International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:12, No:9, 2018

Numerical Approach to a Mathematical Modeling of Bioconvection Due to Gyrotactic Micro-Organisms over a Nonlinear Inclined Stretching Sheet

Madhu Aneja, Sapna Sharma

***Abstract*—The water-based bioconvection of a nanofluid containing motile gyrotactic micro-organisms over nonlinear inclined stretching sheet has been investigated. The governing nonlinear boundary layer equations of the model are reduced to a system of ordinary differential equations via Oberbeck-Boussinesq approximation and similarity transformations. Further, the modified set of equations with associated boundary conditions are solved using Finite Element Method. The impact of various pertinent parameters on the velocity, temperature, nanoparticles concentration, density of motile micro-organisms profiles are obtained and analyzed in details. The results show that with the increase in angle of inclination δ, velocity decreases while temperature, nanoparticles concentration, a density of motile micro-organisms increases. Additionally, the skin friction coefficient, Nusselt number, Sherwood number, density number are computed for various thermophysical parameters. It is noticed that increasing Brownian motion and thermophoresis parameter leads to an increase in temperature of fluid which results in a reduction in Nusselt number. On the contrary, Sherwood number rises with an increase in Brownian motion and thermophoresis parameter. The findings have been validated by comparing the results of special cases with existing studies.

***Keywords*—Bioconvection, inclined stretching sheet, Gyrotactic micro-organisms, Brownian motion, thermophoresis, finite element method.

NOMENCLATURE

α	Thermal diffusivity $\alpha = \frac{k}{\rho C_p}$
δ	angle of inclination
$\frac{Gr}{Re^2}$	Richardson number
γ	average volume of micro-organisms
Ω	Micro-organisms concentration difference parameter
ρ_f	Density of nanofluid
ρ_m	Density of micro-organisms
ρ_p	Density of nanoparticles
σ	Electrical conductivity
τ	Ratio of Effective heat capacitance $\frac{(\rho C)_p}{(\rho C)_f}$
b	Chemotaxis constant
C	Nanoparticle concentration
C_{fx}	Skin friction coefficient
D_B	Brownian Diffusion coefficient
D_m	Micro-organisms diffusivity
D_T	Thermophoresis diffusion coefficient
Ec	Eckert number
g	acceleration due to gravity
k	Thermal conductivity of fluid
Lb	Bioconvection Lewis number
Le	Lewis number
N	Density of motile micro-organisms
n	Nonlinear stretching parameter
N_{nx}	Density number of the motile micro-organisms
N_{ux}	Nusselt number
Nb	Brownian motion parameter
Nr	Buoyancy ratio parameter
Nt	Thermophoresis parameter
Pe	Bioconvection Peclet number
Pr	Prandtl number
q_m	wall mass flux
q_n	wall motile micro-organisms flux
q_w	wall heat flux
Rb	Bioconvection Rayleigh number
Sh_x	Sherwood numbder
T	Temperature
u, v	Interstital velocity component
W_c	Maximum cell swimming speed

M. Aneja is with School Of Mathematics, T.I.E.T, Patiala, Punjab, 147004 India (e-mail: madhu.aneja28@gmail.com).
S. Sharma is with T.I.E.T, Patiala, Punjab, 147004 India (e-mail: sapna.sharma@thapar.edu).

I. INTRODUCTION

THE study of boundary layer flow and heat transfer over nonlinear inclined stretching sheet has gained much interest in last few decades due to its numerous applications in industry and engineering [1]. Some of its significant applications consist of paper production, hot rolling, wire drawing, drawing of plastic films, glass fiber, aerodynamic, extrusion of plastic sheets and in the textile industry. The mechanical properties of the final product strictly depend on the stretching rate and on the rate of cooling.

The study of boundary layer flow over a continuous solid surface moving with constant speed for viscous fluids was initiated by Sakiadis [2]. By considering entrainment of ambient fluid, a different class of boundary layer problem develops, which has a solution substantially different from that of boundary layer flow over a semi-infinite flat plate. The heat and mass transfer on a stretching sheet with suction or blowing for viscous fluids was explored by Gupta and Gupta [3]. Elbashbeshy studied heat transfer over a stretching surface with inconstant heat flux at the surface [4]. The effect of power-law temperature and power-law surface heat flux in the heat transfer characteristics of a continuous linear stretching sheet was examined by Chen and Char [5]. Later, Chiam [6], Hassanien and Gorla [7], Kelson and Desseaux [8] and few others have investigated the steady boundary layer flow of a micropolar fluid due to permeable and non-permeable sheets. Hassanien and Gorla [7] analyzed the heat transfer driven by micropolar fluid from a non-isothermal stretching sheet with suction and blowing.

The investigation mentioned above were confined to Newtonian or Micropolar fluids. The potential importance of nanofluids in industrial applications have motivated studies in this direction. This is due to the fact that the most important issue in the boundary layer flow is the heat transfer characteristics because heat transfer rate affects the product formed in the process. Nanoparticles being solid have high thermal conductivity. Its presence in fluid even in small fraction improve the heat transfer rate. Recently, Khan and Pop [9] investigated the hydrodynamic flow of a nanofluid over a stretching sheet. The analysis of boundary layer flow and heat transfer past a stretching sheet have applications in sheet extrusion for making flat plastic sheets. In this process, it is important to examine the cooling and heat transfer rate for improving the quality of products being formed. The most widely used cooling medium is the conventional fluids such as water and air. However, the rate of heat transfer of such fluids is not adequate for certain sheet materials. Thus, in recent years, it has been proposed to alter flow kinematics that it leads to a slower rate of solidification as compared with water.

