall put problem (2)-(4) to the following system of boundary integral equations:

$$\begin{gathered}\rho(s) - \frac{1}{2\pi}\int_{\Gamma}(\rho(s) - \rho(\xi))\frac{\partial}{\partial n}\ell nR(s,\xi)d\xi = \\ = \frac{\alpha_i}{2\pi\lambda}(T - \int_{\Gamma}\rho(s)\ell nR^{-1}ds)\end{gathered} \ , \tag{12}$$

where

$$R(s,\xi) = ((x(s) - x(\xi))^2 + (y(s) - y(\xi))^2)^{1/2}.$$

For the singular integral operators evaluation, which are included in (12) the discrete operators of the logarithmic potential with simple and double layer are investigated. Their connection and the evaluations in modules term of the continuity (evaluation such as assessments by A. Zigmound are obtained) is shown.

Theorem (main)

Let

$$\int_0 \frac{\omega_\xi(x)}{x} < +\infty$$

And let the equation (12) have the solution $f^* \in C_\Gamma$ (the set of continuous functions on Γ). Then ∃N$_0$∈N= {1, 2…} such that the discrete system ∀N>N$_0$, obtained from (12) by using the discrete double layer potential operator (its properties has been studied), has unique solution $\{\widehat{f}_{j_k}^{(N)}\}, k = \overline{1, m_j}; j = \overline{1, n};$

$$\begin{gathered}|f_{jk}^* - \widehat{f}_{jk}^{(N)}| \le C(\Gamma)(\int_0^{\varepsilon_N}\frac{\omega_\xi(x)\omega_{f^*}(x)}{x}dx + \\ + \varepsilon\int_{\varepsilon_N}^{L/2}\frac{\omega_\xi(x)\omega_{f^*}(x)}{x}dx + \omega_{f^*}(\|\tau_N\|)\int_0^{L/2}\frac{\omega_{f^*}(x)}{x}dx + \\ + \|\tau_N\|\int_{\varepsilon_N}^{L/2}\frac{\omega_{f^*}(x)}{x}dx),\end{gathered}$$

where $C(\Gamma)$ is constant, depending only on $\|\tau_N\|_{N=1}^{\infty}$ --the sequence of partitions of Γ; $\{\varepsilon_N\}_{N=1}^{\infty}$ -- the sequence of positive numbers such that the pair ($\|\tau_N\|,, \varepsilon_N$), satisfies the condition $2 \le \varepsilon\|\tau\|^{-1} \le p$.

Let $\delta \in \left(0, d/2\right)$, where d is diameter Γ, and the splitting τ is that, which is satisfied the condition

$$p' \ge \delta/\|\tau\| \ge 2$$

Then for all $\psi \in C_\Gamma$ (C_Γ - space of all functions continuous on Γ) and $z \in \Gamma$, ($z = x + iy$)

$$\left|(I_{\tau,\delta} f)(z) - \bar{f}(z)\right| \le C(\Gamma)$$

$$\left(\|f\|_C \delta \ln\frac{2d}{\delta} + \omega_f(\|\tau\|) + \|\tau\| \ln\frac{2d}{\delta} + \|f\|_C \omega_Z(\|\tau\|)\right);$$

$$\left|(L_{\Omega,\Gamma} f)(z) - \tilde{f}(z)\right| \le \left(C(\Gamma)\int_0^\Gamma \frac{\omega_f(x)\omega_l(x)}{x^2}dx + \right.,$$

$$\left. + \omega_f(\|\tau\|)\int_\Delta^d \frac{\omega_l(x)}{x}dx + \|\tau\|\int_\Delta^d \frac{\omega_f(x)}{x^2}dx \right)$$

where

$$(L_{\tau,\varepsilon} f)(z) = \sum_{z_{m,e\in\tau(z)}} \left(\frac{f(z_{k,e+1}) + f(z_{k,e})}{2} - f(z) \right) \cdot$$

$$\frac{(y_{k,e+1} - y_{k,e})(x_{k,e} - x) - (x_{k,e+1} - x_{k,e})(y_{k,e} - y)}{\left|z - z_{k,e}\right|^2} + \pi f(z)$$

