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Numerical Analysis of Plate Heat Exchanger Performance in Co-Current Fluid Flow Configuration

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***Abstract*—**For many industrial applications plate heat exchangers are demonstrating a large superiority over the other types of heat exchangers. The efficiency of such a device depends on numerous factors the effect of which needs to be analysed and accurately evaluated.

In this paper we present a theoretical analysis of a co-current plate heat exchanger and the results of its numerical simulation.

Knowing the hot and the cold fluid streams inlet temperatures, the respective heat capacities $\left(\dot{m}Cp\right)$ and the value of the overall heat transfer coefficient, a *1-D* mathematical model based on the steady flow energy balance for a differential length of the device is developed resulting in a set of *N* first order differential equations with boundary conditions where *N* is the number of channels.For specific heat exchanger geometry and operational parameters, the problem is numerically solved using the shooting method.

The simulation allows the prediction of the temperature map in the heat exchanger and hence, the evaluation of its performances. A parametric analysis is performed to evaluate the influence of the *R*-parameter on the ε-NTU values. For practical purposes effectiveness-NTU graphs are elaborated for specific heat exchanger geometry and different operating conditions.

***Keywords*—**Plate heat exchanger, thermal performance, NTU, effectiveness.

I. Introduction

In many applications such as air conditioning, refrigeration, heat recovery and manufacturing industries, heat exchangers are extensively used to transfer energy from one fluid to another. They are commonly used as boilers, condensers, evaporators or car radiators.

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A simple example of such a device is a plate type heat exchanger. In the majority of the industrial applications, the plate heat exchanger is the design of choice because of its distinguishing and attractive features (easy-to-maintain, compact design, light weight) and because of the many advantages it offers. Minimal maintenance, cost effectiveness and especially high efficiency are the most important criteria that are making studies on plate heat exchanger a big challenge for researchers in this field to develop and produce plate heat exchanger achieving the best possible performance in terms of efficiency and economical considerations [1,2,3]. Many studies on multi channels PHE with parallel flow arrangement have been carried out. Some authors have presented and analytically solved the corresponding thermal modelling [4,5].

The objective of the present paper is to present and numerically simulate a simplified model for a plate type heat exchanger in a parallel flow arrangement which satisfactorily predicts its behaviour and accurately evaluates its performance.

II. Heat Exchanger Analysis

For the prediction of the heat exchanger performance, when only the inlet fluid temperatures, the respective mass flow rates and the value of the overall heat transfer coefficient are known, it is suitable to use the NTU -effectiveness method: it is a performance calculation.

The number of transfer units is

$$NTU = \frac{UA}{C_{\min}} \tag{1}$$

where C_{min} is the minimum value of $\dot{m}Cp$ on either the cold or the hot side and U is global heat transfer coefficient. The effective heat transfer area in the plate heat exchanger is calculated by

A=(number of the plates-*2*) × area per plate

The dimensionless heat exchanger effectiveness is defined as the ratio of the actual rate of heat transfer and the rate of maximum heat that could be transferred from one stream to another.

$$\varepsilon = \frac{C_h\left(T_{h_{in}} - T_{h_{out}}\right)}{C_{\min}\left(T_{h_{in}} - T_{c_{in}}\right)} = \frac{C_c\left(T_{c_{out}} - T_{c_{in}}\right)}{C_{\min}\left(T_{h_{in}} - T_{c_{in}}\right)} \tag{2}$$

The R parameter is defined as the ratio of the minimum and maximum of the operating liquids thermal flow rates.

$$R = \frac{C_{\min}}{C_{\max}} \tag{3}$$

with a known effectiveness, the heat transfer rate can readily be determined from the expression:

$$\Phi = \varepsilon C_{\min}\left(T_{h_{in}} - T_{c_{in}}\right) \tag{4}$$

T refers to liquid temperature. Subscripts "h_{in}" and "c_{in}" denote the hot and cold fluid inlet data, respectively.

This work develops a simple modelling of the energy balances in a plate heat exchanger to calculate its thermal effectiveness and the temperature of both fluids at each point in the PHE channels.

III. Plate Heat Exchanger Description

As shown in Fig. 1, the plate heat exchanger considered in this study comprises a stack of thin metal plates. The heat transfer plates separate the two process fluids. The channel is the space established between two adjacent plates, through which the process fluids are distributed and the heat transfer is carried out. The first and last plates have fluid only on one side.