Bioconvection refers to the phenomenon of macroscopic convectional movement of fluid due to the density gradient and is constituted by cumulative swimming of motile micro-organisms. The density of base fluid increases as these self-propelled motile micro-organisms swims in a particular direction, thus causing bioconvection [10], [11]. The study of the growth of bioconvection pattern due to motile micro-organisms was done by Pedley et al. [12]. The micro-organisms which cause bioconvection are categorized as oxytactic or chemotaxis, negative gravitaxis and gyrotactic depends on the force causing their motion. The factors which stimulate these micro-organisms are the oxygen concentration gradient, negative gravity, the displacement between the center of buoyancy and mass, respectively [13]. There is a dissimilarity between micro-organisms and nanoparticles. Micro-organisms are self-propelled while nanoparticles motion is due to Brownian motion and Thermophoresis. It is thus perceptible that micro-organisms move independently without affecting the motion of nanoparticles. The point to be noted here that for the survival of micro-organisms in the base fluid, the nanofluid has to be water-based. There are many applications of bioconvection in bio-microsystems, such as enzymes biosensors [14], chip-size micro-devices for evaluating nanoparticle toxicity [15], the critical functional alveolar-capillary interface of the human lung to evaluate toxic and inflammatory responses of the lung to silica nanoparticles [16]. The study related to nanofluid bioconvection is recently developed. Firstly Kuznetsov [17] studied the onset of bioconvection in a horizontal layer filled with a fluid containing both gyrotactic micro-organisms and nanoparticles. After that, he examined the effect of oxytactic micro-organisms on the characteristics of nanofluid flow [18]. Aziz et al. [19] investigated free convection boundary layer regime passing a horizontal flat sheet of a water-based nanofluid in the presence of micro-organisms. The effect of Navier slip and magnetic field on the heat and mass transfer of a nanofluid containing gyrotactic micro-organisms over a vertical surface was studied by Khan et al. [20]. On the other hand, Mutuku and Makinde [23] examined the boundary layer flow of a water-based nanofluid containing motile gyrotactic micro-organisms over a convectively heat stretching sheet in the presence of a uniform magnetic field.

The present analysis is on bioconvection of a nanofluid due to motile gyrotactic micro-organisms over a non-linear inclined stretching sheet. In this transport phenomenon, the effect of gyrotactic micro-organisms over nonlinear inclined stretching sheet is investigated. Viscous dissipation effects are also considered in this problem. The governing non-linear partial differential equations are solved by efficient Finite Element method. Numerical results for various physical parameters are expressed graphically and discussed.

A. Problem Formulation

We consider a steady, laminar, incompressible viscous boundary layer flow of nanofluid containing gyrotactic micro-organisms past a non-linear stretching sheet. The sheet is inclined from the vertical making an acute angle δ. $x-$ direction is considered as the leading edge of the inclined stretching sheet and flow is confined in the $y-$ direction. The schematic model of the problem along with the coordinates system is shown in Fig 1. The base fluid is taken as water for the survival of micro-organisms in natural conditions. During the bioconvection, the sheet is stretched with velocity $u_w = ax^n$, a is a positive constant and n is called the non-linear stretching parameter. Nanoparticles suspension is diluted due to this there is no agglomeration and accumulation of nanoparticles. It is also assumed that nanoparticles do not affect the direction and velocity of micro-organisms swimming. Variation of density in the buoyancy term is determined by Oberbeck-Boussinesq approximation. It is also considered that heat transfer via radiation is negligible and viscous dissipation is included. Moreover, the velocity of motile micro-organisms, base fluid, and nanoparticles are assumed to be similar.

The governing equations of the flow problem based on conservation of mass, momentum, energy, concentration, and density of motile micro-organisms under above-mentioned considerations are expressed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_f\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu_f\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$+(\rho_f\beta(1-C_\infty)(T-T_\infty) - (\rho_p-\rho_f)(C-C_\infty))gcos\delta$$
$$-g\gamma(\rho_m-\rho_f)(N-N_\infty)cos\delta. \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu\alpha}{k}\left(\frac{\partial u}{\partial y}\right)^2$$
$$+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right)\right\} \tag{3}$$

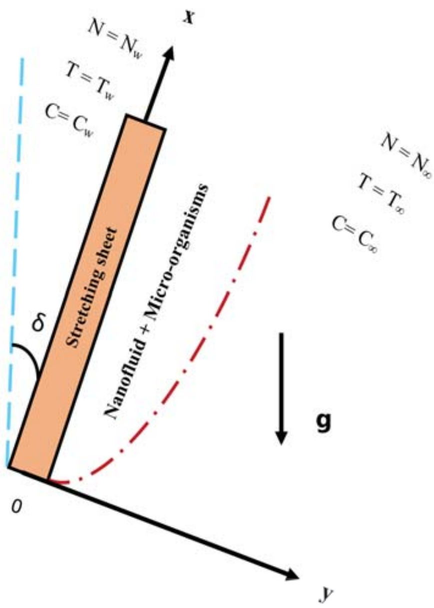

Fig. 1 Schematic model and coordinates system

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{bW_c}{(C_w-C_\infty)}\left[\frac{\partial}{\partial x}\left(N\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right]=$$
$$D_m\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}+2\frac{\partial^2 N}{\partial x\partial y}\right) \quad (5)$$

The last three terms in the momentum equation (2) represent the effect of the inclined stretching sheet. For $\delta = 0^o$, the model reduces to vertical sheet that are discussed in [9], [21]. Note that the angle of inclination δ has explicit effect only on momentum equation while other equations are affected implicitly.

The appropriate boundary conditions are:

$$u = ax^n, \quad v = 0, \quad T = T_w, \quad C = C_w, \quad N = N_w \quad \text{at} \quad x = 0$$

$$u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad C \to C_\infty, \quad N \to N_\infty \quad (6)$$

at $x \to \infty$.

To examine the flow regime, the following similarity variables are introduced [22], [23]

$\eta = y\sqrt{\frac{a(n+1)}{2\nu}}x^{\frac{(n-1)}{2}}$, $u = ax^n f'(\eta)$, $v = -\sqrt{\frac{a(n+1)\nu}{2}}x^{\frac{(n-1)}{2}}\left(f(\eta)+\frac{(n-1)}{(n+1)}\eta f'(\eta)\right)$, $\theta(\eta)=\left(\frac{T-T_\infty}{T_w-T_\infty}\right)$, $\phi(\eta)=\left(\frac{C-C_\infty}{C_w-C_\infty}\right)$, $\chi(\eta)=\left(\frac{N-N_\infty}{N_w-N_\infty}\right)$.

