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Nonlinear Dynamic Analysis of Base-Isolated Structures Using a Partitioned Solution Approach and an Exponential Model

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Abstract**—**The solution of the nonlinear dynamic equilibrium equations of base-isolated structures adopting a conventional monolithic solution approach, i.e. an implicit single-step time integration method employed with an iteration procedure, and the use of existing nonlinear analytical models, such as differential equation models, to simulate the dynamic behavior of seismic isolators can require a significant computational effort. In order to reduce numerical computations, a partitioned solution method and a one dimensional nonlinear analytical model are presented in this paper. A partitioned solution approach can be easily applied to base-isolated structures in which the base isolation system is much more flexible than the superstructure. Thus, in this work, the explicit conditionally stable central difference method is used to evaluate the base isolation system nonlinear response and the implicit unconditionally stable Newmark's constant average acceleration method is adopted to predict the superstructure linear response with the benefit in avoiding iterations in each time step of a nonlinear dynamic analysis. The proposed mathematical model is able to simulate the dynamic behavior of seismic isolators without requiring the solution of a nonlinear differential equation, as in the case of widely used differential equation model. The proposed mixed explicit-implicit time integration method and nonlinear exponential model are adopted to analyze a three dimensional seismically isolated structure with a lead rubber bearing system subjected to earthquake excitation. The numerical results show the good accuracy and the significant computational efficiency of the proposed solution approach and analytical model compared to the conventional solution method and mathematical model adopted in this work. Furthermore, the low stiffness value of the base isolation system with lead rubber bearings allows to have a critical time step considerably larger than the imposed ground acceleration time step, thus avoiding stability problems in the proposed mixed method.

Keywords**—**Base-isolated structures, earthquake engineering, mixed time integration, nonlinear exponential model.

I. Introduction

SEISMIC base isolation has become a widely accepted technique for earthquake protection of buildings and bridges. The concept of base isolation is quite simple: the introduction of a flexible base isolation system between the foundation and the structure allows to move the period of the latter away from the predominant period of the ground motion with the benefit of reducing floor accelerations, story shears and interstory drifts [1], [2].

A conventional monolithic solution approach, characterized by the use of an implicit single-step time integration method adopted with the Newton-Raphson or the pseudo-force iteration procedure, is generally employed to solve the nonlinear dynamic equilibrium equations of seismically isolated structures subjected to earthquake excitation [3]. Existing phenomenological models [4]-[7] and plasticity-based models [8]-[10] can be adopted to predict the dynamic behavior of seismic devices.

Among conventional monolithic solution methods and nonlinear mathematical models, the solution algorithm and analytical model proposed by [4], both implemented in the computer program 3D-BASIS-ME-MB [11], are presented and adopted in this paper because specifically developed for nonlinear dynamic analysis of base-isolated structures with either elastomeric and/or sliding isolation systems. In this monolithic solution approach, the equations of motion are solved using the implicit unconditionally stable Newmark's constant average acceleration method with the nonlinear restoring forces of the seismic isolators being represented as pseudo-forces. An iterative procedure consisting of corrective pseudo-forces is employed within each time step until equilibrium is achieved. The analytical model, based on the set of two first-order ordinary nonlinear differential equations proposed by [12], is able to represent the uniaxial and biaxial behavior of both elastomeric and sliding isolation bearings.

Since the solution of the nonlinear dynamic equilibrium equations using the above-described conventional implicit single-time step integration method and the use of the differential equation model can increase the computational effort very significantly, in this work, a partitioned solution approach [13] and a one dimensional (1D) nonlinear analytical model are proposed in order to reduce numerical computations.

Partitioned time integration methods have been developed by several authors in the last 30 years to allow different time steps or time integration algorithms or both to be used in different spatial subdomains of the mesh [14]-[18]. In the case of seismically isolated structures, the above-mentioned partitioned solution approach can be easily applied being the decomposition of the discrete structural model of such structures driven by physical considerations: The base

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isolation system is much more flexible than the superstructure to decouple the latter from the earthquake ground motion. Thus, an explicit conditionally stable time integration method can be used to evaluate the base isolation system response, and an implicit unconditionally stable time integration method can be adopted to predict the superstructure response with the remarkable benefit in avoiding the iterative procedure within each time step of a nonlinear time history analysis required by conventional implicit time integration methods.

