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Numerical Analysis of Turbulent Natural Convection in a Square Cavity using Large-Eddy Simulation in Lattice Boltzmann Method

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***Abstract*—**In this paper Lattice Boltzmann simulation of turbulent natural convection with large-eddy simulations (LES) in a square cavity which is filled by water has been investigated. The present results are validated by finds of other investigations which have been done with different numerical methods. Calculations were performed for high Rayleigh numbers of Ra=10^8 and 10^9. The results confirm that this method is in acceptable agreement with other verifications of such a flow. In this investigation is tried to present Large-eddy turbulence flow model by Lattice Boltzmann Method (LBM) with a clear and simple statement. Effects of increase in Rayleigh number are displayed on streamlines, isotherm counters and average Nusselt number. Result shows that the average Nusselt number enhances with growth of the Rayleigh numbers.

***Keywords*—**Turbulent natural convection, Large Eddy Simulation, Lattice Boltzmann Method

I. Introduction

ALTHOUGH science has great development in recent years, turbulent flow is still a big problem for scientists and engineers [1]. Many efforts have been performed by now, but because of its complexity it is not still solved. In order to solve turbulent flows, different models are available according to the flow regime and the applied region. One of these models which is considered a lot nowadays is large eddy model. In this model, for the relevant equations integration is done in small distances. So in this way small turbulences which are due to small eddies are omitted. So the equations which are the agent of large eddy's behavior remain. The effect of small eddies on large eddies is considered in the way they are modeled in equations.

In recent decade many engineers focused on Lattice Boltzmann to simulate the fluid flow and heat transfer.

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In conflict with computational fluid dynamic (CFD), Lattice Boltzmann method is based upon microscopic model and mesoscopic kinetic equation, that set of particle behavior in a system is used for simulating continuum of the system.

In Lattice Boltzmann method all calculations are explicit and so it is not needed to solve any set of equations. This method has the ability to be paralleled, because of special identity of the calculations. And also it is so applicable since it is not difficult to apply boundary conditions for complex geometries. The important usages can be: simulation of fluid flow and heat transfer in problems for flows encountering difficult boundary conditions (porous media, moving cycloid surfaces, etc), multiphase flows, non-Newtonian fluid flow (simulation of blood), turbulent flow and etc. Various investigations have been done for laminar flow in square cavity [2] and also for transient and turbulent regimes [3-5]. In natural convection flow, Rayleigh number less than 106 accounts for laminar flow, and Rayleigh number more than it, accounts for turbulent behavior. Many researchers have been studied the laminar natural convection in a square cavity using Lattice Boltzmann [6-7], but because turbulent flow is so complicated and Lattice Boltzmann method is a novel theory, few verifications have been done in such a case using Lattice Boltzmann for turbulent flow [8]. In this study turbulent flow in a square cavity is verified using Lattice Boltzmann method based on Large Eddy Simulation model, equilibrium distribution functions for temperature field is different with other works and first time that it used to this model, it is observed that acceptable agreement exists between the results and achievements from other analysis of fluid flow.

II. Mathematical Formulation

A. Lattice Boltzmann Method

The numerical solution based on Boltzmann equation is named Lattice Boltzmann method which was initially proposed in 1986 by Ferish et al[9] . After that more strongly in 1988 Mc namara and Vzeneti [10], in 1989 Higura and Jimens [11], in 1991 Koelman [12] and in 1992 Chen et al [13] improved it. In Lattice Boltzmann method f and g are two functions called as flow distribution function and temperature distribution function respectively. These functions are utilized to obtain macroscopic characteristics of the flow like velocity, pressure, temperature and etc.In this paper a square grid and D2Q9 model is used for both flow and temperature functions. By detachment of Navier-Stocks equations, governing equations for flow and temperature functions are as follow:

For flow function:

$$f_i(x+c_i\Delta t, t+\Delta t) - f_i(x,t) = \frac{1}{\tau_m}\left[f_i(x,t) - f_i^{eq}(x,t)\right] + \Delta t c_i F \tag{1}$$

That $f_\alpha^{(eq)}$ is the equilibrium distribution functions, and τ_m is relaxation time, which is described in D2Q9 model like:

$$f_\alpha^{eq} = \rho \cdot w_\alpha \left[1 + \frac{3}{c^2} e_\alpha \cdot u + \frac{9}{2c^4}\left(e_\alpha u\right)^2 - \frac{3}{2c^2} u \cdot u\right] \tag{2}$$

In which W_α are:

$$w_\alpha = \begin{cases} 4/9 & \alpha = 0 \\ 1/9 & \alpha = 1,2,3,4 \\ 1/36 & \alpha = 5,6,7,8 \end{cases} \tag{3}$$

And e_α is defined as:

$$e_\alpha = \begin{cases} (0,0) & \alpha = 0 \\ \left(\cos\left[(\alpha-1)\frac{\pi}{4}\right], \sin\left[(\alpha-1)\frac{\pi}{4}\right]\right)\cdot c & \alpha = 1,2,3,4 \\ \left(\cos\left[(\alpha-1)\frac{\pi}{4}\right], \sin\left[(\alpha-1)\frac{\pi}{4}\right]\right)\sqrt{2}\cdot c & \alpha = 5,6,7,8 \end{cases} \tag{4}$$

In which $c = \delta x/\delta t$, δx and δt are length and time constants in the grid respectively.