$(L_{\tau,\varepsilon} f)(z)$ - two-parameter quadrature formula (depending on τ and δ parameters) for logarithmic double layer potential; $\tilde{f}(z)$ - double layer logarithmic potential operator; $C(\Gamma)$ – constant, dependent only from a curve Γ; $\omega_f(x)$ is a module of a continuity of functions f;

$$(I_{\tau,\varepsilon} f)(z) = \sum_{z_{m,e\in\tau(z)}} \frac{f(z_{k,j+1}) + f(z_{k,j})}{2} \cdot$$

$$\cdot \ln \frac{1}{\left|z_{k,j} - z\right|}\left|z_{k,j+1} - z_{k,j}\right|$$

$(I_{\tau,\varepsilon} f)(z)$ - two-parameter quadrature formula (depending on τ and δ parameters) for logarithmic potential simple layer; $\ddot{f}(z)$ -simple layer logarithmic potential operator;

$$z_{k,e} \in \tau,\ z_{k,e} = x_{k,e} + iy_{k,e}$$

$$\tau(z) = \left\{ z_{k,e} \left| z_{k,e} - z \right| > \varepsilon \right\}$$

$$\tau_k = \{z_{k,1}, \dots, z_{k,m_k}\},\ z_{k,1} \le z_{k,2} \le \dots \le z_{k,m_k}$$

$$\|\tau\| = \max_{j=1,m_k}\left|z_{k,j+1} - z_{k,j}\right|$$

Thus are developed effective from the point of view of realization on computers the numerical methods basing on constructed two-parametric quadratute processes for the discrete operators logarithmic potential of the double and simple layer. Their systematic errors are estimated, the methods quadratures mathematically are proved for the approximate solution Fredholm I and II boundary integral equations using Tikhonov regularization and are proved appropriate theorems [1].

3.3. The given calculating technique of the blade temperature field can be applied also to blades with the plug–in deflector. On consideration blades with deflectors in addition to boundary condition of the III kind adjoin also interfaces conditions between segments of the outline partition as equalities of temperatures and heat flows

$$T_v(x,y) = T_{v+1}(x,y) \quad , \tag{13}$$

$$\frac{\partial T_v(x,y)}{\partial n} = \frac{\partial T_{v+1}(x,y)}{\partial n} \quad , \tag{14}$$

where v - number of segments of the outline partition of the blade cross-section; *x, y*- coordinates of segments. At finding of cooler T best values, is necessary to solve the inverse problem of heat conduction.

3.4. For it is necessary at first to find solution of the heat conduction direct problem with boundary condition of the III kind from a gas leg and boundary conditions I kinds from a cooling air leg

$$\frac{\partial T_{\gamma_0}}{\partial n} = \frac{\alpha_0}{\lambda}\left(T_0 - T_{\gamma_0}\right), \tag{15}$$

$$T(x,y)\big|_{\gamma_i} = T_{\gamma i},\ \left(i = \overline{1,M}\right) \tag{16}$$

where $T_{\gamma i}$ -the unknown optimum temperature of a wall of a blade from a leg of a cooling air.

We locate the distribution of temperature $T(x,y)$ as follows

$$T(x,y) = \int_{\gamma_0} \rho_0(s_0)\ln R_0^{-1} ds_0 + \sum_{i=1}^M \int_{\gamma_i} \mu_i(s_i)\frac{\partial}{\partial n}\ln R_i^{-1} ds_i \quad , \tag{17}$$

$$R_i = \left((x - x_i(s_i))^2 + (y - y_i(s_i))^2\right)^{1/2},\ \left(i = \overline{0,M}\right) \tag{18}$$

