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Nonlinear Solitary Structures of Electron Plasma Waves in a Finite Temperature Quantum Plasma

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***Abstract*—**Nonlinear solitary structures of electron plasma waves have been investigated by using nonlinear quantum fluid equations for electrons with an arbitrary temperature. It is shown that the electron degeneracy parameter has significant effects on the linear and nonlinear properties of electron plasma waves. Depending on its value both compressive and rarefactive solitons can be excited in the model plasma under consideration.

***Keywords*—**Electron Plasma Waves, Finite Temperature Model, Modulational Instability, Quantum Plasma, Solitary structure

I. Introduction

THE quantum plasmas which are characterized by high particle density and low temperature are ubiquitous in white dwarfs, neutron stars, galactic plasma, metal nanostructures, intense laser-solid interaction and in many other environments. In recent years propagation of various electrostatic modes such as ion-acoustic waves, electron-acoustic waves, dust-acoustic waves, dust ion-acoustic waves etc. in quantum plasma have been studied by many authors [1]-[12].

Quantum effects in plasmas are usually studied with the help of two well-known formulations, viz. the Wigner-Poisson and the Schrodinger-Poisson formulations. The Wigner-Poisson model is often used in the study of quantum kinetic behaviour of plasma. The Schrödinger-Poisson model describes the hydrodynamic behaviour of plasma particles in quantum scales. The quantum hydrodynamic (QHD) model is derived by taking velocity space moments of the Wigner equations. The QHD model generalizes the classical fluid model for plasma with the inclusion of a quantum correction term also known as the Bohm potential [1]. The model incorporates quantum statistical effects through the equation of state. Because of simplicity, straight forward approach and numerical efficiency the QHD model has been widely used by several authors [1]-[12]. Different approaches for modeling quantum plasmas in electrostatic limit have been reviewed by Manfredi [13].The QHD model as used by most authors is valid for quantum plasmas in the ultra-cold limit.

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But in most practical cases the plasma temperature is finite and not approaching zero. Recently Eliasson and Shukla [14] have developed nonlinear fluid equations taking into account the moments of the Wigner equation and by using the Fermi Dirac equilibrium distribution for electrons with an arbitrary temperature. The model thus developed is expected to describe a finite temperature quantum plasma. The linear and nonlinear properties of electron plasma waves in a quantum plasma have been studied by a few authors in the ultra-cold limit by using QHD model [15-18]. To the best of our knowledge no one has studied this problem including finite temperature effects. The motivation of the present paper is to study the linear and nonlinear properties of electron plasma waves in a finite temperature quantum plasma by using a finite temperature quantum hydrodynamic model.

II. The Finite Temperature QHD Model Equations

The model as developed by Eliasson and Shukla [14] is based on 3D Fermi-Dirac equilibrium distribution for electrons with an arbitrary temperature. Propagation of plane longitudinal electron plasma waves in a collisionless quantum plasma leads to adiabatic compression along one dimension only and hence to a temperature anisotropy of the electron distribution.

In quantum picture the classical incompressibility of phase fluid is violated by quantum tunneling. However to a first approximation one may assume the incompressibility of the electron phase fluid. It may also be assumed that the chemical potential (μ) remains constant during the nonequilibrium dynamics of plasma. Based on these assumptions one may consider the following nonequilibrium particle distribution function:

$$f\left(x,\vec{v},t\right)=\frac{2\left(m/2\pi\hbar\right)^3}{\exp\left\{\left(\beta m/2\right)\left[\left(v_x-v_{ex}\right)^2\eta+v_y^2+v_z^2\right]-\beta\mu\right\}+1} \tag{1}$$

where m is the electronic mass, ħ is the Plank constant divided by 2π, $\beta=1/k_BT_{e0}$, k_B is the Boltzmann constant and T_{e0} is the background temperature, μ is the chemical potential. v_{ex} is the mean velocity of the particles given by

$$v_{ex}(x,t)=\langle v_x\rangle=\frac{1}{n_e}\int v_x f d^3v \tag{2}$$

and $\eta_{ex}(x,t) = T_{e0}/T_{ex}(x,t)=[n_0/n_e(x,t)]^2$ is the temperature anisotropy of the distribution function which is defined from the number density variations where n_0 is given by:

$$n_0 = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{E^{1/2}dE}{\exp\left[\beta(E-\mu)+1\right]} = -\frac{1}{2\pi^2\beta^{3/2}}\left(\frac{2m}{\hbar^2}\right)^{3/2}\Gamma\left(\frac{3}{2}\right)Li_{3/2}\left[-\exp(\beta\mu)\right] \quad (3)$$