After applying similarity transformations, the equations are modified into set of non-linear, coupled ordinary differential equations:

$$f''' + ff'' - \left(\frac{2n}{n+1}\right)(f')^2$$
$$+\left(\frac{2}{n+1}\right)\left(\frac{Gr}{Re^2}\right)(\theta - Nr\phi - Rb\chi)cos\delta = 0, \quad (7)$$

$$\frac{1}{Pr}\theta'' + \theta'(f + Nb\phi') + Nt(\theta')^2 + Ec(f'')^2 = 0, \quad (8)$$

$$\phi'' + Lef\phi' + \left(\frac{Nt}{Nb}\right)\theta'' = 0, \quad (9)$$

$$\chi'' + Lbf\chi' - Pe[\phi''(\Omega+\chi) + \phi'\chi'] = 0. \quad (10)$$

where,

$Nb = \frac{\tau D_B(C_w-C_\infty)}{\alpha}$, $Nt = \frac{\tau D_T(T_w-T_\infty)}{\alpha T_\infty}$, $\frac{Gr}{Re^2} = \frac{(g\beta(1-C_\infty)(T_w-T_\infty)x^3/\nu^2)}{u_w^2x^2/\nu^2}$, $Nr = \frac{(\rho_p-\rho_f)(C_w-C_\infty)}{\rho\beta(1-C_\infty)(T_w-T_\infty)}$, $Rb = \frac{\gamma(\rho_m-\rho_f)(N_w-N_\infty)}{\rho\beta(1-C_\infty)(T_w-T_\infty)}$, $Pr = \frac{\nu}{\alpha}$, $Le = \frac{\nu}{D_B}$, $Lb = \frac{\nu}{D_m}$, $Ec = \frac{u_w^2}{c_p(T_w-T_\infty)}$, $Pe = \frac{bW_c}{D_m}$, $\Omega = \frac{N_\infty}{(N_w-N_\infty)}$

The boundary conditions for system in similarity space can be written as

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1, \quad \phi(0)=1, \quad \chi(0)=1,$$
$$f'(\infty)=0, \quad \theta(\infty)=0, \quad \phi(\infty)=0, \quad \chi(\infty)=0 \quad (11)$$

The prime denotes differentiation with respect to similarity variable η. After applying transformations various thermophysical parameters are obtained which are defined in the Nomenclature.

Shear stress, heat flux, mass flux and motile micro-organisms flux on the surface are τ_w, q_w, q_m and q_n, respectively and can be expressed as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},$$
$$q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}, \quad q_n = -D_m\left(\frac{\partial N}{\partial y}\right)_{y=0}. \quad (12)$$

For practical purpose, the quantities of physical interest are the skin friction coefficient C_{fx}, Nusselt number Nu_x, Sherwood number Sh_x and density number of the motile micro-organisms Nn_x are defined as

$$C_{fx} = \frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{xq_w}{k(T_w-T_\infty)},$$

$$Sh_x = \frac{xq_m}{D_B(C_w-C_\infty)}, \quad Nn_x = \frac{xq_n}{D_m(N-N_\infty)} \quad (13)$$

Putting values of τ_w, q_w, q_m, q_n from (12) and (13), we obtain

$$Re_x^{1/2}C_{fx} = \sqrt{\frac{n+1}{2}}f''(0), \quad Re_x^{-1/2}Nu_x = -\sqrt{\frac{n+1}{2}}\theta'(0),$$

$$Re_x^{-1/2}Sh_x = -\sqrt{\frac{n+1}{2}}\phi'(0), \quad Re_x^{-1/2}Nn_x = -\sqrt{\frac{n+1}{2}}\chi'(0), \quad (14)$$

In above equations, $Re_x = \frac{U_o x}{\nu}$ refers to Reynolds number.

B. Numerical Technique

The system of non-linear ordinary differential equations (7)-(10) subjected to boundary conditions (11) has been solved numerically by finite element method. Momentum equation (7) is the third order equation and is coupled with energy, nanoparticles concentration and density of motile micro-organisms equations by buoyancy term. So, these equations are solved in the coupled form. In order to convert a three order equation into second order, we assume:

$$f^{'} = h. \quad (15)$$

The system then reduces to:

$$h^{''} + fh^{'} - \left(\frac{2n}{n+1}\right)h^2 + \left(\frac{2}{n+1}\right)\left(\frac{Gr}{Re^2}\right)(\theta - Nr\phi - Rb\chi)cos\delta = 0, \quad (16)$$

$$\frac{1}{Pr}\theta^{''} + \theta^{'}(f + Nb\phi^{'}) + Nt(\theta^{'})^2 + Ec(h^{'})^2 = 0, \quad (17)$$

$$\phi^{''} + Lef\phi^{'} + \left(\frac{Nt}{Nb}\right)\theta^{''} = 0, \quad (18)$$

$$\chi^{''} + Lbf\chi^{'} - Pe[\phi^{''}(\Omega + \chi) + \phi^{'}\chi^{'}] = 0. \quad (19)$$

The corresponding boundary conditions (11) transformed to:

$$f(0) = 0, \quad h(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad \chi(0) = 1,$$
$$h(\infty) = 0, \quad \theta(\infty) = 0, \phi(\infty) = 0, \quad \chi(\infty) = 0 \quad (20)$$

C. Results and Discussion

TABLE I
RESULTS FOR SKIN FRICTION COEFFICIENT FOR $Gr = n = Pe = M = Lb = 1, Ec = Rb = \Omega = Nb = 0.1, Pr = 10, \delta = 0^o$ AND DIFFERENT VALUES FOR Nt AND Nr

