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Novel delay-dependent stability criteria for uncertain discrete-time stochastic neural networks with time-varying delays

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Abstract—This paper investigates the problem of exponential stability for a class of uncertain discrete-time stochastic neural network with time-varying delays. By constructing a suitable Lyapunov-Krasovskii functional, combining the stochastic stability theory, the free-weighting matrix method, a delay-dependent exponential stability criteria is obtained in term of LMIs. Compared with some previous results, the new conditions obtain in this paper are less conservative. Finally, two numerical examples are exploited to show the usefulness of the results derived.

Keywords—Delay-dependent stability; Neural networks; Time-varying delay; Linear matrix inequality(LMI).

I. Introduction

RECENTLY, the dynamics of neural networks have been extensively studied, this is mainly to the great potential applications in varies areas such as signal processing, pattern recoganization, static image processing, associative memory and combinatorial optimization [1,2] As is know to all, dynamical behaviors of neural networks are the key to the applications, and the achieved applications heavily depend on the dynamic behaviors of the equilibrium point for neural network, therefore, stability is one of the most important issues related to such behavior. In practice, time delay is frequently encountered in neural networks. Due to the finite speed of information processing the existence of the delays frequently causes oscillation, divergence, or instability in neural networks.In recent years, the stability problem of time-delay neural networks have become a topic of great theoretic and practical importance[3-7,27,29-30]. This issue has gained increasing interest in applications to signal, artificial intelligence.

It is worth pointing out that most neural networks are concerned with continuous-time cases. Since discrete-time neural networks play a more important role than their continuous-time counterparts in today's digital life, moreover, in implementing and applications of neural networks discrete-time neural networks also take a more crucial key than their continuous-time counterparts in that discrete-time analog is often establish to investigate the dynamical characteristics with respect to digital signal transmission [8]. Therefore, both analysis and synthesis problem for discrete-time neural networks have been extensively studied and a great number of important results have been reported in the literature[9-14,28] and the references therein.

It is worth noting that the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes in real nerves systems. So the stochastic disturbance is probably the main resource of the performance degradation of the implemented neural networks. Therefore, the stability for stochastic neural networks with delay have attracted increasing interests and some results related to stochastic disturbances have been published [16-18,20-23,26,31]. In [16] authors have studied the robust exponential stability problem for discrete-time stochastic neural networks, where the LMI approach was developed and a weak assumption on the activation function was considered. Meantime, in [22] authors combined the free-weighting matrix method and established the delay-dependent stability conditions, which proved to be less conservative than [16]. Recently, [17] presented several improved delay-dependent stability results for discrete-time stochastic neural networks by delay partitioning ideal, but it is our observation that there still exists room for further improvement by constructing rational Lyapunov functionals which motivates the present study.

In this paper, the problem of stability analysis for uncertain discrete-time stochastic neural networks with time-varying delays is investigated. By using the discrete-time Jensen inequality, free-weighting matrix method, some sufficient conditions are established to ensure the stochastic neural networks are globally exponential stability in the mean square, which proved to be less conservative than previous results. Finally, two numerical examples are given to demonstrate the effectiveness of the proposed results.

II. Problem statement

Consider the following uncertain discrete-time stochastic neural networks (DSNNs) with time-varying delays described by

$$\begin{aligned} x(k+1) = {} & C(k)x(k) + A(k)f(x(k)) \\ & + B(k)f(x(k-\tau(k))) \\ & + \delta(k, x(k), x(k-\tau(k)))\omega(k) \end{aligned} \tag{1}$$

where $x(k) = [x_1(k), x_2(k), \cdots, x_n(k)]^T \in \Re^n$ is the neuron state vector, $f(x(\cdot)) \in \Re^n$, denotes the neuron activation

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function, $C(k) = C + \Delta C(k)$, $A(k) = A + \Delta A(k)$, $B(k) = B + \Delta B(k)$, $A, B \in \Re^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively. $C = diag(c_1, c_2, \cdots, c_n)$ with $|c_i| < 1$, describes the rate with which the ith neuron will reset its potential state in isolation when disconnected from the networks and external inputs. $C(k)$, $A(k)$, $B(k)$ are the uncertainties of system matrices of the form

$$[C(k), A(k), B(k)] = HF(k)[N_1, N_2, N_3] \tag{2}$$

where H and N_i are known real constant matrices of appropriate dimensions, $F(k)$ is the unknown time-varying matrix function satisfying $F^T(k)F(k) \le I, \forall k \in N^+$, then the system (1) can be rewritten as

$$\begin{aligned} x(k+1) &= Cx(k) + Af(x(k)) + Bf(x(k-\tau(k))) \\ &\quad + Hq_1(k) + \delta(k, x(k), x(k-\tau(k)))\omega(k) \\ q_1(k) &= F(k)p_1(k) \\ p_1(k) &= N_1x(k) + N_2f(x(k)) + N_3f(x(k-\tau(k))) \end{aligned} \tag{3}$$

$\tau(k)$ is time-varying delay and satisfies

$$0 < \tau_1 \le \tau(k) \le \tau_2 \tag{4}$$

where τ_1 and τ_2 are positive integers representing the lower and upper bounds of the time-varying delay. Now we introduce the $\tau_0 = \frac{\tau_2+\tau_1}{2} + \frac{\min\{(-1)^{\tau_2+\tau_1}, 0\}}{2}$, obviously, $\tau(k) \in [\tau_1, \tau_0]$ or $\tau(k) \in (\tau_0, \tau_2]$. So from this partition, our conditions should be considered as two cases.

In the DSNNs (3), $\delta(\cdot,\cdot,\cdot) : \Re \times \Re^n \times \Re^n \to \Re^n$ is the noise intensity function vector, $\omega(k)$ is scalar Wiener process with

$$E[\omega(k)] = 0 \quad E[\omega^2(k)] = 1 \quad E[\omega(i)\omega(j)] = 0 \quad i \ne j \tag{5}$$

In order to obtain our main results, we introduce the following assumptions and definition.

