# New Coordinate System for Countries with Big Territories 

Mohammed Sabri Ali Akresh


#### Abstract

The modern technologies and developments in computer and Global Positioning System (GPS) as well as Geographic Information System (GIS) and total station TS. This paper presents a new proposal for coordinates system by a harmonic equations "United projections", which have five projections (Mercator, Lambert, Russell, Lagrange, and compound of projection) in one zone coordinate system width 14 degrees, also it has one degree for overlap between zones, as well as two standards parallels for zone from 10 S to 45 S . Also this paper presents two cases; first case is to compare distances between a new coordinate system and UTM, second case creating local coordinate system for the city of Sydney to measure the distances directly from rectangular coordinates using projection of Mercator, Lambert and UTM.


Keywords-Harmonic equations, coordinate system, projections, algorithms and parallels.

## I. Introduction

THE theory of united projections was introduced by Prof. Vladimir Podshivalov in 1998, it was aimed for special cases GIS for countries; In 2009-2012 Dr Akresh found the general law for indirect algorithms for five projections, general law for direct algorithms of Russell projection and also for Lagrange projection; The theory is completed for use as new system coordinates for world and big territories [1], [2].

The theory of united projections has local system for big cities; the local system has an advantage in decreasing of distances distortion and very easy to go back to the general of coordinate system [3].

Australia has big territory and use universal transverse Mercator projection by 6,2 and 5 degrees and conformal Lambert projection by 2 standard parallels. State of New South Wales use systems of coordinate( MGA - Map Grid of Australia, MGA zone boundaries across NSW, GDA Lambert - GDA94 Lambert Projection for NSW, Superseded AGD66 Projections, ISG - Integrated Survey Grid, AMG - Australian Map Grid, Others UTM - Universal Transverse Mercator, ANG - Australian National Grid (pre 1966), Lambert - NSW Mapping Agencies (pre GDA); Ellipsoids GRS80 - Geodetic Reference System 1980, WGS84 - World Geodetic System 1984, ANS - Australian National Spheroid Clarke 1858 [4].

Traditional map projections have not relationships between them, as well as many overlaps between zones; these problems hot topic and important. This research tries to solve some of these problems in deferent way.

[^0]
## II.Methodology

The methodology applied for the new coordinate system is to divide all Australia to 3 zones, width of each zone 14 degrees standard parallel, and long of zone begin from equator to 75 degrees south, each zone has central meridian that divides the zone into two conformal sections (first zone $112^{\circ} \mathrm{E}-126^{\circ} \mathrm{E}$, second zone $126^{\circ} \mathrm{E}-140^{\circ} \mathrm{E}$, third zone $140^{\circ} \mathrm{E}$ $154^{\circ} \mathrm{E}$ ), and each zone has three standards parallels for five projections (Mercator, Lambert, Russell, Lagrange, and the compound of projection), as well as overlapping 1 degree between zones; Fig. 1 illustrates the proposal.


Fig. 1 Main Mercator Projection by standards parallels
The projection of Mercator is the main of the five projections because it has overlapping between standards parallels, while four projections (Lambert, Russell, Lagrange, and the compound of the projection) have systems of coordinate for 7 states only, and theory are related to Mercator projection through standards parallels ( $B_{1}=20 \mathrm{~S}, \mathrm{~B}_{2}=25 \mathrm{~S}$, $\mathrm{B}_{3}=36 \mathrm{~S}$ ), and also they haven't overlapping between standards parallels.

The main Mercator projection can be used to determine rectangular coordinates from geographic coordinates that uses statement if in MATH LAB program or other programs, for example if $10 \mathrm{~s}<\mathrm{B}<28 \mathrm{~s}$ go to $\mathrm{B}_{1}$ (first standard parallel -it uses for North Territory and Queensland), if $28 s<\varphi<44 \mathrm{~s}$ go to $B_{3}$ (third standard parallel- used for New South Wales, Victoria and Tasmania) as well as second standard parallel $\mathrm{B}_{2}$ for Western Australia and South Australia; after determining the
value of standard parallel can be used for main Mercator projection [5].