Using expression (17), (18) we shall put problem (2), (15), (16), to the following system of boundary integral equations Fredholm II

$$\rho_0(s_0) - \frac{1}{2\pi}\int_{\gamma_0}\left(\rho_0(s_0) - \rho_0(\xi_0)\right)\frac{\partial}{\partial n}\ln R_0^{-1}(s_0,\xi_0)d\xi_0 +$$

$$+ \frac{1}{2\pi}\sum_{i=1}^M \int_{\gamma_i} \mu_i(s_i)\frac{\partial}{\partial n}\frac{\partial}{\partial n}\ln R_i^{-1} ds_i =$$

$$= \frac{\alpha_0}{2\pi\lambda}\left(T_0 - \int_{\gamma_0}\rho_0(s_0)\ln R_0^{-1} ds_0 - \sum_{i=1}^M \int_{\gamma_i}\mu_i(S_i)\frac{\partial}{\partial n}\ln R_i^{-1} ds_i \right) \tag{19}$$

$$\mu_i(s_i) - \frac{1}{2\pi}\int_{\gamma_i}\left(\mu_i(s_i) - \mu_i(\xi_i)\right)\frac{\partial}{\partial n}\ln R_i^{-1}(s_i,\xi_i)d\xi_i +$$

$$+ \frac{1}{2\pi}\sum_{\substack{i=1\\ j\ne 1}}^M \int_{\gamma_0}\mu_j(s_j)\frac{\partial}{\partial n}\ln R_j^{-1} ds_j +$$

$$+ \frac{1}{2\pi}\int_{\gamma_0}\rho_0(s_0)\ln R_0^{-1} ds_0 = T_{\gamma i} \qquad \left(i = \overline{1,M}\right), \tag{20}$$

where

$$R_i(s_i,\xi_i) = \left((x(s_i) - x_i(\xi_i))^2 + (y(s_i) - y_i(\xi_i))^2\right)^{1/2},\ \left(i = \overline{0,M}\right) \tag{21}$$

and integral

$$\int_{\gamma_i} \mu_i(s_i) \frac{\partial}{\partial n} \frac{\partial}{\partial n} \ln R_i^{-1} ds_i \qquad (22)$$

is not singular.

3.5. The multiples computing experiments with the using BIEM for calculation the temperature fields of nozzle and working blades with various amount and disposition of cooling channels, having a complex configuration, is showed, that for practical calculations in this approach, offered by us, the discretization of the integrations areas can be conducted with smaller quantity of discrete points. Thus the reactivity of the algorithms developed and accuracy of evaluations is increased. The accuracy of temperatures calculation, required consumption of the cooling air, heat flows, losses from cooling margins essentially depends on reliability of boundary conditions, included in calculation of heat exchange.

3.6. Piece-polynomial smoothing of cooled gas-turbine blade structures with automatic conjecture is considered: the method of the least squares, device spline, smooth replenishment, and neural nets are used.

3.6.1. Let the equation of the cooled blade outline segments is the third degree polynomial:

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \qquad (23)$$

The equation of measurements of the output coordinate has a kind:

$$Z_y = a_0 + a_1 x + a_2 x^2 a_3 x^3 + \delta_y \qquad (24)$$

where $Zy = \| z_{1y}, z_{2y}, \ldots, z_{ny} \|^T$ - vector of measurements of output coordinate, n-amount of the points in the consideration interval. For coefficients of polynomial (23) estimate the method of the least squares of the following kind is used

$$\hat{\theta} = (X^T X)^{-1}(X^T Z_y), \qquad (25)$$

$$D_{\hat{\theta}} = (X^T X)^{-1} \sigma^2, \qquad (26)$$

where

$$X = \begin{Vmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ \ldots & \ldots & \ldots & \ldots \\ 1 & x_n & x_n^2 & x_n^3 \end{Vmatrix} \text{ - structural matrix;}$$

$D_{\hat{\theta}}$ - dispersion matrix of errors; $\hat{\theta} = \| a_0, a_1, a_2, a_3 \|^T$ - vector of estimated coefficients.