In discretized velocity region, the amounts for density and momentum are calculated as follow:

$$\rho = \sum_{\alpha=0}^{8} f_\alpha = \sum_{\alpha=0}^{8} f_\alpha^{eq} \tag{5}$$

$$\rho \cdot \mathrm{u} = \sum_{\alpha=1}^{8} \mathrm{e}_\alpha f_\alpha = \sum_{\alpha=1}^{8} \mathrm{e}_\alpha f_\alpha^{eq} \tag{6}$$

And τ_m is :

$$\tau_m = 3\nu + 1/2 \tag{7}$$

For temperature function:

$$g_\alpha(\mathrm{x}_i, t) = g_\alpha(\mathrm{x}_i, t) - \frac{1}{\tau_h}[g_\alpha(\mathrm{x}_i, t) - g_\alpha^{(eq)}(\mathrm{x}_i, t)] \tag{8}$$

Where heat transfer equilibrium distribution functions for D2Q9 model is:

$$g_\alpha^{(eq)} = T w_\alpha [1 + \frac{3}{\mathbf{c}^2}\mathbf{e}_\alpha . \mathbf{u}] \tag{9}$$

And τ_h is :

$$\tau_h = 3\sigma + 1/2 \tag{10}$$

Where temperature is calculated as:

$$T = \sum_{\alpha=0}^{8} g_\alpha = \sum_{\alpha=0}^{8} g_\alpha^{(eq)} \tag{11}$$

B. Large Eddy Simulation Method

In this model the main aim is obtaining ν_t and $\alpha_t = (\frac{\nu_t}{\mathrm{Pr}_t})$ where Pr_t is turbulent Prandtle number which is assumed to be 4. In order to evaluate ν_t we perform as follow:

$$\nu_t = (C\Delta)^2 \left(\left|\bar{S}\right|^2 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\nabla T . \frac{\vec{g}}{\left|\vec{g}\right|}\right)^{1/2} \tag{12}$$

C is considered as Smagorinsky constant and in this paper it is assumed as 0.1 [14] and Δ is gained from $\Delta = \sqrt{(\Delta x)^2 + (\Delta y)^2}$, Δx and Δy are grid extents in X and Y directions.

For $\left|\bar{S}\right|$ we have:

$$\left|\bar{S}\right| = \sqrt{2\bar{S}_{\alpha\beta}\bar{S}_{\alpha\beta}} \tag{13}$$

$$\bar{S}_{\alpha\beta} = \left(\partial_\alpha \bar{u}_\beta + \partial_\beta \bar{u}_\alpha\right)/2 \tag{14}$$

C. Lattice Boltzmann Method based on Large Eddy Simulation Model

Large eddy model is easily applied in Lattice Boltzmann method the way ν_t affects relaxation time [15-17].

$$\nu_{total} = c_s^2(\tau_m - 0.5) = \nu_0 + \nu_t \tag{15}$$

Where ν_{total} and ν_0 are total viscosity and initial viscosity respectively.

$$\tau_m = \frac{(\nu_0 + \nu_t)}{c_s^2} + 0.5 = \frac{\nu_0}{c_s^2} + 0.5 + \frac{\nu_t}{c_s^2} = \tau_0 + \frac{\nu_t}{c_s^2} \tag{16}$$

To obtain ν_t in Lattice Boltzmann method we have:

$$\left|\bar{S}\right| = \frac{3}{2\tau_m}\left|Q\right| \tag{17}$$

$$Q = \sum_{i=0}^{8} e_{i\alpha} e_{i\beta}(f_i - f_i^{eq}) \tag{18}$$

If we put $\left|\bar{S}\right|$ in equation (12):

$$\nu_t = (C\Delta)^2 \left(\frac{9}{4\tau_m^2}\left|Q\right|^2 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\nabla T . \frac{\vec{g}}{\left|\vec{g}\right|}\right)^{1/2} \tag{19}$$

And if we substitute the above equation in (16):

$$\tau_{total} = \tau_0 + \frac{(C\Delta)^2 \left(\frac{9}{4\tau_m^2}\left|Q\right|^2 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\nabla T . \frac{\vec{g}}{\left|\vec{g}\right|}\right)^{1/2}}{c_s^2} \tag{20}$$

To obtain relaxation time in temperature function equation we have:

$$\tau_h = \tau_{D0} + \frac{\alpha_t}{c_s^2} = \tau_{D0} + \frac{\nu_t/\mathrm{Pr}_t}{c_s^2} \tag{21}$$

Where $\tau_{D0} = \frac{\alpha_0}{c_s^2} + 0.5$

Substituting new relaxation time in equation (1) and (8) yields to Lattice Boltzmann equations based on large eddy model.