The 1D Nonlinear Exponential Model (NEM), able to simulate the dynamic response of seismic isolators having a typical symmetric softening force-displacement hysteresis loop within a relatively large displacements range, such as elastomeric and sliding bearings, allows to avoid the numerical solution of the nonlinear differential equations required in the analytical model proposed by [4].

The proposed partitioned solution method and nonlinear analytical model are adopted to analyze a three-dimensional (3D) base-isolated structure with a lead rubber bearing system subjected to earthquake excitation. The numerical results and the computational time are compared with those obtained by using the solution algorithm and the differential equation model proposed by [4] in order to demonstrate the accuracy and the computational efficiency of the proposed solution approach and analytical model.

II. Equations of Motion

In this section, the equations of motion for a typical base-isolated structure subjected to earthquake excitation are formulated. The 3D discrete structural model of such a system can be decomposed into two substructures: the n-story superstructure, considered to remain elastic during the earthquake excitation, and the base isolation system consisting of seismic isolation bearings and a full diaphragm above the seismic devices [19].

In this work, a global coordinate system, denoted with upper case letters X, Y, and Z, is attached to the mass center of the base isolation system. Each floor diaphragm is assumed to be infinitely rigid in its own plane, the columns are assumed to be axially inextensible, the beams are considered to be axially inextensible and flexurally rigid, and the isolation devices are considered rigid in the vertical direction. Thus, the total number of Degrees of Freedom (DOFs) of the 3D structural model of a base-isolated structure is equal to $3n + 3$. The i-th floor diaphragm has three DOFs defined at the diaphragm reference point o_i, which is vertically aligned to the global coordinate system origin O. The DOFs for the i-th floor are the translation u_{ix} along the X-axis, the translation u_{iy} along the Y-axis, and the rotation $u_{i\vartheta}$ about the vertical axis Z; u_{ix} and u_{iy} are defined relative to the ground. The $3n$ superstructure DOFs are listed in the displacement vector $\boldsymbol{u}_s$ whereas the three DOFs of the base isolation system are listed in the displacement vector $\boldsymbol{u}_b$. The i-th diaphragm mass is lumped in its mass center (MC_i) which is also the geometric center.

The earthquake excitation is defined by the horizontal ground acceleration $\ddot{u}_g(t)$ whose line of action is defined by the angle α_g that the epicentral direction forms with the X-axis.

The equations of motion of the 3D discrete structural model of a base-isolated structure are:

$$\begin{bmatrix} \boldsymbol{m}_b & \boldsymbol{0}^T \\ \boldsymbol{0} & \boldsymbol{m}_s \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}}_b \\ \ddot{\boldsymbol{u}}_s \end{Bmatrix} + \begin{bmatrix} \boldsymbol{c}_b + \boldsymbol{c}_1 & \boldsymbol{c}^T \\ \boldsymbol{c} & \boldsymbol{c}_s \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{u}}_b \\ \dot{\boldsymbol{u}}_s \end{Bmatrix} + \begin{bmatrix} \boldsymbol{k}_b + \boldsymbol{k}_1 & \boldsymbol{k}^T \\ \boldsymbol{k} & \boldsymbol{k}_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_b \\ \boldsymbol{u}_s \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{f}_n \\ \boldsymbol{0} \end{Bmatrix} = -\begin{bmatrix} \boldsymbol{m}_b & \boldsymbol{0}^T \\ \boldsymbol{0} & \boldsymbol{m}_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{r}_b \\ \boldsymbol{r}_s \end{Bmatrix} \ddot{\boldsymbol{u}}_g, \tag{1}$$

with

$$\boldsymbol{c} = \begin{bmatrix} -\boldsymbol{c}_1 & \boldsymbol{0} \end{bmatrix}^T, \tag{2}$$

$$\boldsymbol{k} = \begin{bmatrix} -\boldsymbol{k}_1 & \boldsymbol{0} \end{bmatrix}^T, \tag{3}$$

$$\ddot{\boldsymbol{u}}_g = \begin{Bmatrix} \ddot{u}_g \cos(\alpha_g) & \ddot{u}_g \sin(\alpha_g) & 0 \end{Bmatrix}^T, \tag{4}$$

