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Nonlinear Effects in Bubbly Liquid with Shock Waves

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Abstract**—**The paper presents the results of theoretical and numerical modeling of propagation of shock waves in bubbly liquids related to nonlinear effects (realistic equation of state, chemical reactions, two-dimensional effects). On the basis on the Rankine-Hugoniot equations the problem of determination of parameters of passing and reflected shock waves in gas-liquid medium for isothermal, adiabatic and shock compression of the gas component is solved by using the wide-range equation of state of water in the analitic form. The phenomenon of shock wave intensification is investigated in the channel of variable cross section for the propagation of a shock wave in the liquid filled with bubbles containing chemically active gases. The results of modeling of the wave impulse impact on the solid wall covered with bubble layer are presented.

Keywords**—**bubbly liquid, cavitation, equation of state, shock wave

I. Introduction

THE perspectives of invention of new technologies for realization of supercompression of matter guarantee the rising interest for research of mechanics of bubbly liquids in the presence of shock waves [1]–[3]. Small addition of bubbles makes liquid high compressible and provides special features of propagating of acoustic and shock signals in the liquid along with interfacial heat and mass transfer. The variety of nonlinear effects in bubbly liquid in nature and industry originated the importance of theoretical and experimental investigation.

At the present paper theoretical research of shock waves in bubbly liquid is done to describe the nonlinear effects arising in the gas-liquid mixture. Earlier theoretical research of shock wave propagation in gas-liquid media was restricted by the account for only gas or linear liquid compressibility [1]. At the present work the problem of determination of parameters of passing and reflected shock waves from the solid wall in bubble liquid in assumption of isothermal, adiabatic and shock compression of gas fraction [4] is studied on the basis of Rankine-Hugoniot relations by using the wide-ranged equation of state of water in the analytic form [5].

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The effect of shock wave intensification is studied in the tube of the variable cross section filled with liquid, which contains explosive bubbles. The parametric analysis of the problem is carried out for different initial void fractions and geometric characteristics of the tube.

The evolution of impulse of pressure in the bubbly liquid is researched in two-dimensional case. The impact of the wave impulse on the solid wall, covered with bubble layer of limited size, is investigated.

II. The Investigation of Characteristics of Passing and Reflected Shock waves in the Gas-Liquid Media

A. Correlations for Shock Waves in the Gas-Liquid Media

To investigate the characteristics of shock waves we consider the equations of conservation of mass of each phase at the initial and current time moment in volume V containing two-phase mixture with volume concentrations α_l and α_g for liquid and gas phases correspondingly ($\alpha_l = V_l/V$, $\alpha_g = V_g/V$, $\alpha_l + \alpha_g = 1$):

$$m_{g0} = m_{g1} \rightarrow \rho_{g0}\frac{V_{g0}}{V_0} = \rho_{g1}\frac{V_{g1}}{V_0} \rightarrow \frac{\rho_{g0}\alpha_{g0}}{\rho_g} = \alpha_{g1}\frac{\alpha_0}{\rho_{11}}, \quad (1)$$

$$m_{l0} = m_{l1} \rightarrow \rho_{l0}\frac{V_{l0}}{V_0} = \rho_{l1}\frac{V_{l1}}{V_0} \rightarrow \frac{\rho_{l0}\alpha_{l0}}{\rho_{l1}} = \alpha_{l1}\frac{\rho_0}{\rho_1}, \quad (2)$$

from which we can obtain:

$$\frac{\rho_{g0}\alpha_{g0}}{\rho_{g1}} + \frac{\rho_{l0}\alpha_{l0}}{\rho_{l1}} = \frac{V_1}{V_0} = \frac{\rho_0}{\rho_1}. \quad (3)$$

Here $\rho_1 = \rho_{l1}\cdot\alpha_{l1} + \rho_{g1}\cdot\alpha_{g1}$ and $\rho_0 = \rho_{l0}\cdot\alpha_{l0} + \rho_{g0}\cdot\alpha_{g0}$ are the average current and initial values of the mixture density, which are expressed via reduced densities of phases ρ_{li} and ρ_{gi}, where $i = 0, 1$.

In [6] the model of equilibrium disperse media of Campbell-Pitcher is employed, according to which the compression of the gas bubbles is isothermal. The Rakhmatulin model of bubbly liquid [7] proposes that the components of gas-liquid mixture have the same pressure and velocity, but differ in compressibility according to the individual equations of state. In contradiction to [6], [7] at the present paper a wide-range equation of state of water [5] is employed. This equation of state allows to account for nonlinear compressibility of liquid phase and provides good agreement with experimental data on the shock waves characteristics of gas-liquid mixture.