The heat transfer is carried out as liquids flow co-currently through the channels. The cold liquid becomes warmer and the hot liquid cooler.

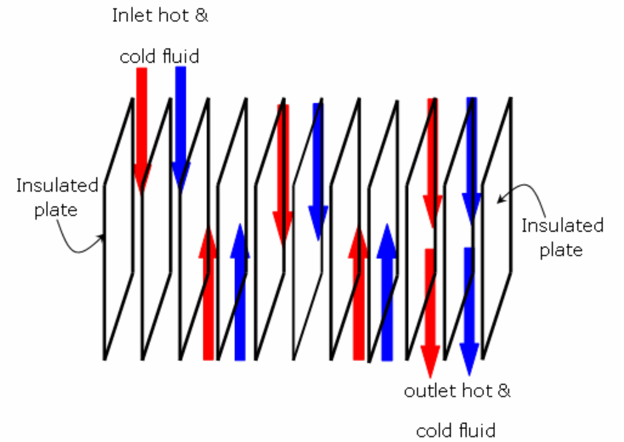

Fig. 1 Simplified schematic of a PHE with nine active plates in parallel flow arrangement.

IV. Mathematical Modelling

Heat exchanger analysis is quite complicated and involves the specification of a set of flow parameters and the geometry.

A. Hypotheses

A set of assumptions must be introduced to develop a simplified mathematical model for the plate heat exchanger. The assumptions are:

- The plate heat exchanger operates under steady state conditions.
- No phase change occurs: both fluids are single phase and are unmixed.
- Heat losses are negligible; the exchanger shell is adiabatic.
- The temperature in the fluid streams is uniform over the flow cross section.
- There is no thermal energy source or sink in the heat exchanger.
- The fluids have constant specific heats.
- The fouling resistance is negligible.

B. Problem formulation and governing equations

While flowing through the PHE the two operating fluids exchange thermal energy through the separating plates.
Applying the steady flow energy conservation equation on an infinitesimal slice within the parallel flow plate heat exchanger (Fig. 2) gives:

For the first channel:

$$\dot{H}_{in} - \dot{H}_{out} + \dot{q}_2 = 0 \tag{5}$$

Where $\left(\dot{q}_2\right)$ is the heat flux per unit length. H is the fluid enthalpy.

$$\dot{q}_2 = UW\Delta x(\langle T_2\rangle - \langle T_1\rangle) \tag{6}$$

W is the plate width.

$$c_1(T_{x+\Delta x} - T_x) = UW\Delta x(\langle T_2\rangle - \langle T_1\rangle) \tag{7}$$

C is the fluid heat flow rate. Subscript 1 refers to the channel number 1.

$$\lim_{\Delta x \to 0} c_1 \frac{T_{x+\Delta x} - T_x}{\Delta x} = UW(T_2 - T_1) \tag{8}$$

$$c_1 \frac{dT_1}{dx} = UW(T_2 - T_1) \tag{9}$$

By the same energy conservation analysis we obtain:
For the second channel

$$c_2 \frac{dT_2}{dx} = UW(T_1 + T_3 - 2T_2) \tag{10}$$

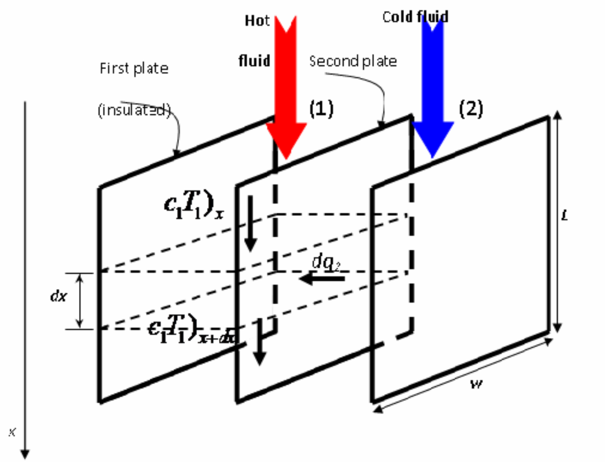

Fig. 1 Control volume for derivation of energy balance in the first channel

Similar equations are also established for the others channels. For a plate heat exchanger of *9* active plates, hence *10* channels, we establish *10* first order differential equations with the corresponding boundary conditions (Table I).