Nr	$Nt = 0.05$	$Nt = 0.1$	$Nt = 0.5$
0.3	1.22202	1.22392	1.224
0.5	1.28574	1.2979	1.3075
0.8	1.38412	1.41402	1.44224
0.4	1.45193	1.49629	1.54296
0.5	1.52195	1.58451	1.66415

The non-linear, coupled ordinary differential Eqs. (16 - 19) with boundary conditions (20) has been analyzed numerically using FEM. The important parameters such as angle of inclination δ, Brownian motion parameter Nb, Thermophoresis parameter Nt, Richardson number $\frac{Gr}{Re^2}$, Buoyancy ratio parameter Nr, Eckert number Ec, bioconvection Rayleigh number Rb, Lewis number Le, bioconvection Lewis number Lb, Peclet number Pe, micro-organisms concentration difference parameter Ω are discussed to see their influence on velocity field $f^{'}(\eta)$, temperature profile $\theta(\eta)$, nanoparticles concentration $\phi(\eta)$ and density of gyrotactic micro-organisms $\chi(\eta)$. For the validation of code, comparison has been made for the reduced Nusselt number with those obtained by Khan and Pop [9] for different values of Nb and Nt which are shown in Table II.

The effect of various thermophysical parameters on the velocity profile is described in Figs. 2-4. It can be seen from

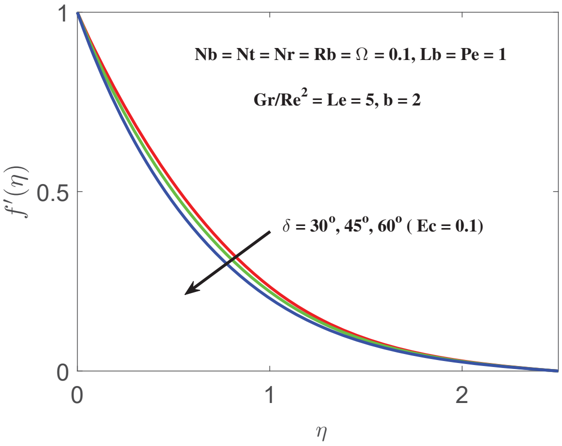

Fig. 2 Impact of angle of inclination δ on $f^{'}(\eta)$

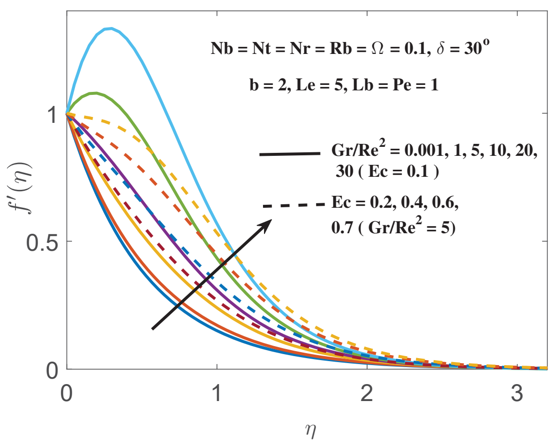

Fig. 3 Impact of $\frac{Gr}{Re^2}$ and Ec on $f^{'}(\eta)$

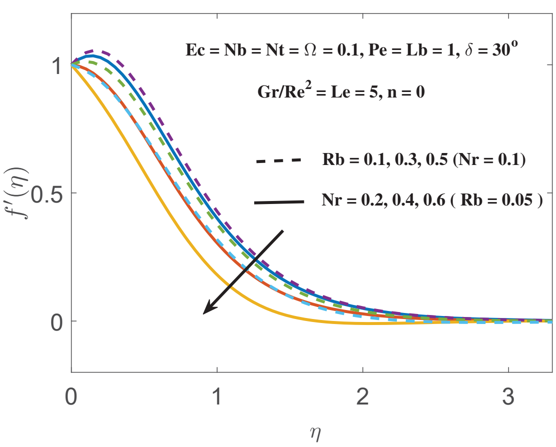

Fig. 4 Impact of Nr and Rb on $f^{'}(\eta)$

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:12, No:9, 2018

TABLE II
COMPARISON OF RESULTS FOR REDUCED NUSSELT NUMBER FOR $Le = Pe = 10$, $n = 1$, $M = Ec = Nr = Rb = Pe = Lb = \Omega = 0, \delta = 0^o$ AND DIFFERENT VALUES FOR Nt AND Nb

	$Nb = 0.1$		$Nb = 0.3$		$Nb = 0.3$	
Nt	Present Results	Khan and Pop [9]	Present Results	Khan and Pop [9]	Present Results	Khan and Pop [9]
0.1	0.956638	0.952493	0.256755	0.2522	0.05596	0.0543
0.2	0.69398	0.6932	0.181657	0.1816	0.0394372	0.0390
0.3	0.524139	0.5201	0.131437	0.1355	0.0299327	0.0291
0.4	0.408426	0.4026	0.100644	0.1046	0.0221499	0.0225
0.5	0.32752	0.3211	0.08291	0.0833	0.0175823	0.0179

TABLE III
RESULTS FOR SHERWOOD NUMBER FOR $Le = Pe = 10$, $n = 1$, $\delta = 0^o$, $M = Ec = Nr = Rb = Pe = Lb = \Omega = 0$ AND DIFFERENT VALUES FOR Nt AND Nb

Nb	$Nt = 0.1$	$Nt = 0.3$	$Nt = 0.5$
0.1	2.11927	2.40624	2.363477
0.2	2.27421	2.48186	2.43115
0.3	2.52916	2.57045	2.48106
0.4	2.78961	2.64549	2.5212
0.5	3.04169	2.70776	2.56326

TABLE IV
RESULTS FOR THE DENSITY OF MOTILE MICRO-ORGANISMS FOR $n = 1, Nb = 1, Ec = 0, Nr = Nt = Gr = 0.5, Lb = Le = 2, Rb = 0.3, \delta = 0^o$, $b = 1$, AND DIFFERENT VALUES FOR Ω AND Pe