Consider the process of propagation of a shock wave in stationary bubbly liquid. To determine the parameters behind the front of the shock wave we use here the equations of conservation of mass, momentum and energy for the mixture written at the interface (correlation of Rankine-Hugoniot) [8] in the following form:

$$D_1 = \sqrt{(p_1 - p_0)\frac{\rho_1}{\rho_0(\rho_1 - \rho_0)}}\ ,\ U_1 = \sqrt{(p_1 - p_0)\frac{(\rho_1 - \rho_0)}{\rho_0\rho_1}}\ ,$$

$$e_1 - e_0 = \frac{1}{2}(p_1 + p_0)\left(\frac{1}{\rho_0} - \frac{1}{\rho_1}\right), \qquad (4)$$

where subscripts 0 and 1 denote the parameters in front of and behind the shock wave.

Consider the situation, when the internal energy of the mixture is the additive function of two components [1]

$$\rho e = e_g\rho_g\alpha_g + e_l\,\rho_l\,\alpha_l\ . \qquad (5)$$

Using the equation of conservation of energy in the case of shock compression of liquid and gas as single phases we can obtain from (1) and (2) the following equations:

$$e_{l1}\frac{\rho_{l1}\alpha_{l1}}{\rho_1} - e_{l0}\frac{\rho_{l0}\alpha_{l0}}{\rho_0} = \frac{1}{2}(p_1 + p_0)\frac{\rho_{l0}\alpha_{l0}}{\rho_0}\left(\frac{1}{\rho_{l0}} - \frac{1}{\rho_{l1}}\right), \qquad (6)$$

$$e_{g1}\frac{\rho_{g1}\alpha_{g1}}{\rho_1} - e_{g0}\frac{\rho_{g0}\alpha_{g0}}{\rho_0} = \frac{1}{2}(p_1 + p_0)\frac{\rho_{g0}\alpha_{g0}}{\rho_0}\left(\frac{1}{\rho_{g0}} - \frac{1}{\rho_{g1}}\right).$$

Summarizing equations (6) we get

$$\frac{e_{g1}\rho_{g1}\alpha_{g1} + e_{l1}\rho_{l1}\alpha_{l1}}{\rho_1} - \frac{e_{g0}\rho_{g0}\alpha_{g0} + e_{l0}\rho_{l0}\alpha_{l0}}{\rho_0} =$$

$$= \frac{1}{2}(p_1 + p_0)\frac{1}{\rho_0}\left((\alpha_{l0} + \alpha_{g0}) - \frac{\rho_0}{\rho_1}(\alpha_{g1} + \alpha_{l1})\right). \qquad (7)$$

It is obvious, that equation (5) along with expression $\alpha_l + \alpha_g = 1$ allows to derive equation (4) from (7).

The noted prove is the reason to use the conservation law of energy for two-phase mixture (4), when it is true for the single phases (6) and the additivity condition (5) is fulfilled. This essentially simplifies the derivation of the parameters of shock waves in mixtures. As it stated in [9] earlier this problem was not considered elsewhere. On the basis on the proven statement we can derive expression for the velocity of the shock wave of two-phase mixture via velocities of separate components.

$$D_1^2 = \frac{1}{\rho_0\left(\frac{\alpha_{0l}}{D_{1l}^2\rho_{0l}} + \frac{\alpha_{0g}}{D_{1g}^2\rho_{0g}}\right)}\ .$$

For the equation of state [5] the shock adiabat of one-component mixture is determined in the form

$$p_{i+1} = \frac{2\rho_{l(i+1)}\left(e^{(p)}(\rho_{l(i+1)}) - e_i\right) - 2\frac{p^{(p)}(\rho_{l(i+1)})}{\Gamma(\rho_{l(i+1)})} - p_0\left(\frac{\rho_{l(i+1)}}{\rho_{li}} - 1\right)}{\left(\frac{\rho_{l(i+1)}}{\rho_{li}} - 1\right) - \frac{2}{\Gamma(\rho_{l(i+1)})}} \qquad (8)$$

Here and further the subscript i = 0 corresponds to the initial unforced state, so, if i + 1 = 1 the parameters of media behind the shock wave front are mentioned. If i = 1 then the parameters behind the front of the reflected shock wave are defined by index i + 1 = 2.

For perfect gas the shock adiabat is used in the form [8]:

$$\frac{p_{i+1}}{p_i} = \frac{\rho_{g(i+1)}\cdot(\gamma+1) - \rho_{gi}\cdot(\gamma-1)}{\rho_{gi}\cdot(\gamma+1) - \rho_{g(i+1)}\cdot(\gamma-1)}\ .$$

When we model the adiabatic compression of the gas fraction in the shock wave we use the Poisson adiabat for perfect gas [8]:

$$\frac{p_{i+1}}{p_i} = \left(\frac{\rho_{g(i+1)}}{\rho_{gi}}\right)^{\gamma}.$$

The volume concentration in the passing shock wave is derived from equations (3) and (4):

$$\alpha_{g1} = \frac{\rho_1 - \rho_{l1}}{\rho_{g1} - \rho_{l1}}\ .$$

As a result of derivation we can obtain the parameters of the mixture after the reflection of the shock wave from the solid wall in the moving coordinates, where mass velocity equals U_1:

$$U_2 = \sqrt{(p_2 - p_1)\frac{(\rho_2 - \rho_1)}{\rho_2\rho_1}}\ ,\ D_2 = \sqrt{(p_2 - p_1)\frac{\rho_2}{\rho_1(\rho_2 - \rho_1)}} \qquad (9)$$