$Li_\gamma(y)$ is the polylogarithm function. In the ultra cold limit i.e. $T\to 0$, we have $\beta\to\infty$ and $\mu\to E_F$.
where

$$E_F = \left(3\pi^2 n_0\right)^{2/3}\left(\hbar^2/2m\right) \quad (4)$$

Now using the zeroth and first moments of the Wigner equation with the Fermi-Dirac distribution function and assuming that the Bohm potential is independent of the thermal fluctuations in a finite temperature plasma one can derive the continuity and momentum equation in the following form:

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e v_{ex})}{\partial x} = 0 \quad (5)$$

$$\frac{\partial v_{ex}}{\partial t} + v_{ex}\frac{\partial v_{ex}}{\partial x} = \frac{e}{m_e}\frac{\partial\phi}{\partial x} - \frac{n_0 V_{Te}^2}{n_e}G\frac{\partial\left(n_e/n_0\right)^3}{\partial x} + \frac{\hbar^2}{2m_e^2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right] \quad (6)$$

where n_e and v_{ex} are respectively the particle density and fluid velocity of electron; ϕ is the electrostatic wave potential and $v_{Te} = \sqrt{k_B T_{Te}/m_e}$ is the themal speed. G is the ratio of two polylogarithm functions given by:

$$G = \frac{Li_{5/2}\left(-\exp[\beta\mu]\right)}{Li_{3/2}\left(-\exp[\beta\mu]\right)} \quad (7)$$

The system is closed by the Poisson equation,

$$\frac{\partial^2\phi}{\partial x^2} = 4\pi e\left(n_e - n_i\right) \quad (8)$$

We now introduce the following normalization:
$x \to x\omega_{pe}/V_{Fe}, t \to t\omega_{pe}, \phi \to e\phi/2k_B T_{Fe}, n_j \to n_j/n_0$ and $u_j \to u_j/V_{Fe}$. Here $\omega_{pe} = \sqrt{4\pi n_0 e^2/m_e}$ the electron plasma oscillation frequency and $v_{Fe} = \sqrt{2k_B T_{Fe}/m_e}$ is the Fermi speed of electrons. Using the above normalization Eqs. (5, 6 and 8) can be recast as:

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e v_{ex})}{\partial x} = 0$$

$$\left(\frac{\partial}{\partial t} + v_{ex}\frac{\partial}{\partial x}\right)v_{ex} = \frac{\partial\phi}{\partial x} - 3G\alpha^2 n_e\frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right] \quad (9)$$

$$\frac{\partial^2\phi}{\partial x^2} = \left(n_e - n_i\right)$$

where $H = \hbar\omega_{pe}/2\,k_B T_{Fe}$ is a nondimensional quantum parameter proportional to the quantum diffraction and $\alpha = \left(V_{Te}/V_{Fe}\right)$. The parameter H is proportional to the ratio between the plasma energy $\hbar\omega_{pe}$ (energy of an elementary excitation associated with an electron plasma wave) and the Fermi energy $k_B T_{Fe}$.

III. Dispersion Characteristics

In order to investigate the nonlinear behaviour of electron plasma waves we make the following perturbation expansion for the field quantities n_e, v_{ex} , n_{ec} and ϕ about their equilibrium values:

$$\begin{bmatrix} n_e \\ v_{ex} \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \varepsilon\begin{bmatrix} n_e^{(1)} \\ v_{ex}^{(1)} \\ \phi^{(1)} \end{bmatrix} + \varepsilon^2\begin{bmatrix} n_e^{(2)} \\ v_{ex}^{(2)} \\ \phi^{(2)} \end{bmatrix} + \ldots\ldots \quad (10)$$

Substituting the expansion (12) in Eqs. (9)-(11) and then linearizing and assuming that all the field quantities vary as $\exp\left[i(kx-\omega t)\right]$, we get for normalized wave frequency ω and wave number k, the following linear dispersion relation

$$\omega^2 = 1 + k^2\left(3G\alpha^2 + \frac{k^2H^2}{4}\right) \quad (11)$$

In the dimensional form the dispersion relation becomes:

$$\omega^2 = \omega_{pe}^2 + 3G\alpha^2 k^2 V_{Fe}^2 + \frac{k^4 V_{Fe}^4 H^2}{4\omega_{pe}^2} = \omega_{pe}^2 + 3Gk^2 V_{Te}^2 + \frac{k^4 V_{Fe}^4 H^2}{4\omega_{pe}^2} \quad (12)$$