For determining geographic coordinates from rectangular coordinates may use statement if $0.173313<(\mathrm{x} / \mathrm{a})<0.485597$ go to $\mathrm{B}_{1}$, x - rectangular coordinate direct of north or south, aearth's polar semi-axis, if $0.485598<(\mathrm{x} / \mathrm{a})<0.763839$ go to $\mathrm{B}_{3}$; as well as if $0.225322<(\mathrm{x} / \mathrm{a})<0.624626$ go to $\mathrm{B}_{2}$, where use main coordinate system for Western Australia

Geodetic projections computation divides into two methods: direct problem and indirect problem as follows.

Direct method; the method uses geographic coordinate transformation $(\varphi, \lambda)$ to rectangular coordinate ( $x, y$ ); the fundamentals of equations are as follows:

$$
\left.\begin{array}{l}
x=X_{0}+C_{1} P_{1}+C_{2} P_{2}+C_{3} P_{3}+\ldots  \tag{1}\\
y=Y_{0}+C_{1} Q_{1}+C_{2} Q_{2}+C_{3} Q_{3}+\ldots
\end{array}\right\},
$$

where: $\mathrm{X}_{0}, \mathrm{Y}_{0}=$ initials coordinates systems for zone projection [1]-[3], [6]; $\mathrm{C}_{\mathrm{j}}=$ coefficients expansion of projection by direct method [1]-[3]; $\mathrm{P}_{\mathrm{j}}, \mathrm{Q}_{\mathrm{j}}=$ elements of harmonic multinomial equations apply to Laplace equations.

An initial coordinates systems for zone projection; can be found from meridian arc, and parallels ellipsoid [1]-[3], [6].

$$
\begin{gather*}
X_{0}=n_{1} B_{p}-n_{2} \sin 2 B_{p}+n_{3} \sin 4 B_{p}-n_{4} \sin 6 B_{p}+n_{5} \sin 8 B_{p}-\ldots  \tag{2}\\
P_{j}=P_{j-1} P_{1}-Q_{j-1} Q_{1}, \quad P_{0}=1  \tag{3}\\
Q_{j}=P_{j-1} Q_{1}+Q_{j-1} P_{1}, \quad Q_{0}=0
\end{gather*}
$$

where $P_{j}=$ different values between latitudes; $\mathrm{Q}_{\mathrm{j}}=$ different values between longitudes.

The different values between latitudes may be calculated from q (isometric latitude), and difference between longitudes begin from $L_{c}$ (center meridian) the given meridian; isometric latitude value can be computed from the following equation [1]-[3]

$$
\begin{equation*}
q=\ln \sqrt{\left(\frac{1+\sin B}{1-\sin B}\right)\left(\frac{1-e \sin B}{1+e \sin B}\right)^{e}} \tag{4}
\end{equation*}
$$

The difference between five projections by harmonic equations "united projections" (Mercator, Lambert, Russell, Lagrange and compound projection) is only in coefficients, each one has his own special coefficients; here will use Mercator projection for direct coefficients are as following:

$$
\begin{aligned}
& C_{1}=\frac{m_{0} \cdot c \cdot \cos B_{0}}{V}, \quad C_{2}=-\frac{C_{1} \cdot \sin B_{0}}{2}, \\
& C_{3}=\frac{C_{1} \cdot \cos ^{2} B_{0}}{6}\left(\tan ^{2} B_{0}-V^{2}\right), \\
& \\
& C_{12}=\ldots .
\end{aligned}
$$