where $\boldsymbol{m}_s$, $\boldsymbol{c}_s$, and $\boldsymbol{k}_s$ are the superstructure mass, damping, and stiffness matrices, respectively. Taking into account that the base isolation system can include linear and nonlinear isolation elements, $\boldsymbol{m}_b$ is the isolation system mass matrix, $\boldsymbol{c}_b$ is the damping matrix of linear viscous isolation elements, $\boldsymbol{k}_b$ is the stiffness matrix of linear elastic isolation elements, and $\boldsymbol{f}_n$ is the resultant nonlinear forces vector of nonlinear elements. In addition, $\boldsymbol{c}_1$ and $\boldsymbol{k}_1$ are the viscous damping and stiffness matrices of the superstructure first story, $\boldsymbol{r}_s$ and $\boldsymbol{r}_b$ are the superstructure and base isolation system influence matrices, respectively, and $\ddot{\boldsymbol{u}}_g$ is the ground acceleration vector.

III. Conventional Solution Method and Analytical Model

Among conventional monolithic solution methods generally used to solve the nonlinear dynamic equilibrium equations of structures subjected to earthquake excitation, the solution algorithm proposed by [4] is presented in the following because specifically developed for the analysis of base-isolated structures. In this monolithic solution approach, the equations of motion are discretized using the implicit unconditionally stable Newmark's constant average acceleration method. Since the nonlinear forces vector $\boldsymbol{f}_n$ is function of both displacement and velocity vectors at time $t + \Delta t$, it is transferred to the right-hand side of (1) and treated as pseudo-forces vector. Thus, an iterative procedure consisting of corrective pseudo-forces is adopted within each time step until equilibrium is achieved. For brevity, in this paper, the above-described implicit time integration method adopted in conjunction with the pseudo-force approach is referred to as the Pseudo-Force Method (PFM).

Nagarajaiah et al. [4] also proposed an analytical model able to represent the uniaxial and biaxial behavior of both elastomeric and sliding bearings. For an elastomeric bearing, the nonlinear restoring forces along the orthogonal directions X and Y are described by:

$$f_{nx} = \alpha \frac{f_y}{y} u_x + (1-\alpha) f_y z_x , \tag{5}$$

$$f_{ny} = \alpha \frac{f_y}{y} u_y + (1-\alpha) f_y z_y , \tag{6}$$

where α is the post-yield to the pre-yield stiffness ratio, f_y is the yield force, y is the yield displacement, u_x and u_y represent the displacements of the isolation device in the X and Y directions, respectively.

For a flat sliding bearing, the nonlinear restoring forces along the two orthogonal directions X and Y are given by:

$$f_{nx} = \mu N z_x , \tag{7}$$

$$f_{ny} = \mu N z_y , \tag{8}$$

in which N is the vertical load carried by the bearing, and μ is the coefficient of sliding friction, which depends on the bearing pressure and the instantaneous velocity of sliding. The dimensionless variables z_x and z_y are governed by the following system of two coupled first-order ordinary nonlinear differential equations proposed by [12]:

$$\begin{Bmatrix} \dot{z}_x \, y \\ \dot{z}_y \, y \end{Bmatrix} = \begin{Bmatrix} A \, \dot{u}_x \\ A \, \dot{u}_y \end{Bmatrix} - \begin{bmatrix} z_x^2 \left[\gamma \, \mathrm{sgn}(\dot{u}_x z_x) + \beta \right] & z_x z_y \left[\gamma \, \mathrm{sgn}(\dot{u}_y z_y) + \beta \right] \\ z_x z_y \left[\gamma \, \mathrm{sgn}(\dot{u}_x z_x) + \beta \right] & z_y^2 \left[\gamma \, \mathrm{sgn}(\dot{u}_y z_y) + \beta \right] \end{bmatrix} \begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \end{Bmatrix}, \tag{9}$$

in which A, β, and γ are dimensionless quantities that control the shape of the hysteresis loop, $\dot{u}_x$ and $\dot{u}_y$ are the velocities that occur at the isolation device in X and Y directions, respectively. The unconditionally stable semi-implicit Runge-Kutta method [20] is proposed by [4] to solve the differential equations governing the behavior of each nonlinear isolation element.