In the moving coordinates the condition of reflecting shock wave from the solid boundary is:

$$|U_1| = |U_2| \rightarrow (p_2 - p_1)\frac{(\rho_2 - \rho_1)}{\rho_2} = (p_1 - p_0)\frac{(\rho_1 - \rho_0)}{\rho_0} \qquad (10)$$

In the fixed coordinates from (9) we can obtain the velocity of the reflected shock wave

$$D_2' = D_2 - U_1 = \sqrt{(p_2 - p_1)\frac{\rho_2}{\rho_1(\rho_2 - \rho_1)}} = \frac{(p_2 - p_1)}{\rho_1 U_1} - U_1 \quad (11)$$

The presented set of algebraic equations (3), (8) – (11) for calculating density, void fraction, pressure and velocity of the shock wave in gas-liquid mixture is solved numerically.

B. The Analysis of Results of Simulation and Experiments

Fig. 1 displays the velocity of the front of the passing D_1 and reflected D_2 shock waves as functions of α_{g0} for the fixed pressure p_1, which were calculated according to the models of adiabatic and shock compression of gas component.

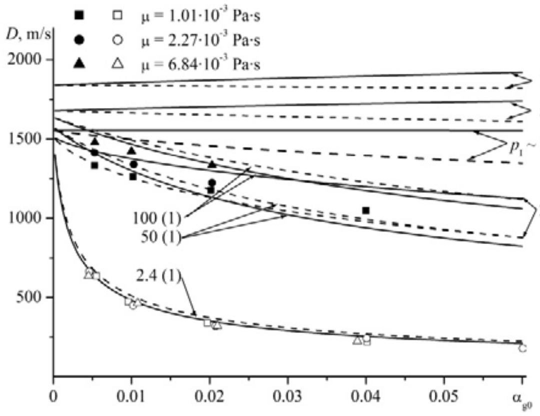

Fig. 1 The dependencies of velocities of passing (1) and reflected (2) shock waves from α_{g0} at different p_1 for models of adiabatic (solid line) and shock (dashed line) compression of gas component. Experimental data [2] are presented for p_1 = 2.4 MPa for passing (□, ○, Δ) and reflected shock waves (●, ■, ▲) in the media with different viscosity

The results of numerical simulation is in good agreement with experimental data for the passing shock waves. As for the reflected waves, in the experiments [2] the velocity of the front of the shock wave varies for different values of liquid viscosity, which depends on the content of glycerine in water. According to the experiments the increase of viscosity leads to the reduction of the velocity of the reflected shock wave. The model of shock compression is preferable to describe the experimental data for low-viscosity liquid. The experimental observations of reflected shock wave in the high-viscosity liquid is better described by the adiabatic compression model. The velocity of the reflected shock wave at pressure $p_1 \approx$ 14.32 MPa, when the adiabatic model of compression of gas component was used, does not depend on the initial void fraction α_{g0}: in Fig. 1 this value is emphasized by line $D \approx$ 1555 m/s.

In Fig. 2 the solid line depicts the calculated dependencies of velocity of passing (1) and reflected (2) shock wave from p_1 at given α_{g0} by using the models of adiabatic compression of gas component.

The obtained results correspond to the results of [10], where the gas-dynamical model was used. The velocity of the front of the passing shock wave decreases, when the initial gas concentration α_{g0} arises. For the reflected shock wave we define more precisely the characteristic point (point I in Fig. 2 at $p_1 \approx 14.32$ MPa), which was received first in [10] by using the gas-dynamic model. This point describes the change of regime of the shock wave flow: up to 14.32 MPa the increase of α_{g0} leads to reduction of velocity D_2 by the compression of gas component. For large amplitudes there observed the contrary dependence: the greater is the initial void fraction α_{g0}, the greater is the velocity of the reflected shock wave. This is closely connected with the amplitude of the pressure in the reflected shock wave and the decreasing of the influence of gas component. For large amplitudes of passing shock wave the governing factor is the nonlinear compressibility of the liquid. The calculations by using the model of shock compression of gas component (dashed line in Fig. 2) give smaller velocity of the reflected shock wave compared with velocity, which is obtained by using the adiabatic model. This happens due to the existence of restriction of degree of shock compression of gas component, which causes the increase of gas content in the reflected shock wave compared with adiabatic model.