For all coefficients of Mercator, Lambert, Russell, Lagrange and compound projection see references [1]-[3], [5]. Indirect method; the method uses rectangular coordinate transformation ( $x, y$ ) to geographic coordinate $(\varphi, \lambda)$; the Fundamentals equations are as following:

$$
\begin{align*}
& q=q_{0}+\sum_{j=1}^{n} C_{j}^{\prime} P_{j}^{\prime}  \tag{6}\\
& L=L_{0}+\sum_{j=1}^{n} C_{j}^{\prime} Q_{j}^{\prime} \\
& P_{j}^{\prime}=P_{j-1}^{\prime} P_{1}^{\prime}-Q_{j-1}^{\prime} Q_{j-1}^{\prime}, P_{0}^{\prime}=1  \tag{7}\\
& Q_{j}^{\prime}=P_{j-1}^{\prime} Q_{1}^{\prime}+Q_{j-1}^{\prime} P_{1}^{\prime}, Q_{0}^{\prime}=0
\end{align*}
$$

Indirect coefficients for all projections can be computed form [1]-[3], [5]. Where determined geographic latitude can be uses iteration value by following equation.

$$
\begin{equation*}
B=2 \arctan \left[\sqrt{\left(\frac{1+e \sin B}{1-e \sin B}\right)^{e}} * \exp (q)\right]-\frac{\pi}{2} \tag{8}
\end{equation*}
$$

Fig. 1 illustrated zone for Main Mercator Projection by standards Parallels, where it has false $\mathrm{Y}_{0}$ coordinate in center meridian $1000,000.000 \mathrm{~m}$ increased where are go to direction of East, while decreased in the direction of West; $\mathrm{B}_{1}, \mathrm{~B}_{2}, \mathrm{~B}_{3}$ standards parallels use for find rectangular coordinates(direct method) and geographic coordinates (indirect method).

## III. CASE Study

This research studies some cases; first case main coordinate system for Australia, second case main coordinate system for NSW and local coordinate system for city Sydney.

## A. New Coordinate System

The first case can be calculate some geographic coordinates ( $\varphi, \lambda$ ) between standards parallels in zone; and also translation to rectangular coordinates in number of zone 1 .
Table I shows results for the four projections. Projection Mercator has overlapping between standards parallels in each zone can be calculated points from two parallels by the same results. The accuracy 0.000 m of Projection Mercator, this accuracy is enough (the accuracy of GPS near to 0.005 m ).

Projection of Lambert and Russell give very good results near Equator better than of Mercator. The best results for decreasing of curves and parallels distortions can be used as standard of parallel near Equator.

TABLE I
ANALYSIS RESULTS BETWEEN PROJECTIONS

| Ellipsoid parameters WGS $84 \mathrm{a}=6378137.00 \mathrm{~m}, \mathrm{~b}=6356752.314 \mathrm{~m}$ |  |  |
| :---: | :---: | :---: |
| $\begin{gathered} \varphi=29^{\circ} 00^{\prime} \mathrm{S}, \lambda=144^{\circ} 00^{\prime} \mathrm{E}, \text { Number of zone } 3, \mathrm{~L}=147^{\circ} 00^{\prime} \mathrm{E} \text {, scale factor } \\ \mathrm{m}_{\text {mercator }}=1.0000000 / \mathrm{m}_{\text {lambert-russell }}=0.99247348 \\ \mathrm{X}_{20}=2212366.2541 \quad \mathrm{X}_{36}=3985542.6703 \end{gathered}$ |  |  |
| Pro. |  | main |
| St. P | $\mathrm{B}_{1}=20^{\circ} \mathrm{S}$ | $\mathrm{B}_{2}=36^{\circ} \mathrm{S}$ |
| X | 3212982.566 | 3212982.566 |
| Y | 707611.974 | 707611.974 |
| S.F.P |  |  |
| C.P |  |  |
| Pro. |  |  |
| St. P | $\mathrm{B}_{1}=20^{\circ} \mathrm{S}$ | $\mathrm{B}_{2}=36^{\circ} \mathrm{S}$ |
| X | 3208530.076 | 3217734.932 |
| Y | 706229.655 | 707822.208 |
| S.F.P | 1.00502647 | 0.99968253 |
| C.P | 2630.621 | 4496.632 |
| Pro. |  |  |
| St. P | $\mathrm{B}_{1}=20^{\circ} \mathrm{S}$ | $\mathrm{B}_{2}=36^{\circ} \mathrm{S}$ |
| X | 3206952.359 | 3218270.001 |
| Y | 708089.145 | 708810.374 |
| S.F.P | 0.9990745 | 0.9966071 |
| C.P | 3187.144 | 4098.138 |
| Pro. |  |  |
| St. P |  |  |
| X |  |  |
| Y |  |  |
| S.F.P |  |  |
| C.P |  |  |