Assumption 1. For any x, $y \in R$, $x \ne y$,

$$l_i^- \le \frac{f_i(x)-f_i(y)}{x-y} \le l_i^+ \tag{6}$$

where l_i^-, l_i^+, are some constants.

Remark 1. The above assumption on the activation function was originally proposed in [9], and wildly used in many papers, see [18,20,21].

Assumption 2. The DSNNs in (3), the activation function satisfies $f(0) \equiv 0$.

According to the Assumption 2, it is obviously that $x(k) = 0$ is a trivial solution of the DSNNs in (3).

Assumption 3. There exists a constant matrix $G \ge 0$, and is assumed to satisfy

$$\begin{aligned} &\delta^T(k, x(k), x(k-\tau(k)))\,\delta(k, x(k), x(k-\tau(k))) \\ &\qquad \le \begin{bmatrix} x(k) \\ x(k-\tau(k)) \end{bmatrix}^T G \begin{bmatrix} x(k) \\ x(k-\tau(k)) \end{bmatrix} \end{aligned}$$

where $G = \begin{bmatrix} G_1 & G_2 \\ & G_3 \end{bmatrix}$.

Throughout the letter, we shall adopt the following definition.

Definition 1. The discrete-time stochastic neural network with time-varying delays (3) is said to be exponential stable in the mean square if there exist two scalars $\alpha > 0$ and $0 < \beta < 1$ such that

$$E\left[\|x(k)\|^2\right] \le \alpha\beta^k \sup_{-\tau_2 \le s \le 0} E\left[\|x(s)\|^2\right] \tag{7}$$

The following Lemmas are needed to develop our main result.

Lemma 1.[12] For any constant matrix $M \in R^{n\times n}, M = M^T > 0$, integer $r_2 \ge r_1$ such that the sums in the following are defined, the

$$\begin{aligned} &-(r_2 - r_1 + 1)\sum_{i=r_1}^{r_2} x^T(i)Mx(i) \\ &\qquad \le -\left(\sum_{i=r_1}^{r_2} x^T(i)\right) M \left(\sum_{i=r_1}^{r_2} x(i)\right) \end{aligned} \tag{8}$$

Lemma 2.[32] Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $\Sigma_2 = \Sigma_2^T > 0$, then $\Sigma_1 + \Sigma_3^T\Sigma_2^{-1}\Sigma_3 < 0$ holds if and only if:

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Lemma 3.[25] For any matrices $Z_1 > 0, Z_2 > 0, M, T$ with appropriate dimensions, such that following matrix inequalities hold.

$$\begin{bmatrix} MZ_1^TM^T & M \\ M^T & Z_1 \end{bmatrix} \ge 0 \qquad \begin{bmatrix} TZ_2^TT^T & T \\ T^T & Z_2 \end{bmatrix} \ge 0 \tag{9}$$

III. Main result

Now, for presentation convenience, in the following we denote

$$\begin{aligned} \Gamma_1 &= diag\left(l_1^-, l_2^-, \cdots, l_n^-\right) \\ \Gamma_2 &= diag\left(l_1^+, l_2^+, \cdots, l_n^+\right) \\ F_1 &= \mathrm{diag}\left(l_1^-l_1^+, l_2^-l_2^+, \cdots, l_n^-l_n^+\right) \\ F_2 &= \mathrm{diag}\left(\tfrac{l_1^-+l_1^+}{2}, \tfrac{l_2^-+l_2^+}{2}, \cdots, \tfrac{l_n^-+l_n^+}{2}\right) \end{aligned}$$

Theorem 1. Suppose that Assumption 1-3 hold. Then the DSNNs (3) is globally robust exponential stable in the mean square if there exist positive-definite matrices $P, Q_i\,(i = 0, 1), E_i\,(i = 0, 1, 2), Z_i\,(i = 1, 2)$, diagonal matrices $D_i\,(i = 1, 2)$, $K > 0, L > 0$, positive scalars $\varepsilon > 0$, and for any matrices $S_i, T_i, M_i\,(i = 1, 2, \cdots 11)$ such that following LMIs hold.

$$P \le \rho I \quad Z_i \le \rho_i I \ i = 1, 2 \quad Q_1 = \begin{pmatrix} Q_{11} & Q_{12} \\ & Q_{13} \end{pmatrix} > 0 \tag{10}$$

$$\begin{pmatrix} \Xi_1 + \Pi_1 I_1(\tau_1)\Pi_1^T + \Pi_2 I_2(\tau_1)\Pi_2^T & \sqrt{\tau_2-\tau_0}M \\ \sqrt{\tau_2-\tau_0}M^T & -Z_1 \end{pmatrix} < 0 \tag{11}$$

$$\begin{pmatrix} \Xi_1 + \Pi_1 I_1(\tau_0)\Pi_1^T + \Pi_2 I_2(\tau_0)\Pi_2^T & \sqrt{\tau_2-\tau_0}M \\ \sqrt{\tau_2-\tau_0}M^T & -Z_1 \end{pmatrix} < 0 \tag{12}$$

$$\begin{pmatrix} \Xi_2 + \Pi_3 I_3(\tau_0)\Pi_3^T + \Pi_4 I_4(\tau_0)\Pi_4^T & \sqrt{\tau_0-\tau_1}T \\ \sqrt{\tau_0-\tau_1}T^T & -Z_2 \end{pmatrix} < 0 \tag{13}$$

$$\begin{pmatrix} \Xi_2 + \Pi_3 I_3(\tau_2)\Pi_3^T + \Pi_4 I_4(\tau_2)\Pi_4^T & \sqrt{\tau_0-\tau_1}T \\ \sqrt{\tau_0-\tau_1}T^T & -Z_2 \end{pmatrix} < 0 \tag{14}$$