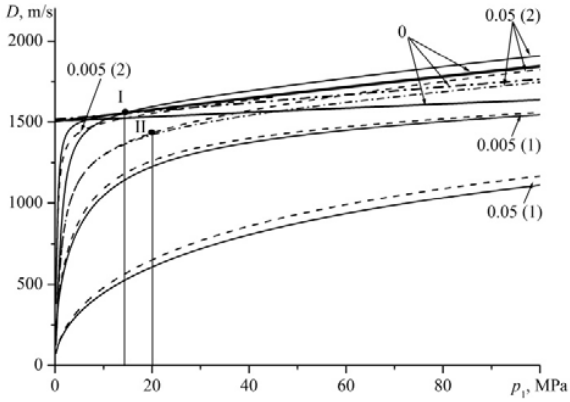

Fig. 2 The dependencies of velocities of passing (1) and reflected (2) shock waves from p_1 at different α_{g0} for models of adiabatic (solid line) and shock (dashed line) compression of gas component (nitrogen). The dash-dotted line marks the solution for the case of shock compression with linear equation of state

When the pressure p_1 increases, the calculations show the growth of D_2 according to the shock compression model while the diminishing of initial gas content, which asymptotically approaches the velocity of shock wave in clean liquid. In Fig. 2 point II marks the pressure value $p_1 \approx 20$ MPa (which corresponds to $p_2 \approx$ 66 MPa). When the pressure exceeds this value the nonlinear compressibility of liquid becomes the dominant factor forcing the velocity D_2 compared with calculations of shock compressibility of water by using the linear equation of state.

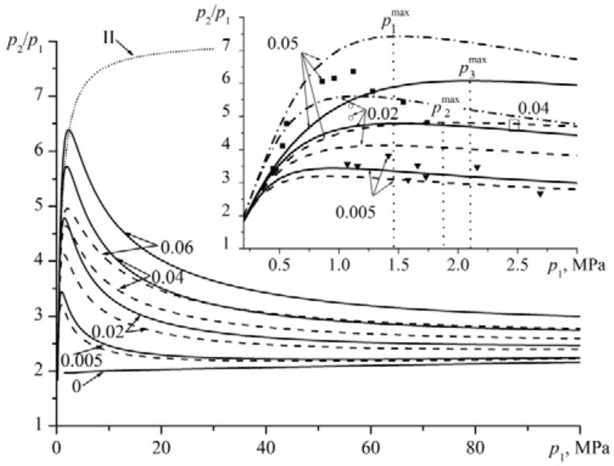

Fig. 3 The ratio p_2/p_1 in the reflected shock wave as function of p_1 for different α_{g0}. The calculations are made by using the models of adiabatic (solid line), shock (dashed line) and isothermal (dash-dotted line) compression of gas component. Line II marks the limit of the compression according to the shock compression model. The experimental data are displayed by symbols: [2]; , ■,▼ [3]

In Fig. 3 the calculated coefficient of reflection p_2/p_1 is presented as function of p_1. The dotted line II displays the degree of reflection of a shock wave in nitrogen, which is the asymptote for the degree of reflection of gas-liquid mixture (water-nitrogen) at the increase of initial void fraction. The additional fragment in Fig. 3 shows the results of calculations according to the model of isothermal compressibility of gas component (dash-dotted line) and experimental data from [2] and [3]. Calculations show the growth of pressure in the reflected shock wave with increase of α_{g0} and its reduction with decrease of p_1. The ratio p_2/p_1 according to adiabatic model in the reflected wave is greater, than in the model of shock compression, for the same α_{g0}. For α_{g0}= 0.05 the maximum value of coefficient of reflection is plotted in the case of isothermal (p_1^{max}), shock (p_2^{max}) and adiabatic (p_3^{max}) models of compression of gas component. The experimental data [3] for α_{g0}= 0.02 and α_{g0}= 0.05 are in good agreement with calculated curves. For $p_1 \approx 1.1$ MPa the experimental points lay below the line corresponding the isothermal model of gas phase compression in the mixture. At higher values of p_1 the experimental points shift closer to the curve, which is obtained by using the adiabatic assumption for gas component up to $p_1 = 1.5$ MPa. At pressures $p_1 > 2.0$ MPa the model of shock compression gives the best approximation of experimental data (Fig. 3 for α_{g0}= 0.04 [2]). For small gas contents (α_{g0}= 0.005) both the models gives the similar results, which correspond to the experiments [3].

III. The Amplification of a Shock Wave in a Channel of Variable Cross Section Filled with Liquid Containing Explosive Bubbles

A. The Problem Formulation

The liquid, which contains bubbles of mixture of chemically active gases, can be treated as a high explosive matter, where detonation waves can occur with amplitude of several hundreds of atmospheres [11]. Such the explosive can be efficiently used in the industrial technologies for a short time increase of the pressure in local zones, but the risk of occurrence of accidents is very high because of possibility of self-burning.

In [12] the problem of influence of the dynamics of gas bubbles with chemically active mixture inside on the contraction of vapor bubbles in deuterated acetone has been solved in the presence of a shock wave in the narrowed channel. It was observed that the shock wave initiates the exothermal reactions inside the gas bubbles, its amplitude increases and causes the supercompression of vapor bubbles.

