

Numerical and Experimental Analysis of Temperature Distribution and Electric Field in a Natural Rubber Glove during Microwave Heating

U. Narumitbowonkul, P. Keangin, P. Rattanadecho

Abstract—The characteristics of temperature distribution and electric field in a natural rubber glove (NRG) using microwave energy during microwave heating process are investigated numerically and experimentally. A three-dimensional model of NRG and microwave oven are considered in this work. The influences of position, heating time and rotation angle of NRG on temperature distribution and electric field are presented in details. The coupled equations of electromagnetic wave propagation and heat transfer are solved using the finite element method (FEM). The numerical model is validated with an experimental study at a frequency of 2.45 GHz. The results show that the numerical results closely match the experimental results. Furthermore, it is found that the temperature distribution and electric field increases with increasing heating time. The hot spot zone appears in NRG at the tip of middle finger while the maximum temperature occurs in case of rotation angle of NRG = 60 degree. This investigation provides the essential aspects for a fundamental understanding of heat transport of NRG using microwave energy in industry.

Keywords—Electric field, Finite element method, Microwave energy, Natural rubber glove.

I. INTRODUCTION

THE heating process is interesting method for produce natural rubber glove (NRG) to vulcanized latex rubber. The current process heating and drying gloves are used to hot-air process. Such a process made heat transfer to the surface, distributing heat unevenly, time-consuming and emissions to the environment.

Microwave energy is one heat source that is an attractive alternative over conventional heating methods because a microwave that penetrates the surface is converted into thermal energy within the material. Several advantages of microwave heating are high speed startup, selective energy absorption, instantaneous electric control, pollution-free environment, change power heating level immediately and high product quality [1]. Thermal conductivity and convection

which on heating or drying in a microwave heating is caused by the change of electromagnetic energy is the kinetic energy of the molecules within the material to be processed [2].

There are the previous studies on experimental and numerical simulation of microwave heating in rubber [3]-[5]. Some research studies of microwave pre-heating in natural rubber using a rectangular wave guide [6]-[7]. Makul and Rattanadecho [8] presented a new method to pre-cure natural rubber-compounding (NRc) by using microwave energy at a frequency of 2.45 GHz with a rectangular wave guide. Khamdaeng et al. [9] were studied deformation behavior of the combined infrared and hot-air vulcanized rubber glove, stress and stretch.

A few studies considered both experimental and numerical simulation of microwave heating in a NRG using a three-dimensional model, especially a detailed study of the parametric effect on heat phenomena. This is because complicated of the dielectric and thermal properties of NRG as affected by the electromagnetic field during heating process.

In this study, the numerical and experimental analysis of temperature distribution and electric field in a NRG during microwave heating at a frequency of 2.45 GHz with microwave oven are investigated. The transient electromagnetic wave propagation equation coupled with the transient heat transfer equation is solved by using the finite element method (FEM). The model of NRG and microwave oven are modeled in a three-dimensional. The influences of position, heating time and rotation angle of NRG on temperature distribution and electric field are considered. The effects of variations of rotation angle of NRG from 0 degree, 30 degree, 60 degree, 90 degree, 120 degree, 150 degree and 180 degree on temperature distribution and electric field are systematically investigated. In addition, the position is chosen from A1 to A5. The presented results provide the necessary aspects for a basic understanding the transport of heat in a NRG during microwave heating in manufacturing of NRG.

II. EXPERIMENTAL APPARATUS

Fig. 1 shows the experimental apparatus for investigated the temperature distribution in a NRG during microwave heating. The frequency of microwave inside the microwave oven is 2.45 GHz. Experimental setup by use ceramic hand-shaped former dipping in natural rubber for thickness 0.5 mm to make NRG. The NRG size M is selected in this study. The NRG placed in center of microwave oven. Dimension of the microwave oven waveguide area is 79 x 43 mm. The

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temperature measured in experimental is using thermocouple K-type during microwave heating [10], [11]. The equipment setup and the position for measuring the temperature in NRG are shown in Figs. 1 (a) and (b), respectively. The recording during microwave operating uses the data logger for record 5 point (A1-A5) of temperature position. The points A1-A5 for measuring the temperature distribution and the dimension of the NRG size M are shown in Fig. 2. The thermocouple K-type is placed inside the NRG for measured temperatures at various heating times.