Estimations of coefficients for the first segment is received with using formula (25). Beginning with second segment, the θ vectors components is calculated on experimental data from this segment, but with the account of parameters found on the previous segments. Thus, each subsequent segment of the blade cross-section outline we shall choose with overlapping. Thus, it is expedient to use the following linear connections between the estimated parameters of the previous segment $\hat{\theta}_{N\ 1}$ and required $\hat{\theta}_N$ for N-th segment:

$$A\theta_N = V, \qquad (27)$$

$$A = \begin{bmatrix} 1 & x_e & x_e^2 & x_e^3 \\ 0 & 1 & 2x_e & 3x_e^2 \\ 0 & 0 & 2 & 6x_e \end{bmatrix}, \qquad (28)$$

$$V = \begin{bmatrix} \hat{a}_{0N-1} + \hat{a}_{1N-1} x_e + \hat{a}_{2N-1} x_e^2 + \hat{a}_{3N-1} x_e^3 \\ \hat{a}_{1N-1} + \hat{a}_{2N-1} x_e + 3\hat{a}_{3N-1} x_e^2 \\ 2\hat{a}_{2N-1} + 6\hat{a}_{3N-1} x_e \end{bmatrix}, \qquad (29)$$

e=(N-1)(n-L); L- number points of overlapping.

The expressions (27)-(29) describe communications, which provide joining of segments of interpolation on function with first and second degrees.

Taking into account the accuracy of measurements, the problem of defining unknown coefficients of the model in this case can be formulated as a problem conditional extremum: minimization of the quadratic form $(Z_y - X\theta)^T \sigma^2 I (Z_y - \theta)$ under the limiting condition (27). Here I is a individual matrix.

For the solution of such problems, usually are using the method of Lagrange uncertain multipliers. In result, we shall write down the following expressions for estimation vector of coefficients at linear connections presence (27):

$$\tilde{\theta}^T = \hat{\theta}^T + (V^T - \hat{\theta}^T A^T)[A(X^T X)^{-1} A^T]^{-1} A (X^T X)^{-1} \qquad (30)$$

$$D_{\tilde{\theta}} = D_{\hat{\theta}} - (X^T X)^{-1} A^T [A (X^T X)^{-1} A^T]^{-1} A (X^T X)^{-1} \sigma^2 \qquad (31)$$

Substituting matrixes A and X and vectors Z_y and V in expressions (25), (26), (30), and (31), we receive estimations of the vector of coefficients for segment of the cooled blade section with number N and also the dispersing matrix of errors.

As a result of consecutive application of the described procedure and with using of experimental data, we shall receive peace-polynomial interpolation of the researched segments with automatic conjecture.

Research showed that optimum overlapping in most cases is the 50%-overlapping.

3.6.2. Besides peace-polynomial regression exist interpolation splines which represent polynomial (low odd degrees - third, fifth), subordinated to the condition of function and derivatives (first and second in case of cubic spline) continuity in common points of the next segments.

If the equation of the cooled gas-turbine blades profile is described cubic spline submitted in obvious polynomial kind (23), the coefficients a_0, a_1, a_2, a_3 determining j-th spline, i.e. line connecting the points $Z_j = (x_j, y_j)$ and $Z_{j+1} = (x_{j+1}, y_{j+1})$, are calculating as follows:

$$\begin{aligned} a_0 &= z_j; \\ a_1 &= z_j'; \\ a_2 &= z_j''/2 = 3(z_{j+1} - z_j)h_{j+1}^{-2} - 2z_j' h_{j+1}^{-1} - z_{j+1}' h_{j+1}^{-1}; \\ a_3 &= z_j'''/6 = 2(z_j - z_{j+1})h_{j+1}^{-3} + z_j' h_{j+1}^{-2} + z_{j+1}' h_{j+1}^{-2}; \end{aligned} \qquad (32)$$

where $h_{j+1} = |z_{j+1} - z_j|$, $j = \overline{1, N-1}$.