At the present paper the research of a compression wave propagation in liquid with explosive bubbles is studied for a round tube of length L and variable cross section $S(x)$. To solve the non-stationary problem the model from [12] is used, in the absence of vapor bubbles:

$$\frac{\partial}{\partial t}(\alpha_L \rho_L S) + \frac{\partial}{\partial x}(\alpha_L \rho_L u S) = 0, \qquad (12)$$

$$\frac{\partial (nS)}{\partial t} + \frac{\partial}{\partial x}(nuS) = 0\,, \qquad (13)$$

$$\frac{\partial}{\partial t}(\alpha_L \rho_L\, uS) + \frac{\partial}{\partial x}(\alpha_L \rho_L\, u^2 S) = -S\frac{\partial p_l}{\partial x} - \tau\Sigma\,, \qquad (14)$$

where u is the velocity of gas-liquid flow, n is the number of bubbles in a unit volume, $\Sigma = 2\sqrt{\pi S}$ is the perimeter of cross section, τ is the tension coefficient [12]. Under the influence of variable pressure the bubbles oscillate, that change the local volume concentration of gas in the mixture.

B. The Results of Numerical Simulation

A shock wave in the undisturbed liquid, which is in the dynamical equilibrium with gas bubbles, is initiated at the left ending of the tube by velocity jump up to value u_1 (simulation of go in piston). The shock wave can increase or go down depends on the parameters of bubbly liquid (radii and concentration of bubbles) and the geometric characteristics of the channel $S(x)$.

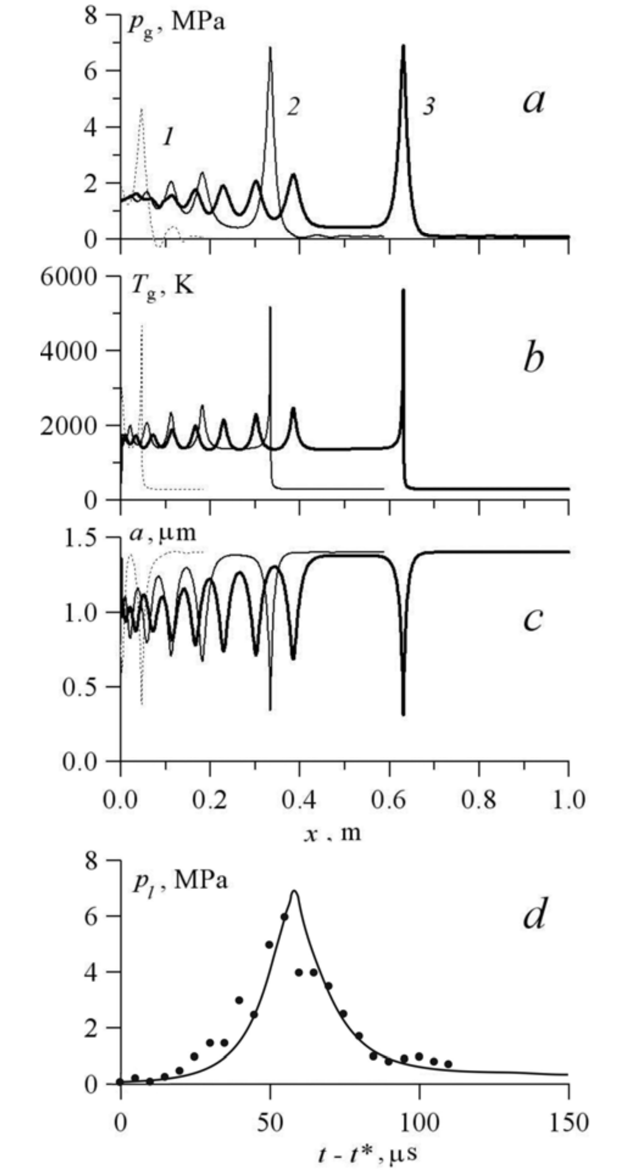

Fig. 4. The spatial distribution of liquid pressure (*a*), gas temperature (*b*) and bubble radius (*c*) for the time moments 0.1 ms (curves 1), 0.4 ms (curves 2) and 0.7 ms (curves 3) at propagation of a detonation wave in the tube of constant cross section. Comparison between experimental (dots) and calculated (solid line) distributions of liquid pressure in time for the fixed cross section of the tube (*d*).

In Fig. 4 the spatial profiles of liquid pressure p_l, gas temperature T_g and bubble radius a at different time moments are presented along the tube of constant cross section, which contains liquid with gas bubbles. The inner of bubbles consists of the mixture of acetylene (C_2H_2) and oxygen (O_2). The results are obtained for the following parameters of the problem [13]: $u_1 = 2$ m/s, $\alpha_{g0} = 0.004$, $a_0 = 1.4$ mm, $p_0 = 0.1$ MPa, $\rho_{l0} = 10^3$ kg/m^3, $T_0 = 293$ K, $\lambda_g = 2.5 \cdot 10^{-2}$ kg·m/(s^3·K), $L = 1$ m, tube diameter $d = 0.1$ m.

According to graphics the detonation wave propagates as an impulse of high pressure (~7 MPa) and temperature (~5000 K), which is pursued by oscillating wave. The amplitude of detonation wave achieves its maximum value after 0.3 milliseconds from the beginning of piston action, the velocities of the front impulse and oscillating wave are 1000 and 700 m/s correspondingly. In Fig. 4*d* the comparison between the calculations and experimental data from [13] is presented for time evolution of pressure during the passing of the detonation shock wave. A good correlation between experimental and numerical data is observed.