The dimensions of a microwave oven cavity are $330 \times 315 \times 225$ mm. (axial: x, y, z, respectively) and used for the numerical simulation too. The microwave system generates a monochromatic wave by magnetron that operates a transverse electric (TE_{10}) mode at a frequency of 2.45 GHz [8]. The microwave is transmitted along the z-direction of the microwave oven cavity. The magnetron with a various microwave power output radiates to the interior of microwave oven. The initial temperature of the NRG measured by thermocouples is $T_0 = 24^\circ\text{C}$ at $t = 0$ s. The microwave power input of 100 W is selected in this study. These microwave power input used in the industry for microwave heating process.

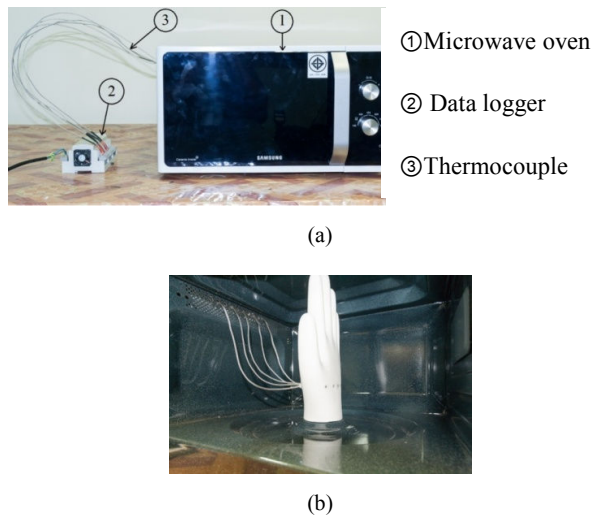


Fig. 1 Experimental apparatus (a) Equipment setup and (b) Position for measuring the temperature in NRG

III. MATHEMATICAL MODELING

The temperature distribution is corresponded to the electric field. This is because when an electric field propagates in the NRG, it is absorbed by the medium and converted into internal heat generation, which causes its temperature to rise. Therefore, the mathematical model consists of transient electromagnetic wave propagation equation coupled with the transient heat transfer equation. The system of governing equations as well as initial and boundary conditions are solved numerically using the FEM via COMSOLTM Multiphysics.

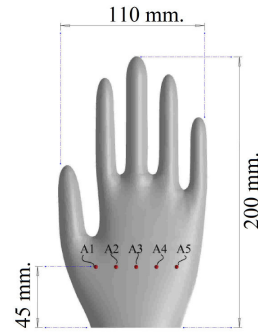


Fig. 2 The points A1-A5 for measuring the temperature distribution and the dimension of NRG

A. Physical Model

A three-dimensional model of NRG and microwave oven in numerical study has same dimensions in experimental study. The physical model for numerical analysis in a NRG during microwave heating is shown in Fig. 3. The NRG is subjected to a uniform microwave via a waveguide. Microwave irradiation can penetrate the NRG surface and is converted into thermal energy within the NRG. This results in a very rapid temperature increase throughout the NRG. In order to obtain a good approximation, a fine mesh is specified in the sensitive areas. To reduce the global mesh size, a mesh convergence test was performed. The number of elements where solution is independent of mesh density is found to be 431,110.

The influences of position, heating time and rotation angle of NRG on temperature distribution and electric field are investigated. The position A1-A5 is considered for measuring the temperature distribution. The heating times during 30 to 180 s are selected. Results are obtained for a range of rotation angle of NRG from 0 to 180 degree. The rotation angles of NRG for numerical analysis are as shown in Fig. 4.