The biaxial interaction can be neglected when the off-diagonal elements of the matrix in (9) are replaced by zeros. This results in a uniaxial model with two independent elements in the two orthogonal directions. For brevity, in this paper, the above-described uniaxial analytical model is referred to as the Bouc-Wen Model (BWM).

IV. Proposed Partitioned Solution Method and Analytical Model

A partitioned solution method can be easily adopted to analyze seismically base-isolated structures being the base isolation system much more flexible than the superstructure to decouple the latter from the earthquake ground motion. In this work, the explicit second order central difference method is proposed to predict the nonlinear response of the base isolation system whereas the implicit second order Newmark's constant average acceleration method, also called trapezoidal rule, is proposed to compute the linear response of the superstructure. Thus, in each time step of a nonlinear time history analysis, the proposed partitioned solution approach requires first the solution of the base isolation system response, then these results are used for the evaluation of the superstructure response.

The proposed Mixed Explicit-Implicit single-time step integration Method (MEIM) is conditionally stable because the central difference method is employed to compute the nonlinear response of the base isolation system. As will be shown in Section V, in seismically base-isolated structures, the typical low stiffness value of the base isolation system generally allows to have a critical time step Δt_{cr} larger than the short time step used to define the ground acceleration accurately. Considering the 3D discrete structural model of a base-isolated structure, the critical time step $\Delta t_{cr} = T/\pi$ can be evaluated considering the lowest natural period given by the following eigenvalue problem:

$$\boldsymbol{k}_b^h \, \boldsymbol{\Phi} = \boldsymbol{m}_b \, \boldsymbol{\Phi} \, \boldsymbol{\Omega}^2 , \tag{10}$$

where $\boldsymbol{k}_b^h$ is the stiffness matrix of the base isolation system assembled using the highest horizontal stiffness of each nonlinear element, $\boldsymbol{\Phi}$ is the modal matrix, and $\boldsymbol{\Omega}^2$ the spectral matrix of the eigenvalue problem.

In the following, a 1D NEM, able to simulate the dynamic behavior of seismic isolators having a continuously decreasing stiffness with increasing displacement, is proposed.

The continuously decreasing tangent stiffness function $k_t(u)$ can be expressed by the following two mathematical expressions, valid for a loading and an unloading curve, respectively:

$$k_t(u) = k_2 + (k_1 - k_2) \, e^{-a(u-u_{min})}, \quad (\dot{u} > 0) \tag{11}$$

$$k_t(u) = k_2 + (k_1 - k_2) \, e^{-a(u_{max}-u)}, \quad (\dot{u} < 0) \tag{12}$$

where k_1 and k_2 are the initial and the asymptotic values of the tangent stiffness, u_{max} and u_{min} are the horizontal displacement values at the most recent point of unloading and loading, respectively, and a is a parameter that defines the transition from k_1 to k_2. Integrating (11) and (12), the following nonlinear hysteretic restoring force is obtained:

$$f_n(u)=f_n(u_{min})+k_2(u-u_{min})-\frac{b}{a}\left[e^{-a(u-u_{min})}-1\right],\ (\dot{u}>0) \quad (13)$$

$$f_n(u)=f_n(u_{max})-k_2(u_{max}-u)+\frac{b}{a}\left[e^{-a(u_{max}-u)}-1\right],\ (\dot{u}<0) \quad (14)$$

where $b=k_1-k_2$.

It is worth noticing that the presented analytical model requires the evaluation of only three parameters, i.e. k_1, k_2, and a, whereas, in the uniaxial analytical model presented in Section III, the number of parameters to be identified is equal to six for both elastomeric and flat sliding bearings.

V. Numerical Application

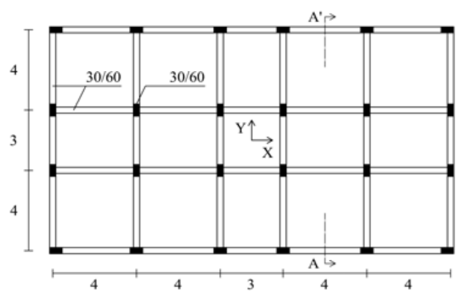
In the following, the nonlinear dynamic response of a 3D base-isolated structure subjected to earthquake excitation is predicted using the PFM and the proposed MEIM. The BWM is adopted when the nonlinear time history analysis is performed with the PFM (PFM-BWM), whereas the proposed NEM is employed when the nonlinear dynamic analysis is carried out using the MEIM (MEIM-NEM). The main aim of the following numerical application is to demonstrate the significant reduction of the computational effort due to the use of the proposed partitioned solution approach and nonlinear analytical model.