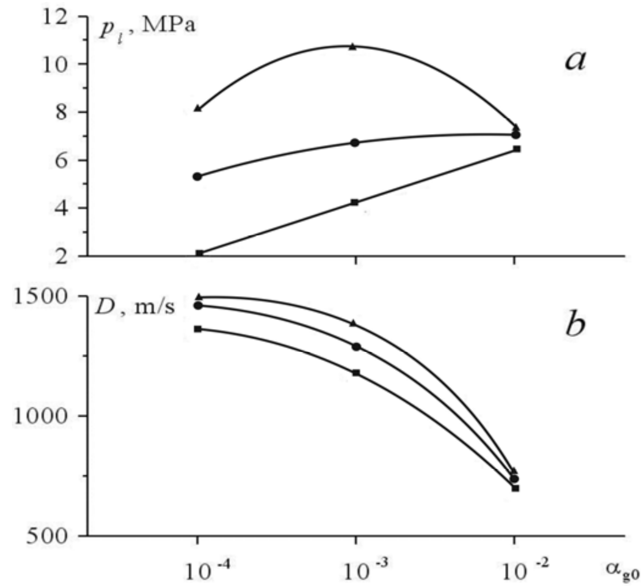

Fig. 5 The maximum pressure in the detonation wave (*a*) and the velocity of its front (*b*) as functions of initial volume concentration of gas α_{g0}. ● – constant cross section, ■ – expanding channel, ▲ – narrowed channel

In Fig. 5 the dependencies of maximum pressure in the wave and velocity of its propagation D from the volume concentration α_{g0} are plotted for three different configurations of the tube:

$$S(x) = \frac{\pi}{4}\left(d_0 + \left(\frac{x}{L}\right)^2 (d_1 - d_0)\right)^2,$$

where $L = 1$ m, $d_0 = d_1 = 0.1$ m (constant cross section), $d_0 =$ 0.2 m, $d_1 = 0.05$ m (narrowed channel), $d_0 = 0.05$ m, $d_1 = 0.2$ m (expanding channel).

The geometry of the channel essentially influences on the maximum amplitude of the detonation wave for $\alpha_g \le 10^{-3}$: the maximum value is achieved for narrowed channel, the minimum value corresponds to the expanding channel. For large void fraction $\alpha_{g0} = 10^{-2}$ the amplitude of the wave is approximately the same for three different configurations of the tube. The velocity of the detonation wave also depends on the geometrical characteristics of the tube, but this dependence is rather weak.

IV. The Propagation of Compression Waves in Bubble Layers of Limited Size

A. The Problem Formulation

Suppose that in the region filled with liquid a cylindrical bubble layer exists, which is situated in parallel direction to the axis z (the longitudinal size is much greater than its transversal size). Consider two-dimensional wave perturbations. Such the perturbations arises, for instance, at plain slash on the liquid with bubble layer of limited size.

Let us use the set of equations of conservation of mass, momentum, energy and bubble concentration in the bubbly liquid supposing the equal velocities of liquid and gas [14]:

$$\frac{\mathrm{d}\rho_i}{dt} + \rho_i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \ (i = l, g)\,,\ \frac{\mathrm{d}n}{dt} + n\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0\,,$$

$$\rho\frac{du}{dt} + \frac{\partial p_l}{\partial x} = 0\,,\ \rho\frac{\mathrm{d}v}{dt} + \frac{\partial p_l}{\partial y} = 0\,, \qquad (15)$$

$$\frac{dp_g}{dt} = -\frac{3\gamma p_g}{a}w - \frac{3(\gamma-1)}{a_0}q\,,\ w = \frac{da}{dt}\,,$$

$$\alpha_l + \alpha_g = 1\,,\ \alpha_g = \frac{4}{3}\pi n a^3\,,\ \rho_i = \rho_i^0\alpha_i\,,\ \rho = \rho_g + \rho_l\,,$$

where γ is the ratio of specific heats for the gas, p_i is the phase pressure, ρ_i^0 is the reduced density of phases, q is the intensity of heat exchange [14], w is the radial velocity of a bubble. The velocities u and v are directed along the axes x and y. Subscripts $i = l$, g mark the parameters of liquid and gas phases correspondingly.

To describe the radial dynamics of a bubble, following the assumption made in [15], we shall suppose that $w = w_R + w_A$, where w_R is described by Rayleigh-Lamb equation, and w_A is defined from solving of the problem of spherical unloading on the sphere surface of radius a in the moving liquid in the acoustic assumption:

$$a\frac{dw_R}{dt} + \frac{3}{2}w_R^2 + 4\nu_l\frac{w_R}{a} = \frac{(p_g - p_l)}{\rho_l^0}\,,\ w_A = \frac{p_g - p_l}{\rho_l^0 C_l \alpha_g^{1/3}}\,, \qquad (16)$$

where ν_l is the kinematic viscosity of liquid, C_l is the speed of sound of liquid ($C_l = 1500$ m/s).