B. Electromagnetic Wave Propagation Analysis

The electromagnetic wave propagation in NRG is modeled on a three-dimensional and is calculated using Maxwell's equations, which describe the interdependence of the electromagnetic field. To simplify the problem, the following assumptions are made:

- 1) The mathematical model of heating of NRG by microwave in the TE_{10} mode is purposed.
- 2) The absorbed energy by air in a microwave oven is negligible.
- 3) The walls of the microwave oven and waveguide are impedance boundary condition.
- 4) The model assumes that dielectric properties of NRG are constant.

The general form of Maxwell's equations for transverse electric wave (TE) mode is derived assuming a harmonic propagation and is simplified to demonstrate the electric field in a NRG [12]:

$$\nabla \times \left(\frac{1}{\mu_r} \nabla \times \vec{E} \right) - k_0^2 \left(\epsilon_r' - \frac{j\sigma}{\omega\epsilon_0} \right) \vec{E} = 0 \quad (1)$$

where \bar{E} is electric field intensity (V/m), μ_r is relative magnetic permeability (H/m), ε_r' is relative dielectric constant, $\varepsilon_0 = 8.8542 \times 10^{-12}$ (F/m) is permittivity of free space, $\omega = 2\pi f$ is angular frequency (rad/s), f is frequency (Hz), σ is electric conductivity (S/m), $\varepsilon_r' = n^2$, n is refractive index and $j = \sqrt{-1}$.

The NRG subjected to microwave radiation in the microwave oven. Therefore, boundary condition for solving electromagnetic wave propagation in the propagation in the TE₁₀ mode as follows:

It is assumed that the NRG is subjected to a uniform microwave power input on the left side corresponded to the position of waveguide of microwave oven in the experimental. Therefore, at the left boundary of the considered domain, an electromagnetic wave propagation simulator employs are the port boundary condition with specified power density, the propagation constant by TE₁₀ mode, as shown in Fig. 3 and as [13]:

$$\beta = \frac{2\pi}{c} \times \sqrt{f^2 - \frac{c^2}{4a^2}} \quad (2)$$

where β is propagation constant, $a = 73$ mm is depth dimension of waveguide, $c = 3 \times 10^8$ m/s is speed of light and $f = 2.45$ GHz is operating frequency of the microwave generator.

The walls surface of the microwave oven and waveguide are impedance boundaries condition.

$$\sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r - j \frac{\sigma}{\omega}}} \hat{n} \times \bar{H} + \bar{E} - (\hat{n} \cdot \bar{E}) \hat{n} = (\hat{n} \cdot \bar{E}) \hat{n} - \bar{E}_s \quad (3)$$

C. Heat Transfer Analysis

Heat transfer analysis of the microwave heating in NRG is modeled in three-dimensional. The mathematical model in this study is shown in Fig. 3. To reduce complexity of the problem, the following assumptions have been offered into the heat transfer analysis:

- 1) Corresponding to electromagnetic field, considering temperature distribution can assume to be three-dimensional.
- 2) There is no phase change of substance in the NRG.
- 3) There is no chemical reaction in the NRG.
- 4) The model assumes that thermal properties of NRG are constant.

The heat transfer analysis is considered only in NRG. The initial condition of NRG is defined as $T_0 = 24^\circ\text{C}$ at $t = 0$ s as corresponds to the initial condition of NRG in experimental. The governing equation describing the heat transfer phenomenon can be written as:

$$\rho C_p \frac{\partial T}{\partial t} - k \nabla^2 T = Q \quad (4)$$

where ρ is density (kg/m³), C_p is heat capacity (J/kg·K), T is temperature (°C), t is time (s), k is thermal conductivity (W/m·K) and Q is external heat source term (W).

In (4), the first and second term on the left-hand side of equation denote transient term and heat conduction term; while, term on the right-hand side of equation denotes external heat source. The external heat source term is equal to the resistive heat generated by the electromagnetic field (microwave power absorbed), which is a function of the electric field, relative dielectric loss factor, loss tangent coefficient and frequency are defined as [8]:

$$Q = 2\pi f \varepsilon_0 \varepsilon_r' (\tan \delta) \bar{E}^2 \quad (5)$$

where $\tan \delta$ is loss tangent coefficient. Our external heat source is evaluated in (5), which is calculated through the electromagnetic wave propagation analysis.