3.6.3. Let us consider other way smooth replenishment of the cooled gas-turbine blade profile on the precisely measured meaning of coordinates in final system of discrete

points, distinguishing from spline-function method and also from the point of view of effective realization on computers.
Let equation cooled blades profile segments are described by the multinomial of the third degree of the type (23). By taking advantage the smooth replenishment method (conditions of function smooth and first derivative are carried out) we shall define its coefficients:

$$\begin{aligned} a_0 &= z_j; \\ a_1 &= (z_{j+1} - z_j)h_{j+1}^{-1}; \\ a_2 &= -((z_{j+2} - z_{j+1})h_{j+2}^{-1} + (z_{j+1} - z_j)h_{j+1}^{-1})h_{j+1}^{-1}; \\ a_3 &= ((z_{j+2} - z_{j+1})h_{j+2}^{-1} - (z_{j+1} - z_j)h_{j+1}^{-1})h_{j+1}^{-2}; \end{aligned} \quad (33)$$

$j=\overline{1, N-1-S}$, $S=1$.
If it's required carry out conditions of function smooth first and second derivatives, i.e. corresponding to cubic splines smooth, we shall deal with the multinomial of the fifth degree (degree of the multinomial is equal 2S + 1, i.e. S = 2).
The advantage of such approach (smooth replenishment) is that it's not necessary to solve system of the linear algebraic equations, as in case of the spline application, though the degree of the multinomial is higher 2.

3.6.4. A new approach of mathematical models' parameters identification is considered. This approach is based on Neural Networks (Soft Computing) [7-9].
Let us consider the regression equations:

$$Y_i = \sum_{j=1}^{n} a_{ij} x_j ; i = \overline{1, m} \quad (34)$$

$$Y_i = \sum_{r,s} a_{rs} x_1^r x_2^s ; r = \overline{0, l}; s = \overline{0, l}; r + s \le l \quad (35)$$

where a_{rs} are the required parameters (regression coefficients).
The problem is put definition of values a_{ij} and a_{rs} parameters of equations (34) and (35) based on the statistical experimental data, i.e. input x_j and x_1, x_2, output coordinates Y of the model.
Neural Network (NN) consists from connected between their neurons sets. At using NN for the solving (34) and (35) input signals of the network are accordingly values of variables $X = (x_1, x_2, ..., x_n)$, $X = (x_1, x_2)$ and output Y.
As parameters of the network are a_{ij} and a_{rs} parameters' values.
At the solving of the identification problem of parameters a_{ij} and a_{rs} for the equations (34) and (35) with using NN, the basic problem is training the last.

We allow, there are statistical data from experiments. On the basis of these input and output data we making training pairs (X, T) for network training. For construction of the model process on input of NN input signals X move and outputs are compared with reference output signals T.
After comparison, the deviation value is calculating by formula

$$E = \frac{1}{2} \sum_{j=1}^{k} (Y_j - T_j)^2$$

If for all training pairs, deviation value E less given then training (correction) parameters of a network comes to end (Fig. 1). In opposite case it continues until value E will not reach minimum.
Correction of network parameters for left and right part is carried out as follows:

$$a_{rs}^{н} = a_{rs}^{c} + \gamma \frac{\partial E}{\partial a_{rs}},$$

where $a_{rs}^{c}, a_{rs}^{н}$ are the old and new values of NN parameters and γ is training speed.
The structure of NN for identifying the parameters of the equation (34) is given on Fig. 2.

IV. Results

The developed techniques of profiling, calculation of temperature fields and parameters of the cooler in cooling systems are approved at research of the gas turbine Ist stage nozzle blades thermal condition. Thus the following geometrical and regime parameters of the stage are used: step of the cascade - $t = 41.5 мм$, inlet gas speed to cascade - $V_1 = 156 м/s$, outlet gas speed from cascade - $V_2 = 512 м/s$, inlet gas speed vector angle - $\alpha_1 = 0.7^0$, gas flow temperature and pressure: on the entrance to the stage - $T_г^* = 1333\,K$, $p_г^* = 1.2095 \cdot 10^6\,Pa$, on the exit from stage - $T_{г1} = 1005\,K$, $p_{г1} = 0.75 \cdot 10^6\,Pa$; relative gas speed on the exit from the cascade - $\lambda_{1ad} = 0.891$.
The geometrical model of the nozzle blades (Fig. 3), diagrams of speed distributions V and convective heat exchange local coefficients of gas $\alpha_г$ along profile contour (Fig. 4) are received.

The geometrical model (Fig. 5) and the cooling tract equivalent hydraulic scheme (Fig. 6) are developed. Cooler basics parameters in the cooling system and temperature field of blade cross section (Fig. 7) are determined.