V. Numerical Methods

We have to integrate a set of *10* coupled first-order ordinary differential equations which are required to satisfy boundary conditions at both boundaries of the system ($x=0$ and $x=L$).
The shooting method is the numerical method used to solve this boundary value problem by reducing it to the solution of an initial value problem . It uses the quality controlled Runge-Kutta method to integrate the ODEs and invokes the multidimensional, globally convergent Newton-Raphson to zero n functions of n variables (n is the number of boundary conditions at $x=L$). The functions are obtained by integrating the *10* differential equations from $x=0$ to $x=L$ [8].
The first step consists in loading geometric data and operational parameters for the plate heat exchanger. The computational procedure requires a data file containing the necessary information about the plate dimension, the inlet temperature, the mass flow rate and the specific heats of both fluids and the global heat transfer coefficient table (1). For the numerical illustration, water is used as the cold and hot fluid.

TABLE I
Differential Equations and Boundary Conditions

Channel	Equation	Boundary condition
1	$c_1 \dfrac{dT_1}{dx} = UW(T_2 - T_1)$	T1(x=0)=Th,in
2	$c_2 \dfrac{dT_2}{dx} = UW(T_1 + T_3 - 2T_2)$	T2(x=0)=Tc,in
3	$c_3 \dfrac{dT_3}{dx} = -UW(T_2 + T_4 - 2T_3)$	T3(x=L) = T1(x=L)
4	$c_4 \dfrac{dT_4}{dx} = -UW(T_3 + T_5 - 2T_4)$	T4(x=L) = T2(x=L)
5	$c_5 \dfrac{dT_5}{dx} = UW(T_4 + T_6 - 2T_5)$	T5(x=0) = T3(x=0)
6	$c_6 \dfrac{dT_6}{dx} = UW(T_5 + T_7 - 2T_6)$	T6(x=0) = T4(x=0)
7	$c_7 \dfrac{dT_7}{dx} = -UW(T_6 + T_8 - 2T_7)$	T7(x=L) = T5(x=L)
8	$c_8 \dfrac{dT_8}{dx} = -UW(T_7 + T_9 - 2T_8)$	T8(x=L) = T6(x=L)
9	$c_9 \dfrac{dT_9}{dx} = UW(T_8 + T_{10} - 2T_9)$	T9(x=0) = T7(x=0)
10	$c_{10} \dfrac{dT_{10}}{dx} = UW(T_9 - T_{10})$	T10(x=0) = T8(x=0)

VI. Results and Discussion

Numerical simulation of the system of the ordinary differential equations previously established is performed using Visual Fortran. Some of the salient results are discussed below.
For the specific operating conditions mentioned in table 1, the corresponding R parameter and NUT are respectively equal to *0.86* and *1.46*. These data injected into the simulation program leads to a heat duty of the HX equal to *45.84* kW.

TABLE II
Units for Magnetic Properties

	Hot side	Cold side
Operating conditions		
Mass flow rate [kg/h]	540	650
Inlet temperature [K]	453	313
Specific heat, *Cp* [J/kg K]	4315	4180
Exchanger data		
Plate length [m]	1	
Plate width [m]	0.15	
Global heat transfer coefficient [w/m^{2}K]	700	

We verified that calculating the heat duty using the hot liquid outlet data or the cold liquid outlet data leads to the same value which is a first illustration of the reliability and the accuracy of the obtained results.

Fig. 3 shows the temperature evolution of both streams. The temperature of the hot liquid initially at *180*°C decreases until *109*°C while the temperature of the cold one increases from *40*°C to *101*°C. Channels are numbered along the flow path. The hot liquid flow through the odd-numbered channels while le cold one flow through the even-numbered ones.