The boundary conditions for analyzing heat transfer are considered as follows:

The outer sides of NRG are considered as electromagnetic heat source boundary condition since it has been influenced by microwave radiation:

$$-\hat{n} \cdot (-k \nabla T) = Q \quad (6)$$

Boundary conditions along the interfaces between NRG and ceramic hand-shaped former are considered as continuity boundary condition. The surface of NRG is assumed to be a thermal insulation boundary condition:

$$-\hat{n} \cdot (-k \nabla T) = 0 \quad (7)$$

In general, the properties of the dielectric can be written in the form of complex permittivity (ε). The complex permittivity is a function of dielectric constant and dielectric loss factor, are defined by [14]:

$$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon_0 (\varepsilon_r' - j\varepsilon_r'') \quad (8)$$

where ε is complex permittivity, ε'' is dielectric constant, ε'' is dielectric loss factor, ε_r' is relative dielectric constant and ε_r'' is relative dielectric loss factor.

TABLE I
THE DIELECTRIC AND THERMAL PROPERTIES OF NRG AND CERAMIC HAND-SHAPED FORMER USED IN THE COMPUTATIONS [15]-[19]

Symbol	QUANTITY	NRG	Ceramic hand-shaped former
σ	Electric conductivity (S/m)	0	0
ε'	Dielectric constant (F/m)	2.2	3.5
ε''	Dielectric loss factor (F/m)	0.1	0.001
k	Thermal conductivity (W/m·K)	0.13	0.24
ρ	Density (kg/m ³)	975	2,200
C_p	Heat capacity (J/kg·K)	1,894	1,004

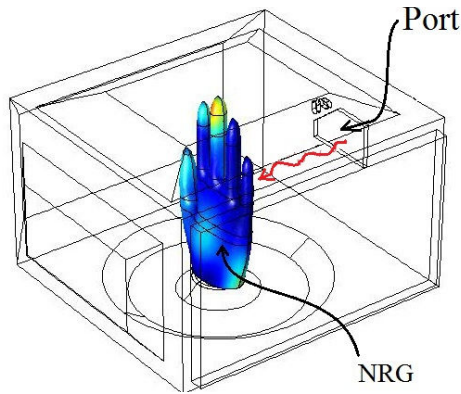


Fig. 3 Physical model for numerical analysis

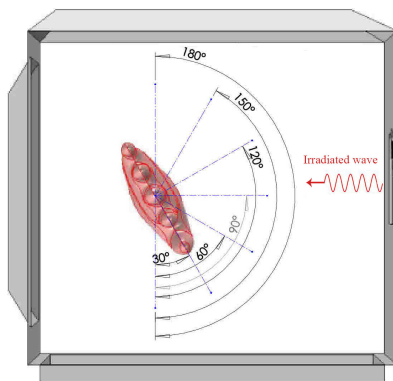


Fig. 4 The rotation angle of NRG for numerical analysis

IV. CALCULATION PROCEDURE

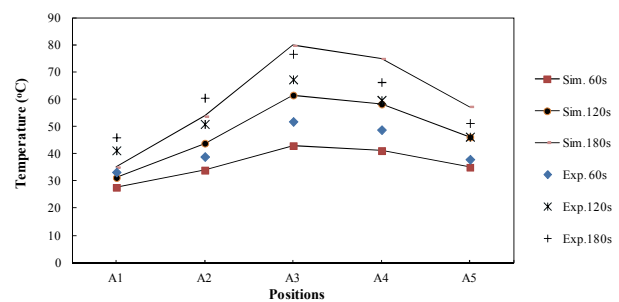
The numerical scheme used in this study is applied to assemble a finite element model. The heat transfer analysis in NRG during microwave heating is expressed through (4)–(7). These equations are coupled to electromagnetic wave propagation equations through (5). Equations (1)–(7) are solved numerically using a FEM via COMSOL™ Multiphysics to demonstrate the temperature distribution and electric field that occurs in NRG exposed to microwave energy. The three-dimensional model of NRG and microwave oven is discretized using triangular elements, and the Lagrange quadratic is used to approximate temperature distribution and electric field variations across each element. The mesh with approximately 431,110 elements is obtained. The model assumes that the dielectric and thermal properties of NRG and ceramic hand-shaped former are constant. The dielectric and thermal properties of NRG and ceramic hand-shaped former are used in the computations are listed in Table I [15]–[19]. The NRG is assumed to be homogeneous and electrically as well as thermally isotropic.