D.Boundary conditions for Flow and Temperature

The geometry of the present problem is shown in Fig 1.a.

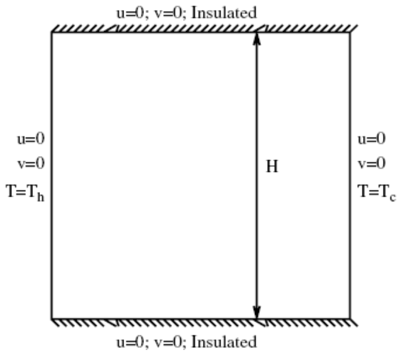


Fig. 1a Geometry of the present study

Implementation of boundary conditions is very important for the simulation. The unknown distribution functions pointing to the fluid zone at the boundaries nodes must be specified. Concerning the no-slip boundary condition, bounce back boundary condition is used on the solid boundaries. In Fig. 1.b the unknown distribution functions, which needs to be determined, are shown as dotted lines. For instance the unknown density distribution functions at the boundary east can be determined by the following conditions:

$$f_{6,n} = f_{8,n} \quad , \quad f_{7,n} = f_{5,n} \quad , \quad f_{4,n} = f_{2,n} \qquad (22)$$

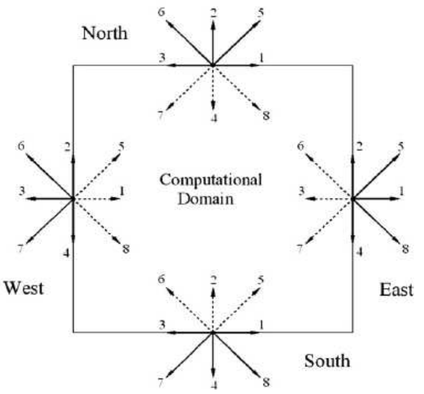


Fig. 1b Boundary conditions with unknown and known nodes

The north and south of the boundaries are adiabatic then bounce back boundary condition is used on them. Temperature at the west and east wall are known, in the west wall $T_H = 1.0$ and in the east wall $T_C = 0$. Since we are using D2Q9, the unknowns distribution functions are g_1, g_5, g_8 at west wall which are evaluated as follows:

$$g_1 = T_H(\omega_1 + \omega_3) - g_3 \qquad (23a)$$

$$g_5 = T_H(\omega_5 + \omega_7) - g_7 \qquad (23b)$$

$$g_8 = T_H(\omega_8 + \omega_6) - g_6 \qquad (23c)$$

And for east wall the unknowns distribution functions evaluated as follows:

$$g_3 = T_C(\omega_1 + \omega_3) - g_1 \qquad (24a)$$

$$g_7 = T_C(\omega_5 + \omega_7) - g_5 \qquad (24b)$$

$$g_6 = T_C(\omega_8 + \omega_6) - g_8 \qquad (24c)$$

III. Results and Discussion

A. Validation

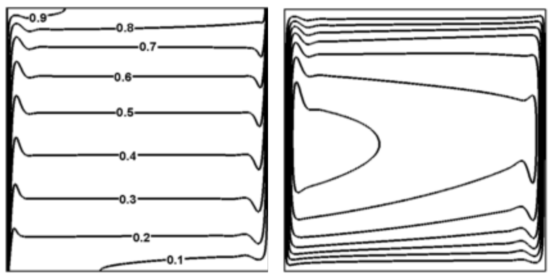
Table I shows the comparison of the average Nusselt numbers for different Rayleigh numbers between present results and finds of Barakoset al [18] and Dixit [19] as a cavity was filled by air with Pr=0.71. Clearly it is seen that the results match previous work.

TABLE I
COMPARISON OF AVERAGE NUSSELT NUMBER WITH PREVIOUS WORKS

Rayleigh Number	Average Nusselt Number(this work)	Average Nusselt Number [18]	Average Nusselt Number [19]
10^8	31.2	32.3	30.5
10^9	58.1	60.1	57.4

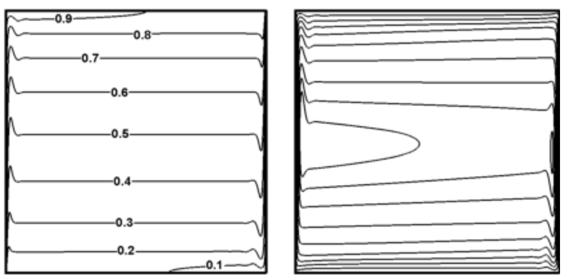
B. Effect of Rayleigh number on the streamline and isotherm

Figs 2 and 3 show the isotherms and the stream lines for Rayleigh numbers of 10^8 and 10^9. As it is clear, results are in good agreement with other methods for numerically analyzing turbulent flow in cavity. When Rayleigh number increases, the symmetry state wastes and the centralization of the streamlines in the core of the cavity tend the hot wall. The heat transfer process increases when Rayleigh number enhances. This process is obvious where two isotherms of T=0.1 and 0.9 move to the cold wall and the hot wall respectively when Rayleigh number rises.