We shall suppose that the liquid is the acoustically compressible and the gas is perfect:

$$p_l = p_0 + C_l^2(\rho_l^0 - \rho_{l0}^0)\,,\ p_g = \rho_g^0 R T_g\,, \qquad (17)$$

where R is the gas constant. Here subscripts 0 denote the parameters of undisturbed state.

The set of equations allows to describe the dynamics of waves with high gradients, when the bubble compression is driven not only by inertia of liquid, but also by acoustic unloading on the bubbles surface via liquid compressibility. In addition if $\alpha_g = 0$ we can derive the wave equation from this set of equations for acoustically compressible liquid. When we investigate the interaction of the waves in clean liquid this fact allows to use through-type methods of calculation.

The set of equations (15)–(17) was solved numerically by explicit scheme. This scheme does not need for artificial viscosity, because the equations contain natural dissipation due to phase transfer and acoustical effects [16].

B. The Results of Numerical Simulation

In Fig. 6 the effects of nonlinearity and two space dimensions are illustrated for the wave impulse in the form:

$$p^0(t, y) = p_0 + \Delta p_0 \exp[\psi(t_*)],\ \psi(t_*) = -\left(\frac{t - t_*/2}{t_0}\right)^2,$$

where p_0 is the initial pressure, Δp_0 is the pressure impulse amplitude, t_* is the duration of the impulse, t_0 is the parameter, which defines the width of initial impulse. The impulse acts through the boundary $x_0 = 0$ and propagates into the channel filled with water, which contains an air bubble layer near the back ending of the calculation region.

In the numerical simulation the following parameters were used: $a_0 = 10^{-3}$ m, $p_0 = 0.1$ MPa, $\rho_{l0}^0 = 10^3$ kg/m^3, $T_0 = 300$ K, $c_g = 1006$ J/(kg·K), $\lambda_g = 2.6 \cdot 10^{-2}$ kg·m/(s^3·K).

The frontal influence of the pressure impulse with time width $t_* = 10^{-4}$ s on the solid wall, which is partly covered with bubble layer, is studied. We shall suppose, that the bubble layer is positioned in the middle of the wall. The parameters of the calculated region are: $l_x = l_y = 0.05$ m, $L_x = 0.5$ m, $L_y = 0.95$ m. The sensor D is placed on the wall behind the layer and has the coordinates $x_0 = 0.5$ m, $y_0 = 0.475$ m. The dash-dotted line corresponds to the case, when the bubble layer is absent. The solid and dotted lines denote the volume concentration of gas in bubble layer $\alpha_{g0} = 10^{-2}$ and 10^{-3} correspondingly.

Fig. 6 shows that bubble layer of limited size essentially decreases the amplitude of the impulse impact on the wall. The impulse of initial amplitude $\Delta p_0 = 0.3$ MPa after passing the bubble layer with volume concentration $\alpha_{g0} = 10^{-2}$ (10^{-3}) has the amplitude 0.05 MPa (0.2 MPa). In the absence of bubble layer the impulse has the amplitude approximately 0.6 MPa. The presence of bubble layer increases the duration of impulse impact on the wall.

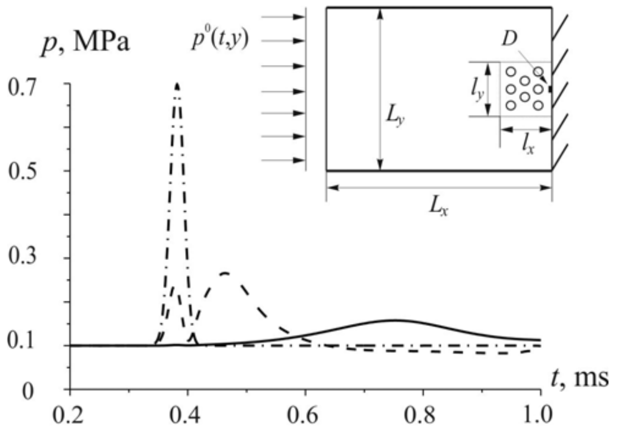

Fig. 6 The frontal influence of pressure impulse on the wall, which is partly covered by bubble layer

Fig. 7 illustrates the compression wave propagation, which is caused by instantaneous pressure jump at the boundaries $x_0 = 0$ and $y_0 = 0$. The boundary conditions are:

$$p^0(t,y) = p^0(t,x) = p_0 + \Delta p_0 .$$

To model this problem we consider the evolution of waves in square region ($L_x = L_y = 1.5$ m), filled with water, accounting for the presence of the bubble layer of squared form ($l_x = l_y = 0.05$ m). The other parameters are the same as in Fig. 6. The oscillograms in Fig. 7 a correspond to the sensor data, which is placed in the middle of bubble layer. The solid and dotted lines describe the situations, when the pressure waves propagate in the liquid with bubble layer and in the clean liquid. The dash-dotted line corresponds to the situation, when the pressure impulse is initiated only at one boundary ($x_0 = 0$). For visualization in Fig. 7 b the profile of pressure at time moment $t = 0.8$ ms is presented.