VI. RESULTS AND DISCUSSION

A. The Comparison of NRG Temperature Distribution of Numerical Results with the Experimental Results

In order to verify the accuracy of the present model, the numerical results are validated against the experimental results

with the same conditions. The comparison of NRG temperature distribution of numerical results with the experimental results based on $P = 100$ W and NRG size M at various position and heating time are displayed in Fig. 5. It can be seen that the temperature distributions in NRG of numerical results are in excellent agreement with the temperature distributions in NRG of experimental results. Certain amounts of mismatch between the numerical results and experimental results are caused by the numerical scheme. The temperature distribution from the numerical results and experimental results gradually increase at position point A1 and A2 to a maximum at position point A3 after that the temperature distribution decreases at position point A4 and A5. Fig. 5 also shows that at position point A3 or approximate the center of NRG has the maximum temperature. In addition, Fig. 5 also indicates that an increase in the heating times results in an increase temperature distributions.

Fig. 5 The comparison of NRG temperature distribution of numerical results with the experimental results based on $P = 100$ W and NRG size M at various position and heating time

B. The Effects of Rotation Angle of NRG

Fig. 6 shows the numerical results of temperature distribution and electric field in NRG during microwave heating based on $P = 100$ W and $t = 180$ s at various rotation angle of NRG. The frequency of microwave is 2.45 GHz and NRG size M are used in this simulation. The rotation angle of NRG of 0 degree, 30 degree, 60 degree, 90 degree, 120 degree, 150 degree and 180 degree are considered. It is observed that the electric field of Fig. 6 at various rotation angles of NRG is similar trends. When the electromagnetic field released from the waveguide of microwave oven, the electromagnetic field will induce an electric field. From the figure, the electric field move from the right-hand side to the left-hand side and move into the NRG. Electric field causes the heat on the NRG. This is because when an electric field propagates in the NRG, it is absorbed by the medium and converted into internal heat generation, which causes its temperature to rise. It is found that the hot spot zone appears in NRG at the tip of middle finger while the maximum temperature occurs in case of rotation angle of NRG = 60 degree.