(a) (b)

Fig. 2 The isotherms (a), The streamlines (b) for Ra=10^8



(a) (b)

Fig. 3 The isotherms (a), The streamlines (b) for Ra=10^9

Fig. 4 illustrates the values of the temperature on the axial midline for both Rayleigh numbers. It is obvious that the value enhances with the augmentation of Rayleigh numbers whereas its state gets more stable by increase in Rayleigh numbers. The cause of this phenomenon can be observed in the isotherm counters which they became stable by the augmentation of Rayleigh numbers.

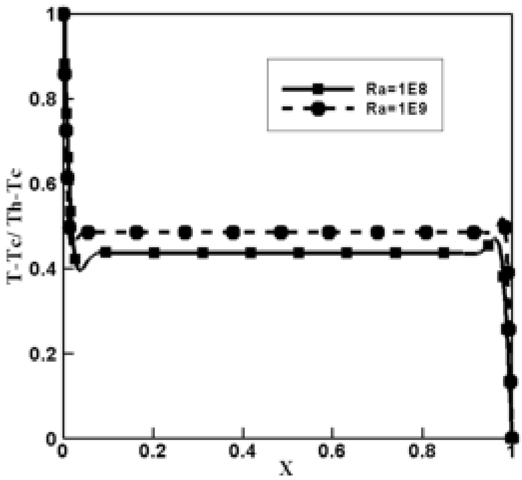


Fig. 4 Temperature distribution at mid-height of cavity for different Ra-values

Figs.5 and 6 show the local Nusselt number on the hot wall and on the cold wall of the cavity respectively for both Rayleigh numbers. The trend of the local Nusselt number is the same for various Rayleigh numbers and just their values increase by the augmentation of Rayleigh number. This trend demonstrates which the most difference of temperature is at the bottom of the cavity whereas the convection process doesn't form completely. On the other hand, Figs.5 and 6 exhibits that the local Nusselt number on the hot wall and on the cold wall have a symmetric manner which shows the accuracy of this method for solving this flow.

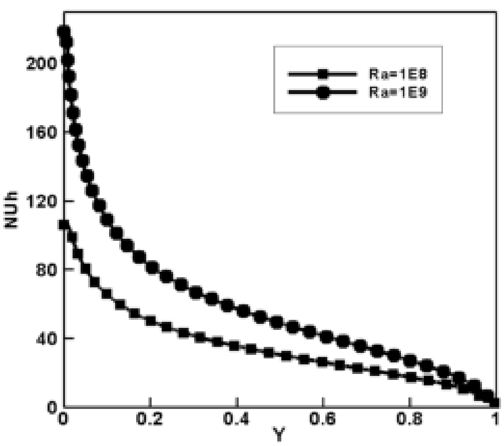


Fig. 5 Nusselt number distributions on the hot wall at Ra=10^8 and 10^9

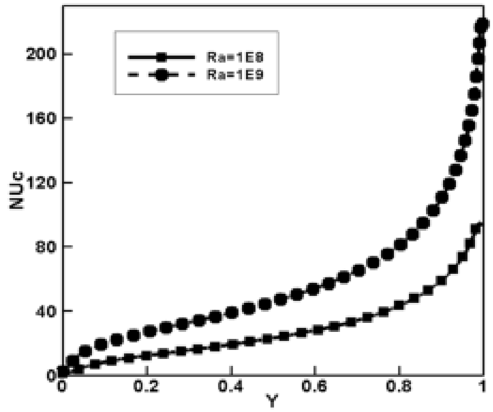


Fig. 6 Nusselt number distributions on the cold wall at Ra=10^8 and 10^9

IV. Conclusions

Turbulent natural convection in a square cavity which are filled with water by Pr=6.2 has been conducted numerically by Lattice Boltzmann Method (LBM). This study has been carried out for Rayleigh numbers of Ra=108 and 109. A proper validation with previous numerical investigations demonstrates that Lattice Boltzmann Method is an appropriate method for turbulent flows problems. The isotherms get more stable when Rayleigh numbers augment. The streamlines in the core of the flow lose the symmetric state which exists at low Rayleigh numbers by increase in Rayleigh numbers.

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