Ω	$Pe = 0.3$	$Pe = 0.5$	$Pe = 0.5$
0.1	2.80479	3.20986	3.59853
0.2	2.84746	3.28579	3.70988
0.4	2.93298	3.43801	3.903
0.6	3.01871	3.55134	4.15565
0.8	3.10416	3.70218	4.30139
1.0	3.18983	3.85296	4.515

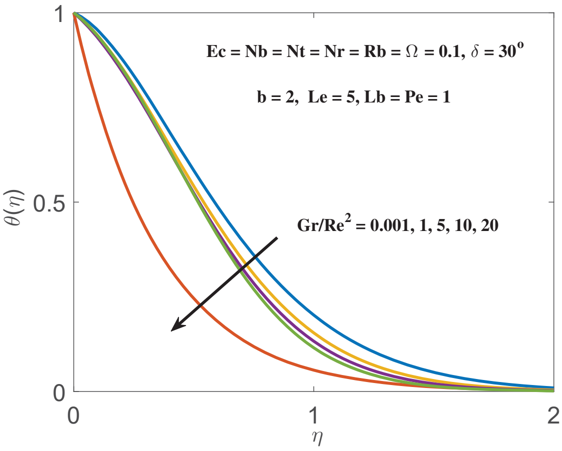

Fig. 6 Impact of $\frac{Gr}{Re^2}$ on $\theta(\eta)$

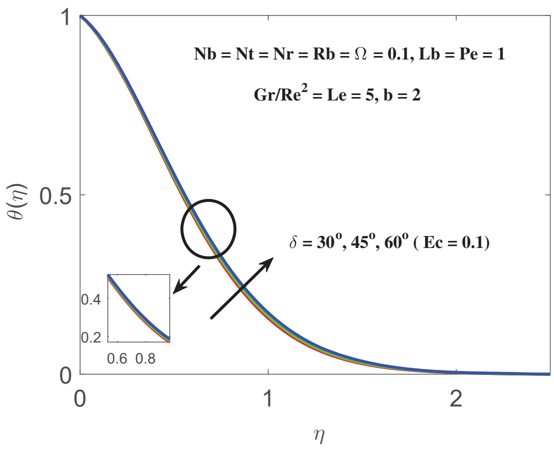

Fig. 5 Impact of angle of inclination δ on $\theta(\eta)$

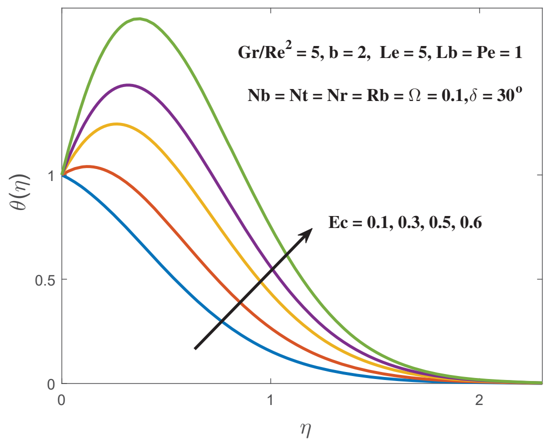

Fig. 7 Impact of Ec on $\theta(\eta)$

Fig. 2 that velocity decreases as the angle of inclination δ increases. This is due to the fact that the angle of inclination δ has a negative effect on buoyancy. Thus momentum boundary layer and velocity decreases. The impact of Richardson number $\frac{Gr}{Re^2}$ and Eckert number Ec on velocity profile is shown in Fig. 3. As the Richardson number increases natural convection becomes dominant. Hence, velocity increases in the adjacency of the sheet due to buoyancy effects. It is also seen in Fig. 3 that velocity also increases with an increase of Eckert number near the wall. This happens because an increase in Ec enhances the viscous dissipation which results into escalating of heating of nanofluid. As the viscosity of nanofluid reduces caused by heating of nanofluid due to viscous dissipation, consequently velocity enhances. It is noticed in Fig. 4 that velocity profile declines due to enhancement in bioconvection Rayleigh number Rb. This behavior is observed because bioconvection dominates the convection due to buoyancy force with increasing bioconvection Rayleigh number Rb. Also, from Fig. 4, it can be seen that flow velocity decreases with an

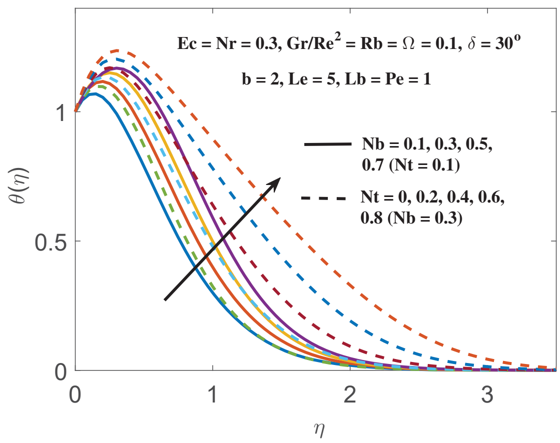

Fig. 8 Impact of Nb and Nt on $\theta(\eta)$

increase in Buoyancy ratio parameter Nr. This is due to the fact that ambient nanoparticles concentration increases which create negative buoyancy.