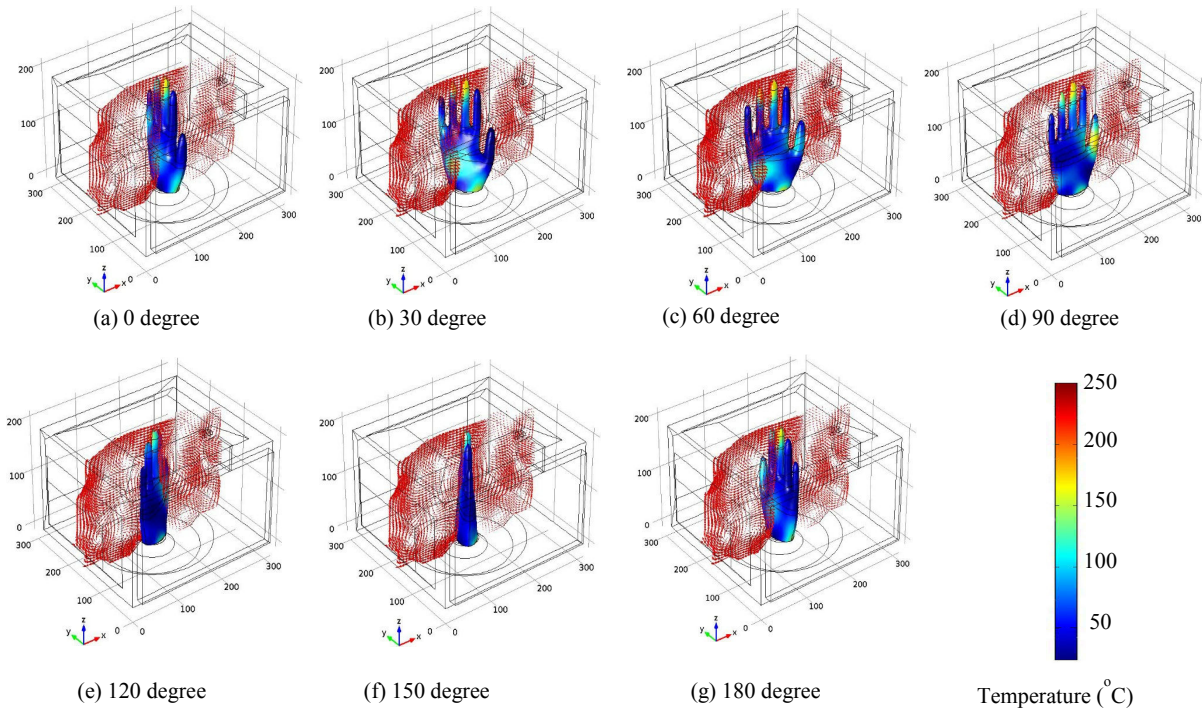


Fig. 6 The electric field and temperature distribution in NRG during microwave heating based on $P = 100$ W and $t = 180$ s at various rotation angle of NRG

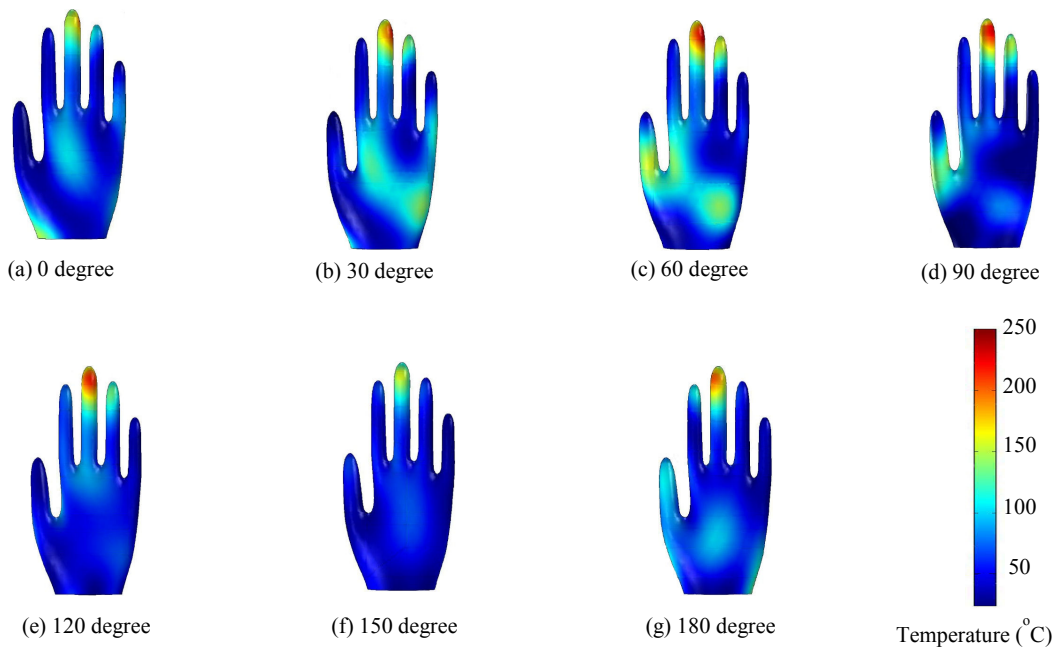


Fig. 7 The temperature distribution in NRG during microwave heating based on $P = 100$ W and $t = 180$ s at various rotation angle of NRG