where

$\Xi_{1(1,1)} = \Xi_{2(1,1)} = C^T PC - P + \rho G_1 + 2\tau_2 Q_0 + \theta Q_{11}$
$+E_0 + E_1 + E_2 - 2\theta\Gamma_1 K + 2\theta\Gamma_2 L + \varepsilon N_1^T N_1 - F_1 D_1$
$+ (\tau_2 - \tau_0)(C-I)^T Z_1 (C-I) + (\tau_2 - \tau_0)\rho_1 G_1$
$+ (\tau_0 - \tau_1)(C-I)^T Z_2 (C-I) + (\tau_0 - \tau_1)\rho_2 G_1$
$\Xi_{1(1,2)} = \Xi_{2(1,2)} = \rho G_2 + (\tau_2 - \tau_0)\rho_1 G_2$
$+ (\tau_0 - \tau_1)\rho_2 G_2 \quad \Xi_{2(1,3)} = -S_1 + T_1 \quad \Xi_{1(1,3)} = -S_1$
$\Xi_{1(1,4)} = M_1 \quad \Xi_{2(1,4)} = -T_1 \quad \Xi_{1(1,5)} = S_1 - M_1$
$\Xi_{2(1,5)} = S_1$
$\Xi_{1(1,6)} = \Xi_{2(1,6)} = C^T PA + \theta Q_{12} + \theta K - \theta L$
$+ (\tau_2 - \tau_0)(C-I)^T Z_1 A + (\tau_0 - \tau_1)(C-I)^T Z_2 A$
$+F_2 D_1 + \varepsilon N_1^T N_2$
$\Xi_{1(1,7)} = \Xi_{2(1,7)} = C^T PB + \varepsilon N_1^T N_3$
$+ (\tau_2 - \tau_0)(C-I)^T Z_1 B + (\tau_0 - \tau_1)(C-I)^T Z_2 B$
$\Xi_{1(1,8)} = \Xi_{2(1,8)} = C^T PH + (\tau_2 - \tau_0)(C-I)^T Z_1 H$
$+ (\tau_0 - \tau_1)(C-I)^T Z_2 B$
$\Xi_{1(1,9)} = \Xi_{2(1,9)} = S_1 \; \Xi_{1(1,10)} = \Xi_{2(1,10)} = -S_1$
$\Xi_{1(1,11)} = \Xi_{2(1,11)} = S_1$
$\Xi_{1(2,2)} = \Xi_{2(2,2)} = \rho G_3 - Q_{11} + 2\Gamma_1 K - 2\Gamma_2 L$
$+ (\tau_2 - \tau_0)\rho_1 G_3 + (\tau_0 - \tau_1)\rho_2 G_3 - F_1 D_2$
$\Xi_{1(2,3)} = -S_2 \quad \Xi_{2(2,3)} = -S_2 + T_2 \quad \Xi_{1(2,4)} = M_2$
$\Xi_{2(2,4)} = -T_2 \quad \Xi_{1(2,5)} = S_2 - M_2 \quad \Xi_{2(2,5)} = S_2$
$\Xi_{1(2,7)} = \Xi_{2(2,7)} = -Q_{12} - K + L + F_2 D_2$
$\Xi_{1(2,9)} = \Xi_{2(2,9)} = S_2 \; \Xi_{1(2,10)} = \Xi_{2(2,10)} = -S_2$
$\Xi_{1(2,11)} = \Xi_{2(2,11)} = S_2$
$\Xi_{1(3,3)} = -S_3 - S_3^T - E_1$
$\Xi_{2(3,3)} = -S_3 - S_3^T - E_1 + T_3 + T_3^T$
$\Xi_{1(3,4)} = -S_4^T + M_3 \quad \Xi_{2(3,4)} = -S_4^T - \left(T_3 - T_4^T\right)$
$\Xi_{1(3,5)} = S_3 - S_5^T - M_3 \quad \Xi_{2(3,5)} = S_3 - S_5^T + T_5^T$
$\Xi_{1(3,6)} = -S_6^T \quad \Xi_{2(3,6)} = -S_6^T + T_6^T$
$\Xi_{1(3,7)} = -S_7^T \quad \Xi_{2(3,7)} = -S_7^T + T_7^T$
$\Xi_{1(3,8)} = -S_8^T \quad \Xi_{2(3,8)} = -S_8^T + T_8^T$
$\Xi_{1(3,9)} = -S_9^T + S_3 \quad \Xi_{2(3,9)} = -S_9^T + T_9^T + S_3$
$\Xi_{1(3,10)} = -S_{10}^T - S_3 \quad \Xi_{2(3,10)} = -S_{10}^T + T_{10}^T - S_3$
$\Xi_{1(3,11)} = -S_{11}^T + S_3 \quad \Xi_{2(3,11)} = -S_{11}^T + T_{11}^T + S_3$
$\Xi_{1(4,4)} = -E_0 + M_4 + M_4^T \; \Xi_{2(4,4)} = -E_0 - \left(T_4 + T_4^T\right)$
$\Xi_{1(4,5)} = S_4 - M_4 + M_5^T \quad \Xi_{2(4,5)} = S_4 - T_5^T$
$\Xi_{1(4,6)} = M_6^T \; \Xi_{2(4,6)} = -T_6^T \quad \Xi_{1(4,7)} = M_7^T$
$\Xi_{2(4,7)} = -T_7^T \quad \Xi_{1(4,8)} = M_8^T \quad \Xi_{2(4,8)} = -T_8^T$
$\Xi_{1(4,9)} = M_9^T + S_4 \quad \Xi_{2(4,9)} = -T_9^T + S_4$