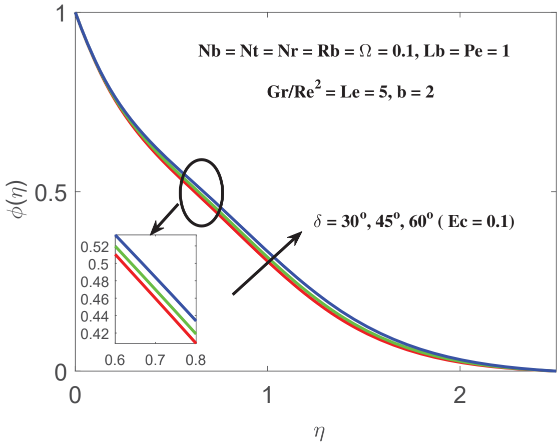

Fig. 9 Impact of angle of inclination δ on $\phi(\eta)$

Figs. 5-8 illustrates the temperature profile for various thermophysical parameters. In Fig 5, it is perceived that at a fixed value of other parameters, the temperature and thermal boundary layer thickness increases by increasing angle of inclination δ. This is because of reduction in buoyancy force as the sheet is inclined. Moreover, it is observed from Fig. 6 that temperature decreases within the thermal boundary layer due to an increase in dimensionless velocity with increasing Richardson number $\frac{Gr}{Re^2}$. Consequently, the thermal boundary layer thickness decreases with increase in Richardson number. It is shown in Fig. 7 that temperature increases due to increase in Eckert number Ec. This is because heat is generated due to viscous dissipation which increases the temperature. From Fig. 8, it reveals that the additional heat is produced by the interaction of nanoparticles and the fluid due to Brownian motion and thermophoresis effects. Consequently, the thermal boundary layer becomes thicker and temperature overshoots