V. Conclusion

The reliability of the methods was proved by experimental investigations heat and hydraulic characteristics of blades in "Turbine Construction" (Laboratory in St. Petersburg, Russia). Geometric model, equivalent hydraulic schemes of cooling tracks have been obtained, cooler parameters and temperature field of "Turbo machinery Plant" enterprise (Yekaterinburg, Russia) gas turbine nozzle blade of the 1st stage have been determined. Methods have demonstrated high efficiency at repeated and polivariant calculations, on the basis of which has been offered the way of blade cooling system modernization.

The application of perfect methods of calculation of temperature fields of elements of gas turbines is one of the actual problems of gas turbine engines design. The efficiency of these methods in the total influences is for operational manufacturability, reliability of engine elements design, and on acceleration characteristics of the engine.

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APPENDIX

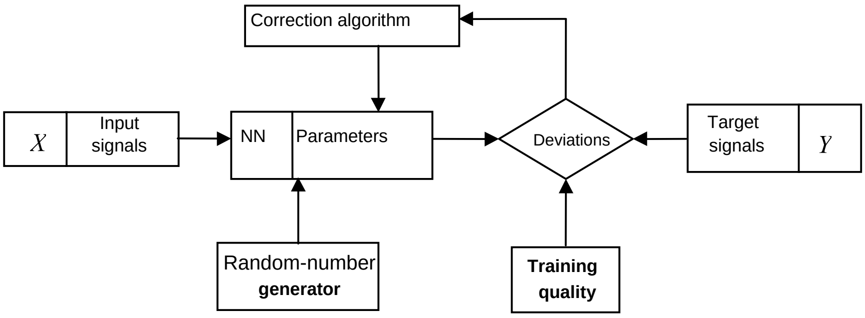

Fig. 1 System for network-parameter (weights, threshold) training (with feedback)