$\Xi_{1(4,10)} = M_{10}^T - S_4 \quad \Xi_{2(4,10)} = -T_{10}^T - S_4$
$\Xi_{1(4,11)} = M_{11}^T + S_4 \quad \Xi_{2(4,11)} = -T_{11}^T + S_4$
$\Xi_{1(5,5)} = S_5 + S_5^T - E_2 - M_5 - M_5^T$
$\Xi_{2(5,5)} = S_5 + S_5^T - E_2 \quad \Xi_{1(5,6)} = S_6^T - M_6^T$
$\Xi_{2(5,6)} = S_6^T \quad \Xi_{1(5,7)} = S_7^T - M_7^T \quad \Xi_{2(5,7)} = S_7^T$
$\Xi_{1(5,8)} = S_8^T - M_8^T \quad \Xi_{2(5,8)} = S_8^T$
$\Xi_{1(5,9)} = S_9^T - M_9^T + S_5 \quad \Xi_{2(5,9)} = S_9^T + S_5$
$\Xi_{1(5,10)} = S_{10}^T - M_{10}^T - S_5 \quad \Xi_{2(5,10)} = S_{10}^T - S_5$
$\Xi_{1(5,11)} = S_{11}^T - M_{11}^T + S_5 \quad \Xi_{2(5,11)} = S_{11}^T + S_5$
$\Xi_{1(6,6)} = \Xi_{2(6,6)} = A^T PA + \theta Q_{13} - D_1 + \varepsilon N_2^T N_2$
$+ (\tau_2 - \tau_0) A^T Z_1 A + (\tau_0 - \tau_1) A^T Z_2 A$
$\Xi_{1(6,7)} = \Xi_{2(6,7)} = A^T PB + \varepsilon N_2^T N_3$
$+ (\tau_2 - \tau_0) A^T Z_1 B + (\tau_0 - \tau_1) A^T Z_2 B$
$\Xi_{1(6,8)} = \Xi_{2(6,8)} = A^T PH + (\tau_2 - \tau_0) A^T Z_1 H$
$+ (\tau_0 - \tau_1) A^T Z_2 H \quad \Xi_{1(6,9)} = \Xi_{2(6,9)} = S_6$
$\Xi_{1(6,10)} = \Xi_{2(6,10)} = -S_6 \quad \Xi_{1(6,11)} = \Xi_{2(6,11)} = S_6$
$\Xi_{1(7,7)} = \Xi_{2(7,7)} = B^T PB - Q_{13} - D_2 + \varepsilon N_3^T N_3$
$+ (\tau_2 - \tau_0) B^T Z_1 B + (\tau_0 - \tau_1) B^T Z_2 B$
$\Xi_{1(7,8)} = \Xi_{2(7,8)} = B^T PH + (\tau_2 - \tau_0) B^T Z_1 H$
$+ (\tau_0 - \tau_1) B^T Z_2 H \quad \Xi_{1(7,9)} = \Xi_{2(7,9)} = S_7$
$\Xi_{1(7,10)} = \Xi_{2(7,10)} = -S_7 \quad \Xi_{1(7,11)} = \Xi_{2(7,11)} = S_7$
$\Xi_{1(8,8)} = \Xi_{2(8,8)} = H^T PH - \varepsilon + (\tau_2 - \tau_0) H^T Z_1 H$
$+ (\tau_0 - \tau_1) H^T Z_2 H \quad \Xi_{1(8,9)} = \Xi_{2(8,9)} = S_8$
$\Xi_{1(8,10)} = \Xi_{2(8,10)} = -S_8 \quad \Xi_{1(8,11)} = \Xi_{2(8,11)} = S_8$
$\Xi_{1(9,9)} = \Xi_{2(9,9)} = -\frac{1}{\tau_1} Q_0 + S_9 + S_9^T$
$\Xi_{1(9,10)} = \Xi_{2(9,10)} = -S_9 + S_{10}^T$
$\Xi_{1(9,11)} = \Xi_{2(9,11)} = S_9 + S_{11}^T$
$\Xi_{1(10,10)} = \Xi_{2(10,10)} = -\frac{1}{\tau_2} Q_0 - S_{10} - S_{10}^T$
$\Xi_{1(10,11)} = \Xi_{2(10,11)} = S_{10} - S_{11}^T$
$\Xi_{1(11,11)} = \Xi_{2(11,11)} = -\frac{1}{\tau_2 - \tau_1} Q_0 + S_{11} + S_{11}^T$
$S = \left[S_1^T, S_2^T, \cdots, S_{11}^T\right]^T \qquad M = \left[M_1^T, M_2^T, \cdots, M_{11}^T\right]^T$
$T = \left[T_1^T, T_2^T, \cdots, T_{11}^T\right]^T \qquad \theta = \tau_2 - \tau_1 + 1$
$\Pi_1 = [0, -I, I, 0, 0, 0, 0, 0, 0, 0, 0]^T$
$\Pi_2 = [0, I, 0, -I, 0, 0, 0, 0, 0, 0, 0]^T$
$\Pi_3 = [0, I, 0, 0, -I, 0, 0, 0, 0, 0, 0]^T$
$\Pi_4 = [0, -I, 0, I, 0, 0, 0, 0, 0, 0, 0]^T$
$\Phi_1 = [0, 0, 0, I, -I, 0, 0, 0, 0, 0, 0]^T$
$\Phi_2 = [0, 0, I, -I, 0, 0, 0, 0, 0, 0, 0]^T$