Fig. 7 shows the temperature distribution in NRG during microwave heating based on $P = 100$ W and $t = 180$ s at various rotation angle of NRG. The frequency of microwave is 2.45 GHz and NRG size M are used in this simulation. The rotation angle of NRG that selected are the same rotation

angle of NRG in Fig. 6. Figs. 7 (a)-(g) show the temperature distribution in NRG at the rotation angle of NRG of 0 degree, 30 degree, 60 degree, 90 degree, 120 degree, 150 degree and 180 degree respectively. It is found that the hot spot zone appears in NRG at the tip of middle finger while the maximum

temperature occurs in case of rotation angle of NRG = 60 degree corresponded to the results in Fig. 6. While, the relationship between temperature distribution and the rotation angle of NRG at various positions is shown in Fig. 8.

VII. CONCLUSIONS

In this study, the numerical and experimental analysis of temperature distribution and electric field in a NRG during microwave heating at a frequency of 2.45 GHz with microwave oven are presented. The transient electromagnetic wave propagation equation coupled with the transient heat transfer equation is solved by using the FEM. A three-dimensional model of NRG and microwave oven are considered in this work. The influences of position, heating time and rotation angle of NRG on temperature distribution and electric field are studied. The numerical model is validated with an experimental study. The results show that the numerical results are in agreement with the experimental results. It is established that the temperature distribution and electric field increases with increasing heating time. The hot spot zone appears in NRG at the tip of middle finger. In addition, it is also found that in case of rotation angle of NRG = 60 degree will result in the maximum temperature than other rotation angles. The presented modeling and fundamental parameters used to study can be provides guidance for the study of heat transport of microwave heating of NRG in industry.

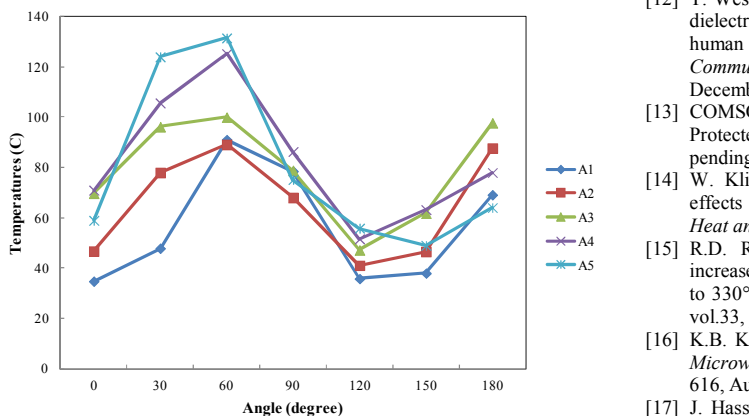


Fig. 8 The relationship between temperature distribution and the rotation angle of NRG at various positions

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REFERENCES

- [1] P. Keangin, T. Wessapan, and P. Rattanadecho, "Analysis of heat transfer in deformed liver cancer modeling treated a microwave coaxial antenna," *Applied Thermal Engineering*, vol.31(16), pp. 3243-3254, November 2011.
- [2] I.W. Turner, and P.G. Jolly, "The effect of dielectric properties on microwave drying kinetics," *Journal of Microwave Power and*

Electromagnetic Energy, H. Poor, *An Introduction to Signal Detection and Estimation*. New York: Springer-Verlag, 1985, ch. 4, vol.25(4), pp. 211-223, 1990.