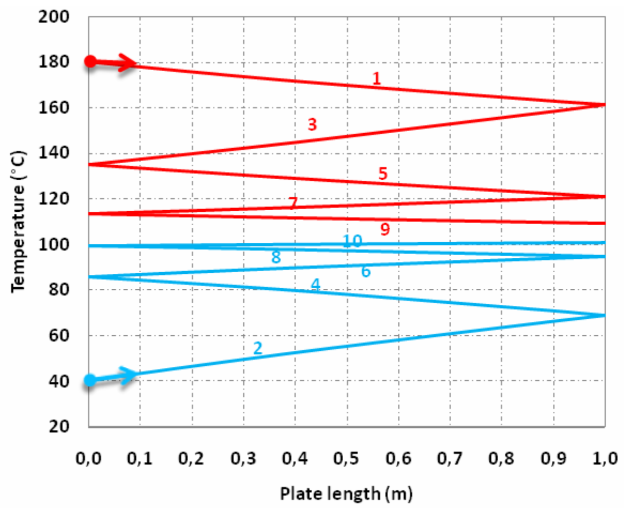

Fig. 3 Temperature profile of the two operating fluid along the PHE channels.

The temperature length curves for the hot and the cold liquid are shown in Fig. 4. This figure illustrates the heat transfer driving force, the temperatures gradient between the hot and the cold liquid, decreases appreciably between the two extremities of the plate heat exchanger. The decrease of temperatures gradient induces, as inspection of Fig. 5 shows, a decrease in the thermal fluxes transferred through each active plate. The heat transfer flux has its maximum value through the first plate and attains its minimum value at the last plate.

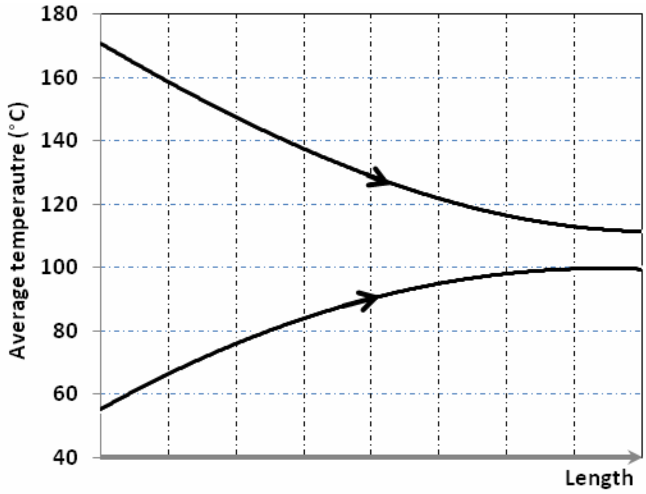

Fig. 4 Average fluids temperature through the HX channels.

Heat flux decreases from a plate to another in the flow direction which is a characteristic of the parallel flow HX.

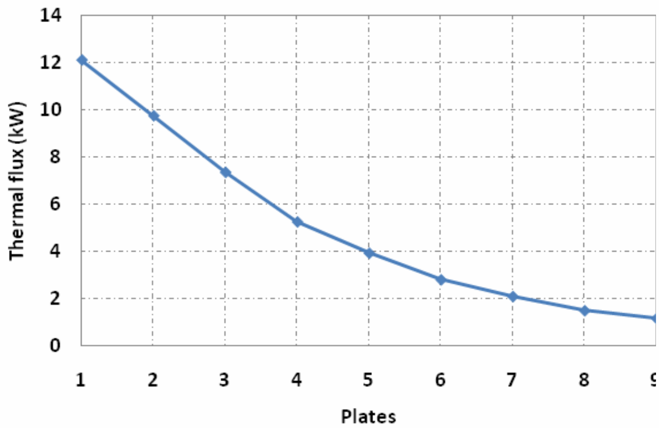

Fig. 5 Thermal fluxes through each active plate.

Fig. 6 illustrates temperature changes through each channel. A general decrease of this gradient is obviously noted.

Fig. 7 provides plots of the PHE effectiveness versus *NUT* for some values of *R* parameter.