Proof: Consider the Lyapunov-Krasovskii functional as follows:

$$v_1(k) = x^T(k) P x(k)$$
$$v_2(k) = \sum_{j=k-\tau_1}^{k-1} \sum_{i=j}^{k-1} x^T(i) Q_0 x(i) + \sum_{j=k-\tau_2}^{k-1} \sum_{i=j}^{k-1} x^T(i) Q_0 x(i) + \sum_{j=k-\tau_2+1}^{k-\tau_1} \sum_{i=j}^{k-1} x^T(i) Q_0 x(i)$$
$$v_3(k) = \sum_{i=k-\tau(k)}^{k-1} \begin{bmatrix} x(i) \\ f(x(i)) \end{bmatrix}^T Q_1 \begin{bmatrix} x(i) \\ f(x(i)) \end{bmatrix} + \sum_{j=k+1-\tau_2}^{k-\tau_1} \sum_{i=j}^{k-1} \begin{bmatrix} x(i) \\ f(x(i)) \end{bmatrix}^T Q_1 \begin{bmatrix} x(i) \\ f(x(i)) \end{bmatrix}$$
$$v_4(k) = \sum_{i=k-\tau_0}^{k-1} x^T(i) E_0 x(i) + \sum_{i=k-\tau_1}^{k-1} x^T(i) E_1 x(i) + \sum_{i=k-\tau_2}^{k-1} x^T(i) E_2 x(i)$$
$$v_5(k) = 2 \sum_{i=k-\tau(k)}^{k-1} (f(x(i)) - \Gamma_1 x(i))^T K x(i) + (\Gamma_2 x(i) - f(x(i)))^T L x(i) + 2 \sum_{j=k-\tau_2+1}^{k-\tau_1} \sum_{i=j}^{k-1} (f(x(i)) - \Gamma_1 x(i))^T K x(i) + (\Gamma_2 x(i) - f(x(i)))^T L x(i)$$
$$v_6(k) = \sum_{j=-\tau_2}^{-1-\tau_0} \sum_{i=k+j}^{k-1} \eta^T(i) Z_1 \eta(i) + \sum_{j=-\tau_0}^{-1-\tau_1} \sum_{i=k+j}^{k-1} \eta^T(i) Z_2 \eta(i) \qquad \eta(i) = x(i+1) - x(i)$$

then along the solution of DSNNs (3), we have

$$E(\Delta v_1(k)) \le E\left(x^T(k)\left(C^T PC - P\right)x(k)\right)$$
$$+2x^T(k) C^T PAf(x(k)) + 2x^T(k) C^T PBf(x(k - \tau(k)))$$
$$+2x^T(k) C^T PHq_1(k) + f^T(x(k)) A^T PAf(x(k))$$
$$+2f^T(x(k)) A^T PBf(x(k - \tau(k)))$$
$$+ 2f^T(x(k)) A^T PHq_1(k)$$
$$+f^T(x(k - \tau(k))) B^T PBf(x(k - \tau(k)))$$
$$+2f^T(x(k - \tau(k))) B^T PHq_1(k) + q_1^T(k) H^T Hq_1(k)$$

$$+\rho\left[\begin{array}{c} x(k) \\ x(k-\tau(k)) \end{array}\right]^T\left[\begin{array}{cc} G_1 & G_2 \\ & G_3 \end{array}\right]\left[\begin{array}{c} x(k) \\ x(k-\tau(k)) \end{array}\right]\Bigg)$$

$$E(\Delta v_2(k)) \le E\Big(2\tau_2 x^T(k) Q_0 x(k)$$
$$-\tfrac{1}{\tau_1}\sum_{i=k-\tau_1}^{k-1} x^T(i) Q_0 \sum_{i=k-\tau_1}^{k-1} x(i)$$
$$-\tfrac{1}{\tau_2}\sum_{i=k-\tau_2}^{k-1} x^T(i) Q_0 \sum_{i=k-\tau_2}^{k-1} x(i)$$
$$-\tfrac{1}{\tau_2-\tau_1}\sum_{i=k-\tau_2+1}^{k-\tau_1} x^T(i) Q_0 \sum_{i=k-\tau_2+1}^{k-\tau_1} x(i)\Bigg)$$

$$E(\Delta v_3(k)) \le E\Big(x^T(k)\theta Q_{11} x(k)$$
$$+2x^T(k)\theta Q_{12} f(x(k)) + f^T(x(k))\theta Q_{13} f(x(k))$$
$$-x^T(k-\tau(k)) Q_{11} x(k-\tau(k))$$
$$-2x^T(k-\tau(k)) Q_{12} f(x(k-\tau(k)))$$
$$-f^T(x(k-\tau(k))) Q_{13} f(x(k-\tau(k)))\Big)$$

$$E(\Delta v_4(k)) = E\Big(x^T(k)(E_0+E_1+E_2)x(k)$$
$$-x^T(k-\tau_0)E_0 x(k-\tau_0) - x^T(k-\tau_1)E_1 x(k-\tau_1)$$
$$-x^T(k-\tau_2)E_2 x(k-\tau_2)\Big)$$

$$E(\Delta v_5(k)) \le E\Big(2\theta(f(x(k)) - \Gamma_1 x(k))^T K x(k)$$
$$-2f^T(x(k-\tau(k)))Kx(k-\tau(k))$$
$$+2x^T(k-\tau(k))\Gamma_1 K x(k-\tau(k))$$
$$+2\theta(\Gamma_2 x(k) - f(x(k)))^T L x(k)$$
$$+2f^T(x(k-\tau(k)))Lx(k-\tau(k))$$
$$-2x^T(k-\tau(k))\Gamma_2 L x(k-\tau(k))\Big)$$

$$E(\Delta v_6(k)) = E\Big((\tau_0-\tau_1)\eta^T(k) Z_2 \eta(k)$$
$$+(\tau_2-\tau_0)\eta^T(k) Z_1\eta(k) - \sum_{i=k-\tau_0}^{k-1-\tau_1}\eta^T(i) Z_2\eta(i)$$
$$-\sum_{i=k-\tau_2}^{k-1-\tau_0}\eta^T(i) Z_1\eta(i)\Bigg)$$