A. Analyzed 3D Base-Isolated Structure

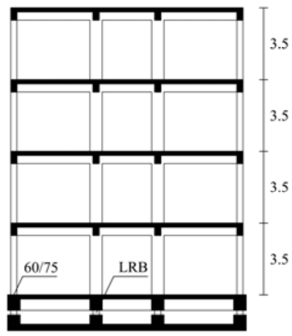
The superstructure is a four-story reinforced concrete structure with plan dimensions 19 m x 11 m, and story height h = 3.5 m. The weight of the superstructure is 9921.24 kN and the first three natural periods are 0.33 s, 0.33 s, and 0.26 s, respectively. Each superstructure diaphragm mass includes the contributions of the dead load and live load on the floor diaphragm and the contributions of the structural elements and of the nonstructural elements between floors.

The base isolation system, having a total weight of 3006.44 kN, consists of an orthogonal mesh of foundation beams having rectangular cross section with dimensions 60 cm x 75 cm, and 24 identical Lead Rubber Bearings (LRBs), positioned centrically under all columns.

As a result of the kinematic constraints assumed in Section II, the total number of DOFs, defined relative to the ground, is equal to 15. The typical floor plan and a section of the analyzed 3D base-isolated structure are shown in Fig. 1.



(a)



(b)

Fig. 1 Four-story reinforced concrete base-isolated structure. (a) typical floor plan; (b) section A-A'

TABLE I
Analytical Models Parameters

BWM	f_y [N]	y [m]	α	A	β	γ
	45400	0.017	0.10	1	0.5	0.5
NEM	k_1 [N/m]	k_2 [N/m]	a			
	4513478	265499	50			

The base isolation system has been designed in order to provide an effective isolation period T_{eff} = 2.50 s and an effective viscous damping υ_{eff} = 0.15 at the design displacement d_d = 0.50 m. Each elastomeric bearing has a yield force f_y = 45400.3 N, a yield displacement y = 0.017 m and a post-yield to pre-yield stiffness ratio α = 0.10.

B. Analytical Models Parameters

Table I shows the parameters of the two analytical models adopted to simulate the dynamic behavior of each LRB.

Fig. 2 illustrates the theoretical force-displacement hysteresis loops produced by use of the BWM and the NEM. They are obtained, as done in experimental tests, by applying a sinusoidal harmonic displacement having amplitude equal to 0.50 m and frequency of 0.40 Hz. It can be seen that the two analytical models adopting the parameters listed in Table I can reproduce hysteresis loops having the same area and effective stiffness.

C. Dynamic Response of the 3D Base-Isolated Structure

Harmonic ground motion, having amplitude $\ddot{u}_{g0}$ = 2.5 m/s^2, frequency ω_g = 2π rad/s, and time duration t_d = 20 s, is imposed with an angle α_g equal to π/6. The time step of the harmonic earthquake excitation is chosen equal to 0.005 s because normally 200 points per second are used to define accurately an acceleration record [3].

Table II gives the Nonlinear Time History Analyses (NLTHAs) results obtained using the PFM-BWM and the proposed MEIM-NEM, both implemented on the same computer (Intel® Core™ i7-4700MQ processor, CPU at 2.40 GHz with 16 GB of RAM) by using the computer program Matlab and verified using SAP2000. In the PFM-BWM, the adopted convergence tolerance value is equal to 10^{-8}, and the unconditionally stable semi-implicit Runge-Kutta method [20] is employed to solve the differential equations governing the behavior of each nonlinear isolation element with a number of steps equal to 50.