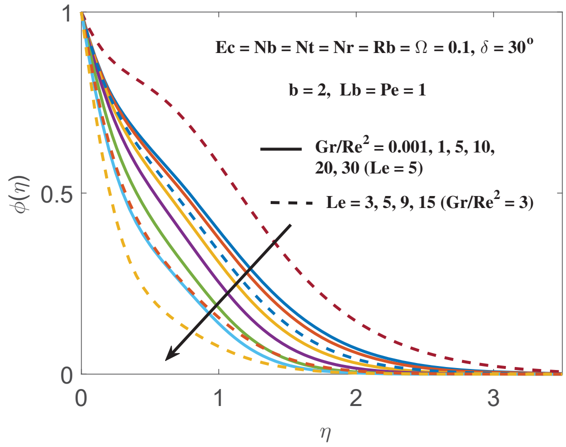

Fig. 10 Impact of $\frac{Gr}{Re^2}$ and Le on $\phi(\eta)$

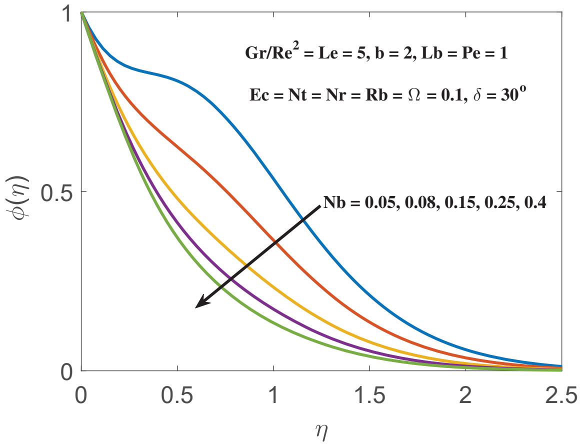

Fig. 11 Impact of Nb on $\phi(\eta)$

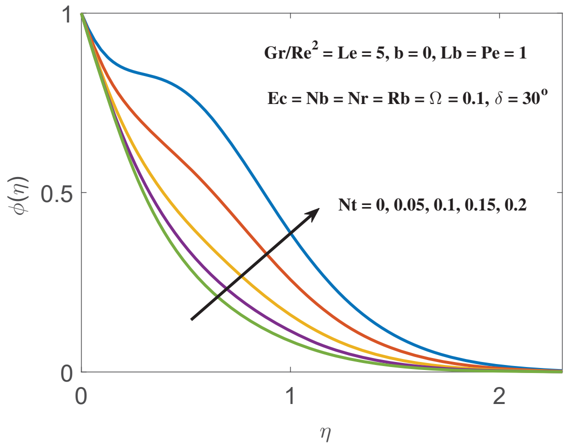

Fig. 12 Impact of Nt on $\phi(\eta)$

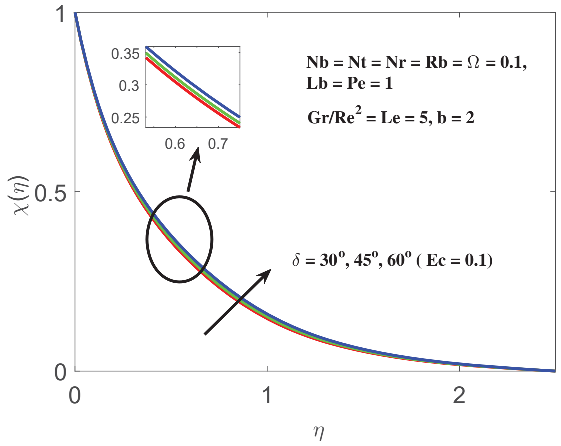

Fig. 13 Impact of angle of inclination δ on $\chi(\eta)$

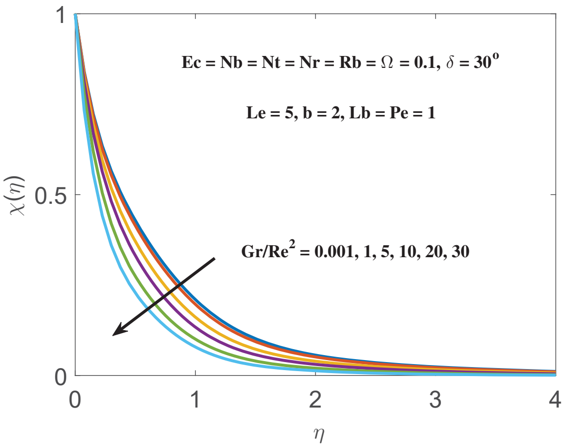

Fig. 14 Impact of $\frac{Gr}{Re^2}$ on $\chi(\eta)$

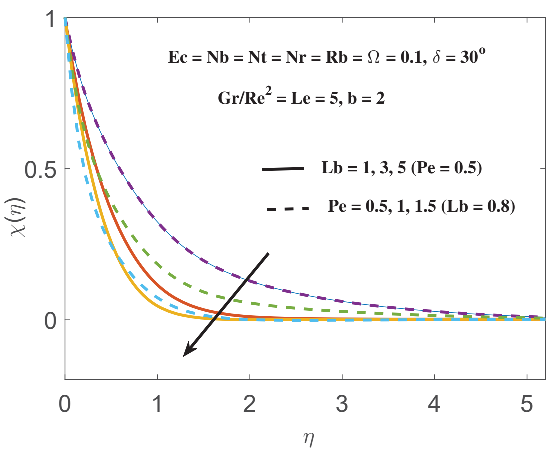

Fig. 15 Effect of Lb and Pe on $\chi(\eta)$

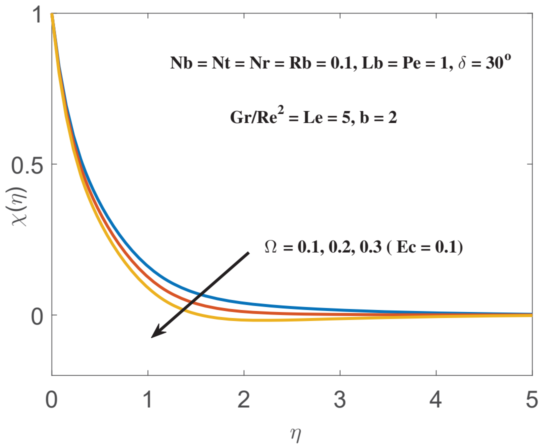

Fig. 16 Effect of Ω on $\chi(\eta)$

in the vicinity of the stretching sheet for increasing value of Nb and Nt.

The variation of nanoparticles concentration for different pertinent parameters such as Richardson number $\frac{Gr}{Re^2}$, Lewis number Le, Brownian motion parameter Nb, Thermophoresis parameter Nt and angle of inclination δ are shown in Figs. 9-12. The increase in angle of inclination δ enhances the concentration of nanoparticles as shown in Fig. 9. This is because when an angle of inclination $\delta = 0^o$, the sheet is in the vertical direction. Hence, the maximum gravitational force acts on the flow. As the angle of inclination elevates the strength of gravitational force decreases, due to this buoyancy force decreases which causes the increase in nanoparticle concentration. In Fig. 10, it can be seen that nanoparticle concentration decreases within the boundary layer with the increase in Richardson number because the velocity increases. The nanoparticle concentration boundary layer also depends on Lewis number Le. It is observed that nanoparticles concentration decreases with increasing Lewis number. In fact, Brownian motion coefficient decreases with arise in transverse distance because of this reason nanoparticles volume fraction decreases promptly. Fig. 11 depicts that the increasing values of Brownian motion parameter Nb cause a lower nanoparticles concentration profile. Fig. 12 elucidates how thermophoresis parameter Nt effects the nanoparticles concentration distribution. By increasing thermophoresis parameter Nt, the nanoparticles concentration increases in the vicinity of the stretching sheet. So, it can be concluded that the nanoparticles boundary layer becomes thick with increasing values of Nt.

The effect of bioconvection and nanofluid parameters on the density of motile gyrotactic micro-organisms are described in Figs. 13-16. It can be seen that variation in density of motile micro-organisms depends upon Richardson number $\frac{Gr}{Re^2}$, bioconvection Lewis number Lb, Peclet number Pe, bioconvection constant Ω, angle of inclination δ. It is noticed in Figs. 13 and 14 that effect of angle of inclination δ and Richardson number $\frac{Gr}{Re^2}$ shows similar behavior on the density of motile micro-organisms as nanoparticles concentration.

Increase in an angle of inclination enhances the density of motile micro-organisms while increasing Richardson number $\frac{Gr}{Re^2}$ decreases the boundary layer thickness of motile micro-organisms, as a result, the motile micro-organisms flux increases. Fig. 15 elucidates that density of motile micro-organisms declines with the rise in bioconvection Lewis number Lb because bioconvection Lewis number behave alike as regular Lewis number Le, with increase in bioconvection Lewis number Lb and Peclet number Pe micro-organisms diffusion decreases. Moreover, it is shown in Fig. 16, bioconvection constant Ω shows the negligible effect on the density of motile micro-organisms.

The effect of various thermophysical parameters on the local skin friction, local Nusselt number, local Sherwood number and local density number of motile micro-organisms is shown in Tables I-IV. Table I shows that the local skin friction coefficient increases with the increase in Buoyancy ratio parameter Nr and Thermophoresis parameter Nt. Since the increment in value of these parameters boosts the resistance of the nanofluid flow containing motile micro-organisms. It is elucidated in Table II that local Nusselt decreases with increase in Brownian motion parameter Nb and Thermophoresis parameter Nt. This is because of zig-zag motion of nanoparticles in fluid give rise to increase in temperature and hence heat transfer decreases. The effect of the pertinent parameter on local Sherwood number is displayed in Table III. It is observed that the rate of mass transfer increases with increase in Brownian motion parameter Nb and Thermophoresis parameter Nt, due to rise in the gradient of nanoparticles concentration profiles. From Table IV, an increase in the motile micro-organisms flux is noted with an increment in micro-organisms concentration difference parameter Ω and Peclet number Pe. This is due to the fact that the concentration of motile micro-organisms decreases within the boundary layer.

II. Conclusion

The boundary layer flow of a water-based nanofluid containing motile gyrotactic micro-organisms past an inclined stretching sheet is studied numerically. A similarity solution is obtained which depends upon various thermophysical parameters. Brownian motion and thermophoresis effects are accounted in the problem being investigated. The main observations of the present study are as follows:

- The dimensionless velocity decreases when the angle of inclination of the stretching sheet increases while the reverse pattern is observed for Richardson number, bioconvection Rayleigh number, Eckert number and Buoyancy ratio parameter.
- Increasing angle of inclination, Eckert number, Brownian motion parameter, Thermophoresis parameter, Buoyancy ratio parameter enhances the temperature near the wall. In contrast, temperature decreases with rising in Richardson number.
- The increment in Richardson number, Lewis number, Brownian motion parameter declines nanoparticles concentration. On the other hand, nanoparticles concentration rises with an increase in angle of inclination, Thermophoresis parameter.
- It is observed that the density of motile micro-organisms increases with enhancement in the angle of inclination and Bioconvection constant whereas opposite behavior is observed with increase in Richardson number, Bioconvection Lewis number and Peclet number.
- The skin friction C_{fx} increases with increase in Thermophoresis parameter Nt and Buoyancy ratio parameter Nr, keeping other pertinent parameters fixed.
- The rate of heat transfer reduces while local Sherwood number rises with an increase in Brownian motion parameter Nb and Thermophoresis parameter Nt.
- Density number of motile micro-organisms raises with enhancement in micro-organisms concentration difference parameter Ω and Peclet number Pe.