Case one: if $\tau_1 \le \tau(k) \le \tau_0$, then

$$-\sum_{i=k-\tau_0}^{k-1-\tau_1}\eta^T(i) Z_2\eta(i) = -\sum_{i=k-\tau_0}^{k-1-\tau(k)}\eta^T(i) Z_2\eta(i)$$
$$-\sum_{i=k-\tau(k)}^{k-1-\tau_1}\eta^T(i) Z_2\eta(i)$$
$$= -\tfrac{1}{\tau_0-\tau_1}(\tau(k)-\tau_1)\sum_{i=k-\tau(k)}^{k-1-\tau_1}\eta^T(i) Z_2\eta(i)$$
$$-\Big(1-\tfrac{1}{\tau_0-\tau_1}(\tau(k)-\tau_1)\Big)\sum_{i=k-\tau(k)}^{k-1-\tau_1}\eta^T(i) Z_2\eta(i)$$
$$-\tfrac{1}{\tau_0-\tau_1}(\tau_0-\tau(k))\sum_{i=k-\tau_0}^{k-1-\tau(k)}\eta^T(i) Z_2\eta(i)$$
$$-\Big(1-\tfrac{1}{\tau_0-\tau_1}(\tau_0-\tau(k))\Big)\sum_{i=k-\tau_0}^{k-1-\tau(k)}\eta^T(i) Z_2\eta(i)$$
$$\le \xi^T(k)\Pi_1 I_1(\tau(k))\Pi_1^T\xi(k)$$
$$+\xi^T(k)\Pi_2 I_2(\tau(k))\Pi_2^T\xi(k)$$

where

$$I_1(\tau(k)) = \left[-\tfrac{2}{\tau_0-\tau_1} + \tfrac{1}{(\tau_0-\tau_1)^2}(\tau(k)-\tau_1)\right] Z_2$$
$$I_2(\tau(k)) = \left[-\tfrac{2}{\tau_0-\tau_1} + \tfrac{1}{(\tau_0-\tau_1)^2}(\tau_0-\tau(k))\right] Z_2$$
$$\xi^T(k) = \Big[x^T(k), x^T(k-\tau(k)), x^T(k-\tau_1),$$
$$x^T(k-\tau_0), x^T(k-\tau_2), f^T(x(k)),$$
$$f^T(x(k-\tau(k))), q_1^T(k), \sum_{i=k-\tau_1}^{k-1} x^T(i),$$
$$\sum_{i=k-\tau_2}^{k-1} x^T(i), \sum_{i=k-\tau_2+1}^{k-\tau_1} x^T(i)\Big]$$

Now from the Lemma 3, we know that

$$\sum_{i=k-\tau_2}^{k-\tau_0-1}\left[\begin{array}{c}\xi(k)\\ \eta(i)\end{array}\right]^T\left[\begin{array}{cc} MZ_1^{-1}M^T & M \\ M^T & Z_1\end{array}\right]\left[\begin{array}{c}\xi(k)\\ \eta(i)\end{array}\right] > 0$$

that is

$$\sum_{i=k-\tau_2}^{k-\tau_0-1}\xi^T(k) M Z_1^{-1} M^T \xi(k) + 2\xi^T(k) M\eta(i)$$
$$+\eta^T(i) Z_1\eta(i)$$
$$= (\tau_2-\tau_0)\xi^T(k) M Z_1^{-1} M^T\xi(k)$$
$$+2\xi^T(k) M\sum_{i=k-\tau_2}^{k-\tau_0-1}\eta(i)$$
$$+\sum_{i=k-\tau_2}^{k-\tau_0-1}\eta^T(i) Z_1\eta(i) > 0$$

From the definition of the $\eta(i)$, obviously we can have following equality

$$2\xi^T(k) M\left(x(k-\tau_0) - x(k-\tau_2) - \sum_{i=k-\tau_2}^{k-\tau_0-1}\eta(i)\right) = 0$$

that is equivalent to the

$$2\xi^T(k) M\Phi_1\xi(k) - 2\xi^T(k) M\sum_{i=k-\tau_2}^{k-\tau_0-1}\eta(i) = 0$$

then combination the above discussion, we can have the upper bound of the $E(\Delta v_6(k))$

$$E(\Delta v_6(k)) \le E\Big((\tau_0-\tau_1)\eta^T(k) Z_2\eta(k)$$
$$+(\tau_2-\tau_0)\eta^T(k) Z_1\eta(k) + \xi^T(k)\Pi_1 I_1(\tau(k))\Pi_1^T\xi(k)$$
$$+\xi^T(k)\Pi_1 I_1(\tau(k))\Pi_1^T\xi(k)$$
$$+(\tau_2-\tau_0)\xi^T(k) M Z_1^{-1} M^T\xi(k) + 2\xi^T(k) M\Phi_1\xi(k)\Big)$$

In order to derive less conservative results, we add the following zero equation with free-matrix S

$$2\xi^T(k) S\Big(x(k-\tau_2) - x(k-\tau_1)$$
$$+\sum_{i=k-\tau_1}^{k-1} x(i) - \sum_{i=k-\tau_2}^{k-1} x(i)$$
$$+\sum_{i=k-\tau_2+1}^{k-\tau_1} x(i)\Bigg) = 0 \qquad (15)$$

From the (6), for any positive-definite diagonal matrix D_1 and D_2, it follows that

$$\left[\begin{array}{c} x(k) \\ f(x(k)) \end{array}\right]^T\left[\begin{array}{cc} -F_1D_1 & F_2D_1 \\ & -D_1 \end{array}\right]\left[\begin{array}{c} x(k) \\ f(x(k)) \end{array}\right] \ge 0$$
$$\left[\begin{array}{c} x(k-\tau(k)) \\ f(x(k-\tau(k))) \end{array}\right]^T\left[\begin{array}{cc} -F_1D_2 & F_2D_2 \\ & -D_2 \end{array}\right]$$
$$\left[\begin{array}{c} x(k-\tau(k)) \\ f(x(k-\tau(k))) \end{array}\right] \ge 0 \qquad (16)$$