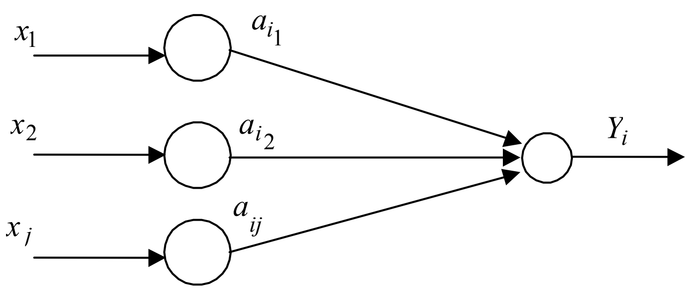

Fig. 2 Neural network structure for multiple linear regression equation

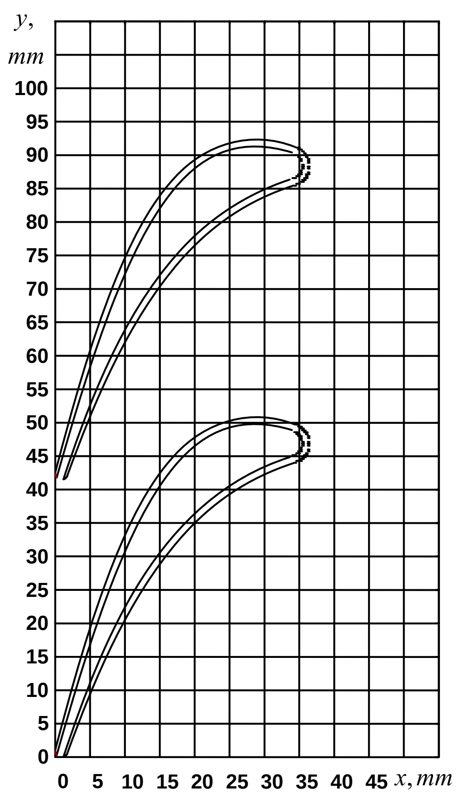

Fig. 3 The cascade of profiles of the nozzle cooled blade

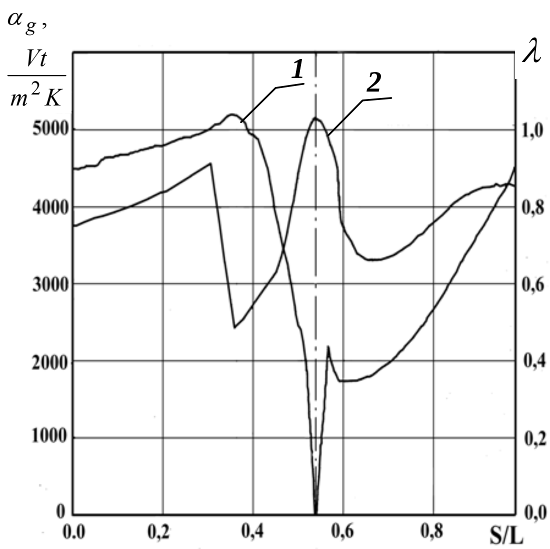

Fig. 4 Distribution of the relative speeds λ (1) and of gas convective heat exchange coefficients α_Γ (2) along the periphery of the profile contour

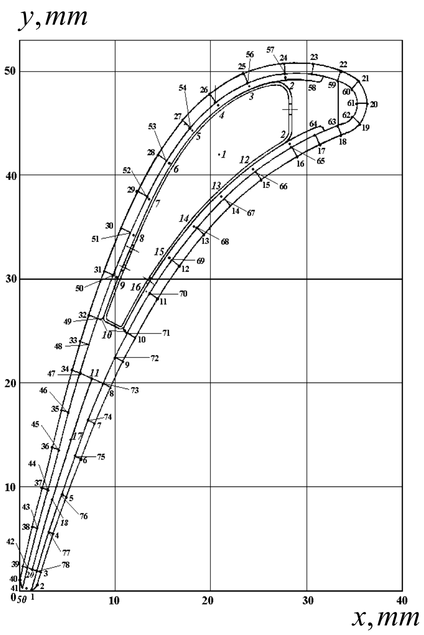

Fig. 5 Geometrical model with foliation of design points of contour (1-78) and equivalent hydraulic schemes reference sections *(1-50)* of the experimental nozzle blade

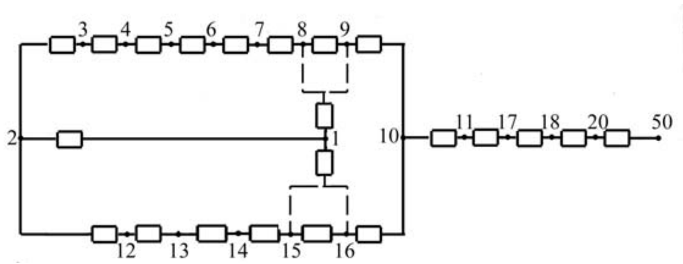

Fig. 6 The equivalent hydraulic scheme of experimental nozzle blade cooling system

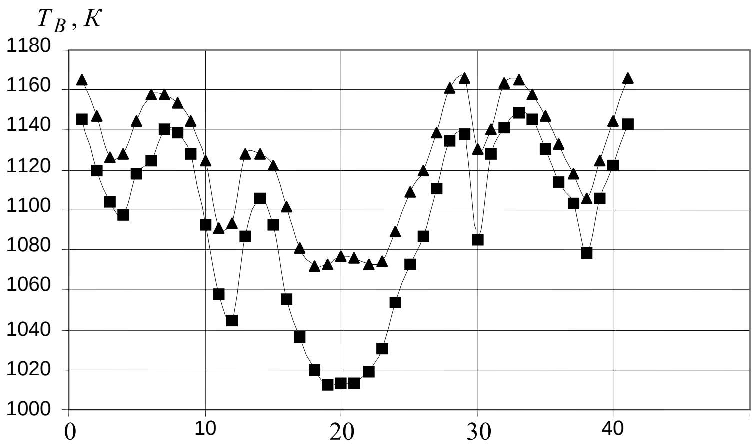

Fig. 7 Distribution of temperature along outside▲) and internal ■) contours of the cooled nozzle blade