Notes: St. P - standard parallel for zone (reference latitude ); S.F.P - scale factor point ,it shows distortions of distance in position of point; C.P- curve of parallel comparative of equator, where curve of parallel in equator equal of zero (a straight line ).

## B. Main Coordinate System for NSW and Local Coordinate

 System for City SydneyThe main coordinate system for each state has been important for cities. The Position of center of city Sydney $\varphi=33^{\circ} 53^{\prime} 455^{\prime \prime} \mathrm{S}, \lambda=151^{\circ} 10^{\prime} 45^{\prime \prime} \mathrm{E}$, position of Sydney in universal transverse Mercator UTM between zone 56.

The local system gives a good results in the present time and the local coordinate system determine rectangular coordinate system for each city within high accuracy in distances measurements " without Sampson correction method", and it's equal distances measured by indirect geodetic problems. The relation between local system and general system see Fig. 2 as follows [1]-[3]:

$$
\begin{array}{ll}
d X=\frac{m_{0}}{m_{0}^{\prime}} d x, & d Y=\frac{m_{0}}{m_{0}^{\prime}} d y, \\
X=X_{0}+d X, & Y=Y_{0}+d Y \tag{9}
\end{array}
$$

where: $m_{0}$ - ideal scale factor for main projection; $m_{0}{ }_{0}$ - ideal scale factor for local projection; $X_{0}, Y_{0}$ - initials coordinates systems for main projection; $d x, d y$ - coordinates system for local projection; $d X, d Y$ - coordinates system for main projection.

Fig. 2 illustrates tree projections Mercator, Russell and Lambert by main case for NSW and local case for city of Sydney; Yellow font represents minimum distortion of distances for Lambert projection main and the local system; Red font represents min distortion of distances for Russell projection main and the local system.


Fig. 2 Projection of Mercator and Lambert with locals systems
TABLE II
Locals Systems and UTM Analysis Results
Projection of Lambert- Local coordinate system for city Sydney