From (2) and (3), we know that $q_1^T(k) q_1(k) \le p_1^T(k) p_1(k)$, then there exist a positive scalar ε satisfying the following inequality

$$\varepsilon\left(p_1^T(k) p_1(k) - q_1^T(k) q_1(k)\right) \ge 0 \qquad (17)$$

that is

$$\varepsilon\Big(x^T(k) N_1^T N_1 x(k) + 2x^T(k) N_1^T N_2 f(x(k))$$
$$+2x^T(k) N_1^T N_3 f(x(k-\tau(k)))$$
$$+f^T(x(k)) N_2^T N_2 f(x(k))$$
$$+2f^T(x(k)) N_2^T N_3 f(x(k-\tau(k)))$$
$$+f^T(x(k-\tau(k))) N_3^T N_3 f(x(k-\tau(k)))$$
$$-q_1^T(k) q_1(k)\Big) \ge 0$$

Now combining above discussion, we have a upper bound as

$$E(\Delta v(k)) \leq E\left(\xi^T(k)\left(\Xi_1^* + \Pi_1 I_1(\tau(k))\Pi_1^T + \Pi_2 I_2(\tau(k))\Pi_2^T\right)\xi(k)\right)$$
$$\Xi_1^* = \Xi_1 + (\tau_2 - \tau_0) M Z_1^{-1} M^T$$

Then if we want to have

$$\Xi_1^* + \Pi_1 I_1(\tau(k))\Pi_1^T + \Pi_2 I_2(\tau(k))\Pi_2^T < 0$$

for $\tau_1 \leq \tau(k) \leq \tau_0$, which is equivalent to handle following two LMIs by the convex combination theorem.

$$\Xi_1^* + \Pi_1 I_1(\tau_1)\Pi_1^T + \Pi_2 I_2(\tau_1)\Pi_2^T < 0$$
$$\Xi_1^* + \Pi_1 I_1(\tau_0)\Pi_1^T + \Pi_2 I_2(\tau_0)\Pi_2^T < 0$$

that are equivalent to (11) and (12) hold by the Lemma 2. Therefore, if the LMIs (10)-(12) hold, there exist a positive scalar $\lambda_1 > 0$ satisfying $E(\Delta v(k)) \leq -\lambda_1 E\|x(k)\|^2$.

Case two: if $\tau_0 < \tau(k) \leq \tau_2$

$$-\sum_{i=k-\tau_2}^{k-\tau_0-1}\eta^T(i) Z_1 \eta(i) = -\sum_{i=k-\tau_2}^{k-\tau(k)-1}\eta^T(i) Z_1 \eta(i)$$
$$-\sum_{i=k-\tau(k)}^{k-\tau_0-1}\eta^T(i) Z_1 \eta(i)$$
$$= -\frac{1}{\tau_2-\tau_0}(\tau_2 - \tau(k))\sum_{i=k-\tau_2}^{k-\tau(k)-1}\eta^T(i) Z_1 \eta(i)$$
$$-\frac{1}{\tau_2-\tau_0}(\tau(k) - \tau_0)\sum_{i=k-\tau_2}^{k-\tau(k)-1}\eta^T(i) Z_1 \eta(i)$$
$$-\frac{1}{\tau_2-\tau_0}(\tau_2 - \tau(k))\sum_{i=k-\tau(k)}^{k-\tau_0-1}\eta^T(i) Z_1 \eta(i)$$
$$-\frac{1}{\tau_2-\tau_0}(\tau(k) - \tau_0)\sum_{i=k-\tau(k)}^{k-\tau_0-1}\eta^T(i) Z_1 \eta(i)$$
$$\leq \left[x^T(k-\tau(k)) - x^T(k-\tau_2)\right] I_3(\tau(k))$$
$$\left[x(k-\tau(k)) - x(k-\tau_2)\right]$$
$$+\left[x^T(k-\tau_0) - x^T(k-\tau(k))\right] I_4(\tau(k))$$
$$\left[x(k-\tau_0) - x(k-\tau(k))\right]$$
$$= \xi^T(k)\Pi_3 I_3(\tau(k))\Pi_3^T\xi(k)$$
$$+\xi^T(k)\Pi_4 I_4(\tau(k))\Pi_4^T\xi(k)$$

With the similar method as coping in the case one by the Lemma 3, we can have

$$E(\Delta v_6(k)) \leq E\left((\tau_0 - \tau_1)\eta^T(k) Z_2 \eta(k)\right.$$
$$+(\tau_2 - \tau_0)\eta^T(k) Z_1 \eta(k) + \xi^T(k)\Pi_3 I_3(\tau(k))\Pi_3^T\xi(k)$$
$$+\xi^T(k)\Pi_4 I_4(\tau(k))\Pi_4^T\xi(k)$$
$$\left.+(\tau_0 - \tau_1)\xi^T(k) T Z_2^{-1} T^T\xi(k) + 2\xi^T(k) T\Phi_2\xi(k)\right)$$

Then under this condition,we have a upper bound as

$$E(\Delta v(k)) \leq E\left(\xi^T(k)\left(\Xi_2^* + \Pi_3 I_3(\tau(k))\Pi_3^T + \Pi_4 I_4(\tau(k))\Pi_4^T\right)\xi(k)\right)$$
$$\Xi_2^* = \Xi_2 + (\tau_0 - \tau_1) T Z_2^{-1} T^T$$

Similar to the case one, if we want to have

$$\Xi_2^* + \Pi_3 I_3(\tau(k))\Pi_3^T + \Pi_4 I_4(\tau(k))\Pi_4^T < 0$$

for $\tau_1 < \tau(k) \leq \tau_2$, which are equivalent to handle following two LMIs by the convex combination theorem.

$$\Xi_2^* + \Pi_3 I_3(\tau_0)\Pi_3^T + \Pi_4 I_4(\tau_0)\Pi_4^T < 0$$
$$\Xi_2^* + \Pi_3 I_3(\tau_2)\Pi_3^T + \Pi_4 I_4(\tau_2)\Pi_4^T < 0$$

that are equivalent to (13) and (14) hold by the Lemma 2. Therefore, if the LMIs (10),(13) and (14) hold, there exist a positive scalar $\lambda_2 > 0$ satisfying $E(\Delta v(k)) \leq -\lambda_2 E\|x(k)\|^2$.