The degeneracy parameter G determines the transition between the ultra cold and thermal cases. In the low twmperature limit $\beta\mu\to\infty$, $\mu \approx E_F \equiv (mV_{Fe}^2)/2$ and $G \approx 2\beta E_F/5$, then the dispersion relation (11) takes the form

$$\omega^2 = \omega_{pe}^2 + \frac{3}{5}k^2 V_{Fe}^2 + \frac{k^4 V_{Fe}^4 H^2}{4\omega_{pe}^2} \quad (13)$$

which is similar to the dispersion relation for electron plasma waves in a quantum plasma obtained by using one dimensional QHD Model. In high temperature limit $\beta\mu\to -\infty$ so that $G\to 1$ and then the dispersion relation (12) reduces to the Bohm-Gross dispersion relation for electron plasma waves in a hot plasma

$$\omega^2 = \omega_{pe}^2 + 3k^2 V_{Te}^2 + \frac{k^4 V_{Fe}^4 H^2}{4\omega_{pe}^2} \quad (14)$$

In many cases of practical interest the last term on the RHS may be neglected and then one gets the well known Bohm-Gross dispersion relation of electron plasma waves in a hot plasma. It may be noted that in the frequency range where $\omega^2 >> 1$ the dispersion relation (11) reduces to the form:

$$\omega \approx \sqrt{3G}\alpha k + \frac{H^2k^3}{8\sqrt{3G}\alpha} \quad (15)$$

Fig.1 shows the linear dispersion characteristics for different values of G.

The electron degeneracy parameter is found to increase the slope of the dispersion curve. i.e.as the value of G increases the wave frequency increases for a given k.

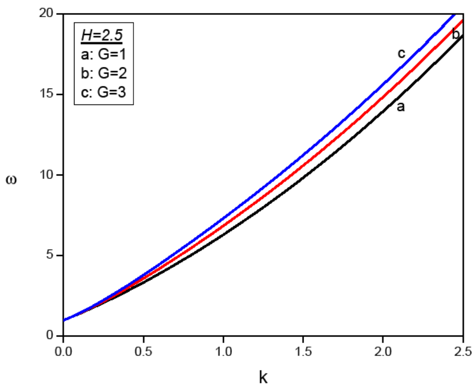

Fig. 1 Linear dispersion curve for different value of G

IV. Derivation of the KdV Equation

In order to study the nonlinear behaviour of electron plasma waves we use the standard reductive perturbation technique and the usual stretching of the space and time variables:

$$\xi=\varepsilon^{1/2}(x-Vt) \quad \text{and} \quad \tau=\varepsilon^{3/2}t \tag{16}$$

where V is the normalized linear velocity of the wave and ε is the smallness parameter measuring the dispersion and nonlinear effects.

Now writing the Eqs. (9) in terms of these stretched co-ordinates ξ and τ and then applying the perturbation expansion (10) and solving for the lowest order equation with the boundary condition $n_e^{(1)}$, $u_e^{(1)}$, and $\phi^{(1)} \to 0$ as $|\xi| \to \infty$, the following solutions are obtained:

$$n_{ex}^{(1)}=\frac{\phi^{(1)}}{3G\alpha^2-V_0^2},\; v_{ex}^{(1)}=\frac{V_0\phi^{(1)}}{3G\alpha^2-V_0^2}. \tag{17}$$

and then going for the next higher order terms in ε and following the usual method we obtain the desired Korteweg de Vries (KdV) equation:

$$\frac{\partial\phi}{\partial\tau}+A\phi\frac{\partial\phi}{\partial\xi}+B\frac{\partial^3\phi}{\partial\xi^3}=0 \tag{18}$$

$$A=\frac{\left(3G\alpha^2+3V^2\right)}{2V\left(3G\alpha^2-V^2\right)} \tag{19}$$

$$B=\frac{\left(3G\alpha^2-V^2\right)^2-H^2/4}{2V} \tag{20}$$

To find the solution of Eq. (18) we transform the independent variables ξ and τ into one variable η = ξ - M τ where M is the normalized constant speed of the wave frame. Applying the boundary conditions that as η → ± ∞; $\phi, D_\eta\phi, D_\eta^2\phi \to 0$ the possible stationary solution of Eq. (20) is obtained as:

$$\phi=\phi_m \sec h^2\left(\frac{\eta}{\Delta}\right) \tag{21}$$

where the amplitude ϕ_m and width Δ of the soliton are given by:

$$\phi_m=3\frac{M}{A} \tag{22}$$

and $$\Delta=\sqrt{\frac{4B}{M}} \tag{23}$$

For the existence of soliton solution we require B > 0. It requires that $3G\alpha^2 < V^2 - (H/2)$ or $3G\alpha^2 > V^2 + (H/2)$. The nature of the solitary waves (i.e. whether the system will support compressive or rarefactive solitary waves) depends on the sign of A.

If A is positive (or negative) a compressive (or rarefactive) solitary wave is excited. Thus for $3G\alpha^2 < V^2 - (H/2)$ rarefactive soliton and for $3G\alpha^2 > V^2 + (H/2)$ compressive soliton is formed. From Eq (20) it is clear that the dispersive coefficient B vanishes for two critical values of the diffraction parameter H, given by

$H_{c1} = 2\,(3G\alpha^2 - V^2)$ for $3G\alpha^2 > V^2$ (24a)
$H_{c2} = 2\,(V^2 - 3G\alpha^2)$ for $3G\alpha^2 < V^2$ (24b)

At these values of H no soliton solution is possible. For H< H_{c1} compressive solitons and for H < H_{c2} rarefactive solitons are obtained.

V. Results and Discussion

Using the nonlinear quantum fluid equations for electrons with an arbitrary temperature and the standard reductive perturbation technique both the linear and nonlinear properties of electron-plasma waves has been investigated.

The electron degeneracy parameter G is shown to influence the linear and nonlinear properties of the electron plasma waves in a significant way. Fig 1 shows that the wave frequency increases with increase in the degeneracy parameter G. The model plasma under consideration can support both compressive and rarefactive types of soliton. Soliton amplitude and width are found to depend significantly on the degeneracy parameter G. Fig. 2 shows that both the amplitude and width of the compressive solitons increase with increase in G.

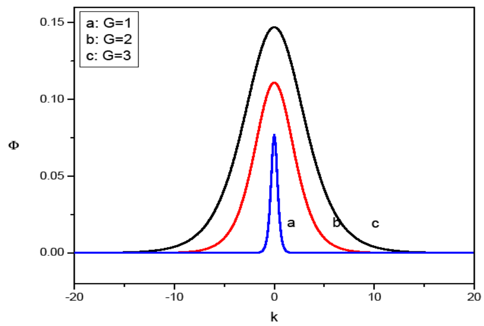

Fig. 2 Compressive solitary wave profiles for different values of degeneracy parameter G

It is shown that the amplitude and width of the rarefactive soliton decreases with increase in the value of G (Fig.3).

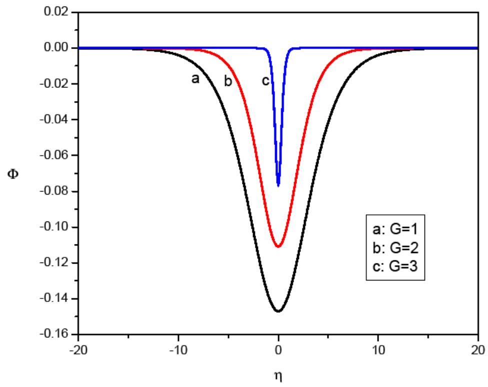

Fig. 3 Rarefactive solitary wave profiles for different values of degeneracy parameter G

As the degeneracy parameter G determines the transition from ultra cold to thermal cases it is important to know its value. Table I shows the values of G for certain practical plasmas. Finally we would like to point out that the investigation presented here may be helpful in the understanding of the basic features of long wavelength electron plasma waves in dense and hot plasmas such as can be found in white dwarfs, neutron stars and intense laser-solid plasma experiments.

TABLE I
VALUES OF ELECTRON DEGENERACY PARAMETER FOR DIFFERENT PLASMAS

Type of Plasma	Density (m^{-3})	Temperature (K)	G
Tokamak	10^{20}	10^{18}	1
Inertial Confinement Fusion	10^{32}	10^{8}	1
Metal and Metal clusters	10^{28}	10^{4}	1. 4
Jupiter	10^{32}	10^{4}	1. 4
White Dwarf	10^{35}	10^{8}	4

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