Acknowledgments

The authors acknowledge Department of Science and Technology (DST), India for providing financial assistance.

References

[1] E. M. Abo-Eldahab, M. A. E. Aziz, Blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption, International Journal of Thermal Sciences, vol. 43 (7), 2004, pp. 709 – 719.

[2] B. C. Sakiadis, Boundary-layer behavior on continuous solid surfaces: I. boundary-layer equations for two-dimensional and axisymmetric flow, AIChE Journal, vol. 7 (1), 1961, pp. 26–28.

[3] P. S. Gupta, A. S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, The Canadian Journal of Chemical Engineering, vol. 55 (6), 1977, pp. 744–746.

[4] E. M. A. Elbashbeshy, Heat transfer over a stretching surface with variable surface heat flux, Journal of Physics D: Applied Physics, vol. 31 (16), 1998, pp. 19-51.

[5] C.-K. Chen, M.-I. Char, Heat transfer of a continuous, stretching surface with suction or blowing, Journal of Mathematical Analysis and Applications, vol. 135 (2), 1988, pp. 568 – 580.

[6] T. C. Chiam, Micropolar fluid flow over a stretching sheet, ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift fr Angewandte Mathematik und Mechanik, vol. 62 (10), 1982, pp. 565–568.

[7] I. Hassanien, R. S. R. Gorla, Mixed convection boundary layer flow of a micropolar fluid near a stagnation point on a horizontal cylinder, International Journal of Engineering Science, vol. 28 (2), 1990, pp. 153 – 161.

[8] N. Kelson, A. Desseaux, Effect of surface conditions on flow of a micropolar fluid driven by a porous stretching sheet, International Journal of Engineering Science vol. 39 (16), 2001, pp. 1881 – 1897.

[9] W. Khan, I. Pop, Boundary-layer flow of a nanofluid past a stretching sheet, International Journal of Heat and Mass Transfer, vol. 53 (1112), 2010, pp. 2477 – 2483.

[10] N. Hill, T. Pedley, Bioconvection, Fluid Dynamics Research, vol. 37 (12), 2005, pp. 1 – 20, biofluiddynamics.

[11] A. Avramenko, A. Kuznetsov, Stability of a suspension of gyrotactic microorganisms in superimposed fluid and porous layers, International Communications in Heat and Mass Transfer, vol. 31 (8), 2004, pp. 1057 – 1066.

[12] T. J. Pedley, N. A. Hill, J. O. Kessler (1988), The growth of bioconvection patterns in a uniform suspension of gyrotactic micro-organisms, Journal of Fluid Mechanics, vol. 195, 1988, pp. 223237.

[13] S. Ghorai, N. A. Hill (2000), Wavelengths of gyrotactic plumes in bioconvection, Bulletin of Mathematical Biology, vol. 62 (3), 2000, pp. 429–450.

[14] Li H, Liu S, Dai Z, Bao J, Yang X, Applications of nanomaterials in electrochemical enzyme biosensors, Sensors, vol. 9, 2009, pp. 8547–8561.
[15] H. S. Z. A. Munir, J. Wang, Dynamics of capturing process of multiple magnetic nanoparticles in a flow through microfluidic bioseparation system, IET Nanobiotechnol, vol. 3, 2009, pp. 55–64.
[16] D. Huh, B. D. Matthews, A. Mammoto, M. Montoya-Zavala, H. Y. Hsin, D. E. Ingber, Reconstituting organ-level lung functions on a chip, Science, vol. 328 (5986), 2010, pp. 1662–1668.
[17] A. Kuznetsov, The onset of nanofluid bioconvection in a suspension containing both nanoparticles and gyrotactic microorganisms, International Communications in Heat and Mass Transfer, vol. 37 (10), 2010, pp. 1421 – 1425.
[18] A. Kuznetsov, D. Nield, Double-diffusive natural convective boundary-layer flow of a nanofluid past a vertical plate, International Journal of Thermal Sciences, vol. 50 (5), 2011, pp. 712 – 717.
[19] A. Aziz, W. Khan, I. Pop, Free convection boundary layer flow past a horizontal flat plate embedded in porous medium filled by nanofluid containing gyrotactic microorganisms, International Journal of Thermal Sciences, vol. 56, 2012, pp. 48 – 57.
[20] W. Khan, O. Makinde, {MHD} nanofluid bioconvection due to gyrotactic microorganisms over a convectively heat stretching sheet, International Journal of Thermal Sciences, vol. 81, 2014, pp. 118 – 124.
[21] S. A. M. Mehryan, F. Moradi Kashkooli, M. Soltani, K. Raahemifar, Fluid flow and heat transfer analysis of a nanofluid containing motile gyrotactic micro-organisms passing a nonlinear stretching vertical sheet in the presence of a non-uniform magnetic field; numerical approach, PLOS ONE, vol. 11 (6), 2016, pp. 1–32.
[22] P. Rana, R. Bhargava, Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study, Communications in Nonlinear Science and Numerical Simulation, vol. 17 (1), 2012, pp. 212 – 226.
[23] W. N. Mutuku, O. D. Makinde, Hydromagnetic bioconvection of nanofluid over a permeable vertical plate due to gyrotactic microorganisms, Computers & Fluids, vol. 95, 2014, pp. 88 – 97.