As it can be seen from Fig. 7 a (dotted line) the maximum pressure of about 0.3 MPa is achieved at the bisector of two boundaries, when the pressure impulse of 0.1 MPa is initiated at these boundaries. The sensor detects the rising of pressure up to 0.3 MPa, and afterwards this value falls down to 0.2 MPa as the sum of pressure impulses at the boundaries has the same value.

Consider the realization of such the wave picture more precisely. After instantaneous jump of pressure from 0.1 MPa up to 0.2 MPa at the boundaries $x_0 = 0$ and $y_0 = 0$ the propagation of waves start in the direction of the boundaries $x_0 = L_x$ and $y_0 = L_y$, and the total amplitude reaches the maximum value of 0.3 MPa. Simultaneously from the point of convergence of the two waves the compression waves start to propagate in the direction of boundaries $x_0 = 0$ and $y_0 = 0$

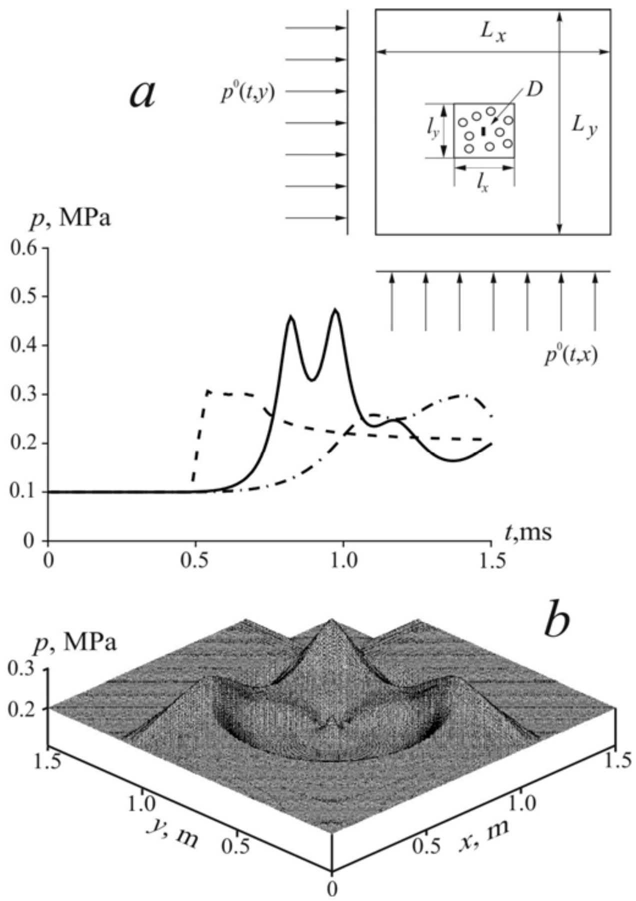

Fig. 7 The evolution of two wave impulses impacting via related boundaries $x_0 = 0$ and $y_0 = 0$

Reflecting from these boundaries, as they are the free boundaries, the secondary waves drop the pressure amplitude from 0.3 MPa down to 0.2 MPa. In the case than the middle of the channel is occupied by the bubble layer the sensor at this point detects the impulse of pressure, which amplitude is two times greater than the pressure in the absence of bubble layer. The sensor detects the rarefaction wave passing after compression wave. In the case, when the wave is originated from the single boundary $x_0 = 0$ and the bubble layer is put in the middle of the region (dash-dotted line in Fig. 7 a), the sensor detects the pressure amplitude, which equals the amplitude of pressure, arising from two initial impulses in the absence of bubble layer.

V. Conclusion

The investigation of characteristics of passing and reflected shock waves in gas-liquid media shows that the model of shock compression of gas component gives the best approximation with experimental data on reflection of shock waves in slight viscosity liquid. The experiments with large viscosity are better described by the model of adiabatic compression. For a small gas contents the shock compression and adiabatic models give the similar results, which are in good agreement with experimental data.

The characteristic point for the velocities of reflected waves at pressure p_1 =14.32 MPa is obtained for the adiabatic compression model, which defines the constant value of velocity of the shock wave front and does not depend on the initial void fraction. The influence of nonlinear compressibility of liquid becomes important for the reflected wave at p_1> 20 MPa.

It was obtained that the detonation wave arises in the liquid, which contains the explosive bubbles. The detonation wave in the tube consists of single wave with high amplitude and subsequent oscillating wave. The amplitude increases with growth of the void fraction up to 10^{-3}. At large volume concentration it stops to increase, and in the case of expanding channel the amplitude even drops.

As a result of research of compression waves propagation in two-dimensional region it was found that the bubble layer can drop or rise the amplitude of the initial pressure impact. The type of influence of bubble layer on the amplitude of the pressure impulse depends on the size of bubble layer, void fraction and bubble radius. The bubble layer, which is placed close to the solid wall, influences the wave propagation in some distance.

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