- [3] V. Bovtun, W. Stark, J. Kelm, V. Porokhonsky, and Y. Yakimenko, "Microwave dielectric properties of rubber compounds undergoing vulcanization," *KGK-Kautschuk and Gummi Kunststoffe*, vol.55, pp. 673-678, 2001.
- [4] D. Martin, D. Ighigeanu, E. Mateescu, G. Craciun, and A. Ighigeanu, "Vulcanization of rubber mixtures by simultaneous electron beam and microwave irradiation," *Radiation Physics and Chemistry*, vol.65(1), pp. 63-65, August 2002.
- [5] H. Zou, L. Shuhuan, S. Yan, W. Hanguang S. Zhang, and M. Tian, "Determining factors for high performance silicone rubber microwave absorbing materials," *Journal of Magnetism and Magnetic Materials*, vol.323(12), pp. 1643-1651, June 2011.
- [6] N. Doo-ngam, P. Rattanadecho, and W. Klinklai, "Microwave pre-heating of natural rubber using a rectangular wave guide (MODE: TE10)," *Songklanakar Journal Science and Technology*, vol.29(6), pp. 1599-1608, November 2007.
- [7] J.F. Gerling, "Waveguide components and configurations for optimal performance in microwave heating system," *Gerling Applied Engineering, Inc.*, pp. 1-8, 2000.
- [8] N. Makul, and P. Rattanadecho, "Microwave pre-curing of natural rubber-compounding using a rectangular wave guide," *International Communication in Heat and Mass Transfer*, vol.37(7), pp. 914-923, April 2010.
- [9] T. Khamdaeng, N. Panyoyai and T. Wongsiriamnuay "Material Parameter of Rubber Glove Vulcanized Using Combine Infrared and Hot-Air Heating," *American Journal of Applied Sciences*, vol. 11 (4), pp. 648-655, 2014
- [10] W. E. Olmstead and M. E. Brodwin, "A model for thermocouple sensitivity during microwave heating, International Journal of Heat and Mass Transfer," *International Journal of Heat and Mass Transfer*, vol. 40(7), pp. 1559-1565, May 1997.
- [11] D. Potter, "Measuring temperature with thermocouples – a tutorial," *National Instruments Corporation*, pp. 1-15, 1996.
- [12] T. Wessapan, S. Srisawatthisukul and P. Rattanadecho, "The effects of dielectric shield on specific absorption rate and heat transfer in the human body exposed to leakage microwave energy," *International Communications in Heat and Mass Transfer*, vol.38, pp. 255-262, December 2010.
- [13] COMSOL Multiphysics Reference Guide 1998–2012 COMSOL Protected by U.S. Patents 7,519,518; 7,596,474; and 7,623,991. Patents pending.
- [14] W. Klinbun, K. Vafai and P. Rattanadecho, "Electromagnetic field effects on transport through porous media," *International Journal of Heat and Mass Transfer*, vol.55(1-3), pp. 325-335, January 2012.
- [15] R.D. Rands, W.J. Ferguson and J.L. Prather, "Specific heat and increases of entropy and enthalpy of the synthetic rubber GR-S from 0° to 330° K," *Journal of Research of the National Bureau of Standards*, vol.33, pp. 63-70, July 1944.
- [16] K.B. Khalid, "Microwave dielectric properties of hevea rubber latex," *Microwave Conference, APMC 92. 1992 Asia-Pacific*, vol.2, pp.611-616, August 1992.
- [17] J. Hassan, K. Khalid, W. Mohammad and D.W. Yusoff, "Microwave dielectric properties of hevea rubber latex in the temperature range of -30°C to 50°C," *Pertanika Journal of Science & Technology*. vol.5(2), pp. 179-190, 1997.
- [18] J. Sridee, "Rheological properties of natural rubber latex, A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Engineering in Polymer Engineering Suranaree University of Technology Academic Year 2006," ISBN 974-533-588-6, 2006.
- [19] P. Ortiz-Serna, R. Diaz-Calleja, M.J. Sanchis, G. Floudas, R.C. Nunes, A.F. Martins and L.L. Visconte, L. L., "Dynamics of natural rubber as a function of frequency, temperature, and pressure," *A dielectric spectroscopy investigation, Macromolecules*, vol.43(11), pp. 5094-5102, May 2010.



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