| Projection of Lambert- Local coordinate system for city Sydney |  |  |
| :---: | :---: | :---: |
| $\mathrm{X}_{36}=3985542.6703 \mathrm{~m}, \mathrm{~m}_{\text {lambert-russell }}=0.99247348, \mathrm{Y}_{0}=1000000 \mathrm{~m} \mathrm{~m}_{0}=0.9993451$ |  |  |
|  | A | B |
|  | $\begin{aligned} & \varphi=33^{\circ} 55^{\circ} 00^{\prime \prime} \mathrm{S}, \\ & \lambda=151^{\circ} 08^{\circ} 00^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \varphi=33^{\circ} 58^{\prime} 00^{\prime \prime} \mathrm{S}, \\ & \lambda=151^{\circ} 14^{\prime} 00{ }^{\prime \prime} \mathrm{E} \end{aligned}$ |
| X | 3762621.591 | 3768559.070 |
| Y | 1382114.074 | 1391112.246 |
| S.F.P | 0.9999974 | 0.99996658 |
| S - form X,Y | $\mathrm{S}_{\mathrm{A}-\mathrm{B}}=10780.573 \mathrm{~m} \pm 0.194 \mathrm{~m}$ |  |
| S- from I. G.P | $\mathrm{S}_{\mathrm{A}-\mathrm{B}}=10780.768 \mathrm{~m}$ |  |
| Projection of Russell - Local coordinate system for city Sydney$\begin{aligned} & \mathrm{X}_{36}=3985542.6703 \mathrm{~m}, \mathrm{~m}_{\text {lambert-russell }}==0.99247348, \mathrm{Y}_{0}=1000000 \mathrm{~m} \mathrm{~m} \\ & \mathrm{~A}=0.99876701 \\ & \mathrm{~B} \end{aligned}$ |  |  |
|  |  |  |
| X | 3762574.561 | 3768502.618 |
| Y | 1381886.889 | 1390891.354 |
| S.F.P | 0.9999750 | 1.000002 |
| S - from X,Y | $\mathrm{S}_{\mathrm{A}-\mathrm{B}}=10780.642 \mathrm{~m} \pm 0.126 \mathrm{~m}$ |  |
| S- from I. G.P m | $\mathrm{S}_{\mathrm{A}-\mathrm{B}}=10780.768 \mathrm{~m}$ |  |
| projection UTM Mercator $\mathrm{m}_{0}=0.9996$ zone 56 |  |  |
|  | A | B |
| X | 3754485.036 | 3759866.257 |
| Y | 327437.617 | 336778.668 |
| S.F.P | 0.9999671 | 0.99992842 |
| S - form X,Y | $\mathrm{S}_{\mathrm{A}-\mathrm{B}}=10780.203 \mathrm{~m} \pm 0.565 \mathrm{~m}$ |  |
| S- from I. G.P m | $\mathrm{S}_{\mathrm{A}-\mathrm{B}}=10780.768 \mathrm{~m}$ |  |

Table II illustrates the analysis of results between Lambert projection- locals systems, Russell projection- locals systems and UTM; where Locals systems give good results and better than of UTM; UTM in these positions gives a good result, but in other positions distortion of distances between $\pm 1.00$ 50.00 m for medium distances (without uses method Sampson).

## IV. Conclusion

The new coordinate system by five projections with local systems is better than the old coordinates systems by traditional projections "UTM, LCP" for countries and world, and results follows.

1) The new coordinate system by five projections has an advantage of decreasing distances distortions and better than of UTM and Lambert conformal projection;
2) Lambert projection and Russell best near equator as that decreased of curves parallels distortions;
3) The new coordinate system by five projections can be securing the marines transportation and marines navigation;
4) The local coordinates systems give high resolution for detecting positions.

## References

[1] M. Akresh, Advance geodesy and cartographic for GIS, 1st ed. Aalmustakbe, Libya, 2012, pp. 23-185
[2] U. Padshyvalau, The theoretical basis for forming coordinate environment for GIS, 1st ed., PSU, Novopolotsk, 1998, pp. 8-52.
[3] M.S Akresh, "Development of scientific and technical foundations and technology of forming a coordinate system for geographic information systems in the Libya" Ph.D dissertation Dept. applied geodesy, Polotsk State Univ., Novopolotsk, Belarus, 2010.
[4] http:www.nsw.gov.au
[5] M. Akresh, "New Methodology for Direct Algorithms in Russell Projection-Stereographic Projection," journal of Earth science and engineering David publishing company, vol. 2, no. 4, pp. 253-256, April 2012.
[6] V. Morozov, Course spheroid geodesy, 2nd ed., Nedra, Ministry of Education, Moscow, 1979, pp.213-253.

Mohammed Sabri Ali Akresh is staff member at University of Tripoli, Engineering Faculty, Civil Engineering, Libya; Bachelor B.sc and master M.sc 1993 from Baku Faculty, Majoring in Hydrographic 5.00 score of 5.00 Master M.sc 1999 from University of Tripoli, Majoring in High Geodesy 3.7 score of 4.00; Ph.D 2010 from Polotsk State University- Belarus, majoring in advanced Geodesy 4.9 score of 5.0 ; he is now working for degree of doctor of sc.


[^0]:    M.S Akresh is with the Civil Engineering Department, University of Tripoli, Alfarnaj, Libya (e-mail: sab20084@mail.ru).