TABLE II
NLTHAs RESULTS WITH Δt = 0.005 s

			$u_x^{(MC_b)}$ [m]		$u_y^{(MC_b)}$ [m]		$\ddot{u}_x^{(MC_4)}$ [g]		$\ddot{u}_y^{(MC_4)}$ [g]	
	tct [s]	*tctp*	max	min	max	min	max	min	max	min
PFM-BWM	373.24	-	0.071	-0.065	0.099	-0.146	0.329	-0.342	0.591	-0.562
MEIM-NEM	1.25	0.33%	0.073	-0.060	0.095	-0.140	0.323	-0.331	0.512	-0.527

TABLE III
NLTHAs RESULTS WITH Δt = 0.001 s

			$u_x^{(MC_b)}$ [m]		$u_y^{(MC_b)}$ [m]		$\ddot{u}_x^{(MC_4)}$ [g]		$\ddot{u}_y^{(MC_4)}$ [g]	
	tct [s]	*tctp*	max	min	max	min	max	min	max	min
MEIM-NEM	5.73	1.53%	0.073	-0.060	0.095	-0.140	0.325	-0.325	0.514	-0.510

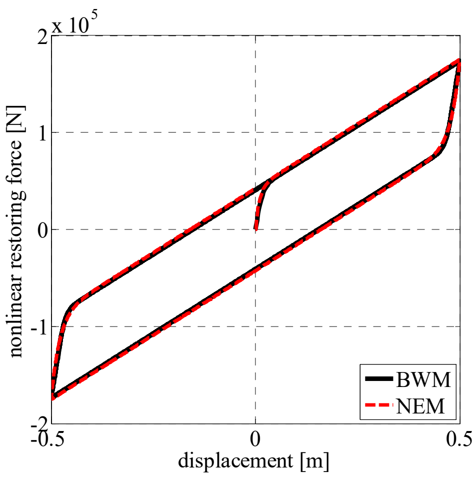


Fig. 2 Simulated force-displacement hysteresis loops

The comparison of the maximum and minimum values of the MC_b displacements and MC_4 accelerations in X and Y directions, obtained using the PFM-BWM and the MEIM-NEM, reveals that the proposed partitioned solution approach and analytical model provide numerical results that are close enough to those obtained adopting the PFM-BWM.

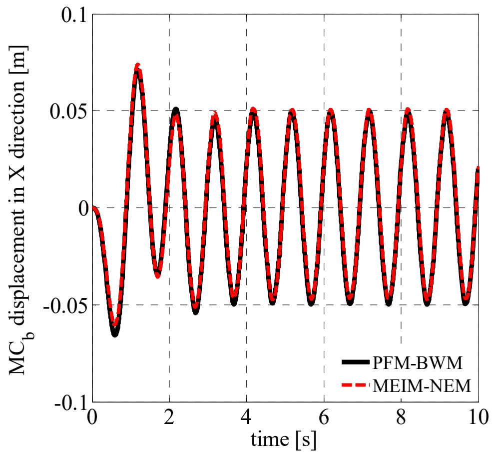
As regards the stability of the MEIM, the critical time step Δt_{cr}, evaluated considering the lowest natural period given by the eigenvalue problem in (10), is equal to 0.12 s. It is clear that the low stiffness value of the base isolation system allows to have a critical time step considerably larger than the imposed ground acceleration time step, thus avoiding stability problems.

As far as the computational efficiency is concerned, the total computational time, *tct*, required by the MEIM-NEM is significantly reduced in comparison to the PFM-BWM. It must be noted that the comparisons using the *tct* are meaningful only qualitatively because it depends on the CPU speed, memory capability and background processes of the computer used to obtain the previous results. To this end, in order to normalize the computational time results, Table II also shows the percentage of the MEIM-NEM *tct* evaluated with respect to the PFM-BWM *tct* as follows:

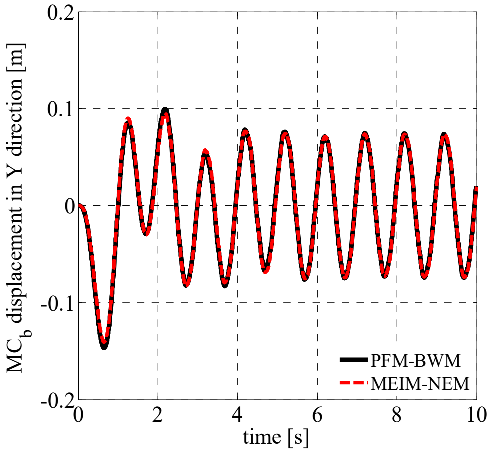
$$\text{MEIM-NEM } tctp[\%] = \frac{\text{MEIM-NEM } tct}{\text{PFM-BWM } tct} \cdot 100.$$

In addition, according to the numerical results listed in Table III, the proposed MEIM-NEM, performed with a smaller time step, that is, Δt = 0.001 s, requires less computational effort than the PFM-BWM even if the latter is performed using the larger time step (Δt = 0.005 s). Indeed, in this case, the MEIM-NEM *tctp*, referred to the PFM-BWM *tct* evaluated adopting Δt = 0.005 s, is equal to 1.53 %.