PHE Effectiveness is calculated using the hot and the cold liquids outlet temperatures determined by the numerical simulation. It can be observed from Fig. 7 shows for a specified value of *R* parameter the effectiveness of the *PHE* monotonically increases with an increase of *NTU*. But bigining from a value of NTU equal to 3, the plate heat exchanger effectiveness approaches a nearly constant value regardless of the *R* parameter value. Fig. 8 illustrates the evolution of the PHE effectiveness versus NUT for various values of parameter R; we compare simulated PHE effectiveness to the its effectiveness evaluated by the subsequent relation available in literature for parallel flow heat exchanger [6]. The comparison shows excellent agreement between them.

$$\varepsilon = \frac{1-\exp(-NTU\,(1+R))}{1+R} \tag{11}$$

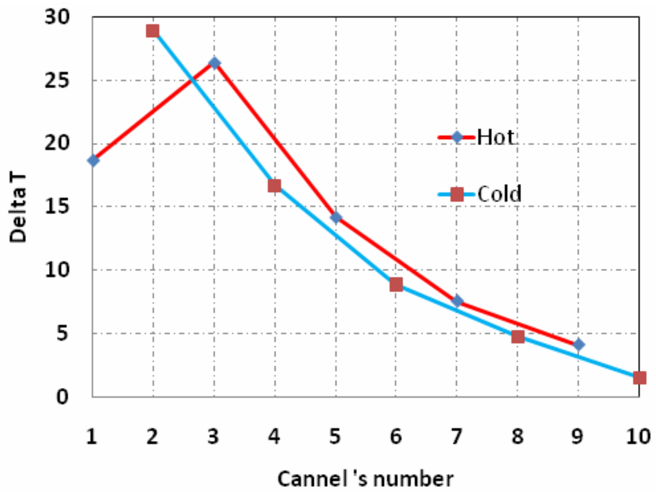

Fig. 6 Temperature change of the two operating fluids through each channel.

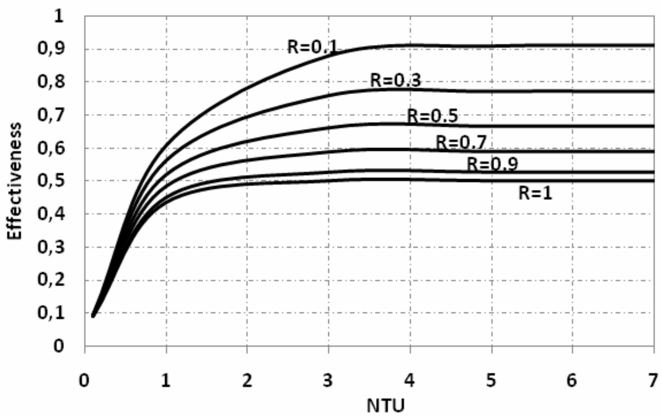

Fig. 7 Evolution of the PHE effectiveness versus NUT for various values of R parameter

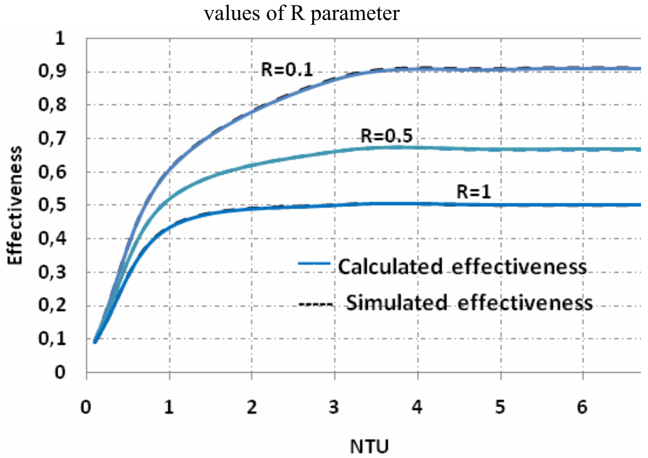

Fig. 8 Evolution of the PHE effectiveness versus NUT for various values of R parameter (Comparison between calculated and simulated values)

VII. CONCLUSION

A numerical analysis of the thermal performance of a plate type heat exchanger with parallel flow configuration is performed. The computation is based on the effectiveness-*NTU* model. The numerical results illustrate the evolution of the most important parameters of the plate heat exchanger. A parametric analysis is presented which brings out the effect of *NTU* and the *R* parameter, the heat capacity rate ratio, on the performance of the PHE. To check the validity of the presented simplified model established to describe the energy balances in the PHE and the numerical scheme adopted, simulated performance has been compared to the performance evaluated by theoretical relations. Comparison shows an excellent agreement between them. The temperature gradients through each channel and heat fluxes through each active plate are also evaluated.

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