Now combining the case one and the case two, we can easy to know that if the LMIs (10)-(14) hold, we will have

$$E(\Delta v(k)) \leq -\min(\lambda_1, \lambda_2) E\|x(k)\|^2 \qquad (18)$$

Furthermore, with the similar method in the [18], we can obtain that system (3) is globally robust exponentially stable in the mean square. This completes the proof of the Theorem 1. ■

Remark 2. In this paper, based on the convex combination theorem, Theorem 1 proposes a delay-dependent stability criterion for uncertain stochastic neural networks with time-varying delays can be achieved by solving some LMIs. Free-weighing matrices S_i, M_i and T_i are introduced into the LMI condition are to reduce conservatism for system (3).

Remark 3. By introduced τ_0, we divided two kinds of cases to discuss our results at each subintervals $\tau_1 \leq \tau(k) \leq \tau_0$ and $\tau_0 < \tau(k) \leq \tau_2$, which is different from the method of [16,22], the main advantage of this method is that it makes full use of the information on the considered time-delay $\tau(k)$, meantime we through a numerical example show that Theorem 1 provide an improved result compared with the recent ones in [16,22].

IV. Examples

In this section, we will give two examples to show the effectiveness of the conditions given here.

Example 1. Consider the uncertain discrete-time stochastic neural network (3) with:

$$C = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.9 \end{bmatrix} \qquad A = \begin{bmatrix} 0.4 & -0.7 \\ 0.1 & 0.005 \end{bmatrix}$$
$$B = \begin{bmatrix} -0.2 & 0.6 \\ -0.5 & -0.1 \end{bmatrix} \qquad H = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}$$
$$N_1 = N_2 = N_3 = 0.1$$

The activation function satisfy Assumption 1 with $\Gamma_1 = diag(0,0)$ and $\Gamma_2 = diag(0.5, 0.5)$. Choosing $G_1 = G_2 = 0.001I$ and $G_3 = 0.002I$ in Theorem 1. Table 1 show the corresponding maximum allowable value of τ_2 for given τ_1, one can see that stability criteria propose in this paper significantly improve the existing results of [16,22], and the feasibility is depicted as Fig.1.

Example 2. Consider the uncertain DSNNs with the following parameters:

$$C = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.1 \end{bmatrix} \qquad A = \begin{bmatrix} 0.12 & 0.24 \\ -0.15 & 0.2 \end{bmatrix}$$
$$B = \begin{bmatrix} -0.25 & 0.1 \\ 0.02 & 0.09 \end{bmatrix} \qquad H = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}$$
$$N_1 = \begin{bmatrix} 0.15 & 0.1 \\ 0 & -0.7 \end{bmatrix} \qquad N_2 = \begin{bmatrix} 0.1 & 0.3 \\ -0.2 & 0.05 \end{bmatrix}$$
$$N_3 = \begin{bmatrix} 0.13 & 0.06 \\ -0.05 & 0.15 \end{bmatrix} \qquad G_1 = G_2 = G_3 = 0$$

The activation function satisfy Assumption 1 with $\Gamma_1 = diag(0.1, 0.2)$ and $\Gamma_2 = diag(1, 1.1)$. Obviously when $G_1 = G_2 = G_3 = 0$ in the Theorem 1, which is equivalent to the criteria of uncertain discrete-time neural networks (DNNs) with time-varying delays. For $\tau_1 = 2$, by [13,14,12], the upper bound of the time-varying delay $\tau(k)$ is 6, 10 and 12, respectively. By the Theorem 1 in this paper, we obtain $\tau_2 = 41$. Namely, when $\tau_1 = 2$ and $\tau_2 = 41$, the stability condition in the Theorem 1 is applicable but those in [13,14,12] are not applicable for this example. The further comparison is listed in Table2, from which one can see that the criterion proposed in Theorem 1 is less conservative than those obtained in [13,14,12].

V. Conclusion

In this letter,a improved delay-dependent global robust exponential stability criterion for uncertain stochastic discrete-time neural networks with time-varying delay is proposed. A suitable Lyapunov functional has been proposed to derive some less conservative delay-dependent stability criteria by using the free-weighting matrices method and the convex combination theorem. Finally, two numerical examples have been given to demonstrate the effectiveness of the proposed method.

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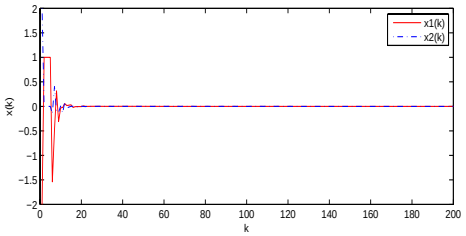

Fig.1. Dynamics response of system (3).

TABLE I
CALCULATED THE MAXIMUM τ_2 FOR GIVEN τ_1 FOR EXAMPLE 1.

τ_1	2	4	6	8	10
[16]	4	6	8	9	11
[22]	11	12	14	16	18
Theorem 1	27	29	31	33	35

TABLE II
CALCULATED THE MAXIMUM τ_2 FOR GIVEN τ_1 FOR EXAMPLE 2.

τ_1	2	4	6	8	10
[13]	6	8	10	12	14
[14]	10	12	14	16	18
[12]	12	14	16	18	20
Theorem 1	41	43	45	47	49