Figs. 3 and 4 illustrate, respectively, the displacement time history of the MC_b and the acceleration time history of the MC_4 for a time duration of the harmonic earthquake excitation t_d = 10 s. The good agreement is evident between responses computed using the PFM-BWM and the proposed MEIM-NEM.

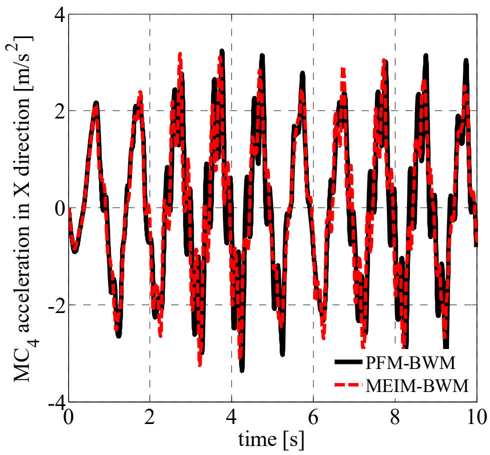


(a)

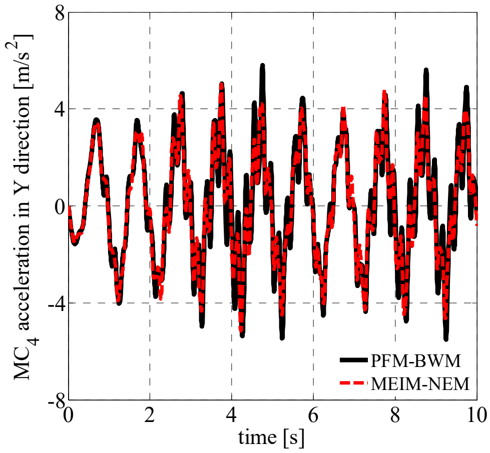


(b)

Fig. 3 Displacement time history of the base isolation system mass center in (a) *X* and (b) *Y* directions



(a)



(b)

Fig. 4 Acceleration time history of the superstructure fourth story mass center in (a) *X* and (b) *Y* directions

VI. Conclusions

A MEIM and a NEM have been proposed in order to reduce numerical computations in the nonlinear time history analysis of base-isolated structures.

In the proposed solution algorithm, the nonlinear response of the base isolation system is computed first using the explicit central difference method, then the implicit Newmark's constant average acceleration method is adopted to evaluate the superstructure linear response. Thus, the MEIM does not require an iterative procedure for each time step of a nonlinear dynamic analysis.

The presented NEM is able to simulate the dynamic behavior of seismic isolators avoiding the solution of a nonlinear differential equation required in differential equation models.

From the numerical results presented in the paper, the following conclusions can be drawn:

(1) The presented MEIM and NEM provide results that are close enough to those obtained adopting the PFM and the BWM, for both two values of time step used in the nonlinear time history analyses of the analyzed 3D base-isolated structure with LRBs;

(2) The low stiffness value of the base isolation system with LRBs allows to have a critical time step considerably larger than the imposed ground acceleration time step, thus avoiding stability problems in the proposed MEIM;

(3) The *tct* required by the MEIM-NEM is significantly reduced in comparison to the PFM-BWM: the MEIM-NEM *tctp*, evaluated with respect to the PFM-BWM *tct* for a Δt = 0.005 s, is equal to 0.33 %. In addition, the MEIM-NEM, performed with a smaller time step, that is, Δt = 0.001 s, requires less computational effort than the PFM-BWM even if the latter is performed using the larger time step (i.e., Δt = 0.005 s): indeed, the MEIM-NEM *tctp*, referred to the PFM-BWM *tct* evaluated adopting Δt = 0.005 s, is equal to 1.53 %.

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