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# New Approaches on Stability Analysis for Neural Networks with Time-Varying Delay

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*Abstract*—Utilizing the Lyapunov functional method and combining linear matrix inequality (LMI) techniques and integral inequality approach (IIA) to analyze the global asymptotic stability for delayed neural networks (DNNs),a new sufficient criterion ensuring the global stability of DNNs is obtained.The criteria are formulated in terms of a set of linear matrix inequalities,which can be checked efficiently by use of some standard numercial packages.In order to show the stability condition in this paper gives much less conservative results than those in the literature,numerical examples are considered.

*Keywords*—Neural networks, Globally asymptotic stability , LMI approach , IIA approach , Time-varying delay.

## I. INTRODUCTION

NEURAL networks have attracted many researchers attention during the past decades and have found successful applications in many various areas. such as signal processing,static image processing,combinatorial optimization and associative memory [1,2].the occurrence of time delays is unavoidable during the processing and transmission of the signals because of the finite switching speed of amplifiers in electronic networks or finite speed for signal propagation in biological networks ,the existence of time delay may cause instability and oscillation of neural networks.Therefore stability analysis of delayed neural networks has been extensively investigated by many researchers [3-30].

In this regard, many sufficient conditions ensuring global asymptotic stability and global exponential stability for delayed neural networks have been derived [3-25]. However in most of the known results,the time-varying delay varies from 0 to an upper bound.In fact,the lower bound of time-varying delay is not restricted to be zero. A typical example of dynamic with interval time-varying delays is networked control systems [18]. [19] pointed that the stability conditions are hardly improved by using the same Lyapunov-Krasovskii functional, delay-partitioning approach, which was firstly introduced by Gu [21], has attracted by many researchers. Now, some researchers found many new approaches on stability analysis for neural networks with time-varying delay. Such as by estimating more tighter upper bounds,introducing new Lyapunov functional,dividing delay interval and so on.

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Motivated by this mentioned above, in this paper, two new delay-dependent stability criteria for neural networks with interval time-varying delay will be proposed by dividing the delay interval $[\varsigma_0, \varsigma_m]$ into four itervals $[\varsigma_0, \frac{\varsigma_0+\varsigma(t)}{2}]$, $[\frac{\varsigma_0+\varsigma(t)}{2}, \varsigma(t)]$,$[\varsigma(t), \frac{\varsigma_m+\varsigma(t)}{2}]$,$[\frac{\varsigma_m+\varsigma(t)}{2}, \varsigma_m]$, constructing new Lyapunov-Krasovskii functional which contains some new integral and triple-integral terms and establishing some new zero equalities,two new delay-dependent stability criteria for neural networks with interval time-varying delay will be proposed by employing different approaches.Finally numerical examples are given to show the effectiveness and less conservativeness of the proposed methods.

Notations: The notations in this paper are quite standard. $I$ denotes the identity matrix with appropriate dimensions,$R^n$ denotes the $n$ dimensional Euclid space, and $R^{m\times n}$is the set of all $m \times n$ real matrices, $*$ denotes the elements below the main diagonal of a symmetric block matrix. For symmetric matrices $A$ and $B$,the notation $A > B$(respectively,$A \geq B$ ) means that the matrix $A - B$ is positive definite (respectively, nonnegative).

## II. PROBLEM STATEMENT

Consider the following neural networks with interval time varying delays:

$$\dot{z}(t) = -Cz(t) + Ag(z(t)) + Bg(z(t - \varsigma(t))) + I_0 \tag{1}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T \in R^n$ is the neuron state vector,$g(z(t)) = [g_1(z_1(t)), g_2(z_2(t)), \ldots, g_n(z_n(t))]^T \in R^n$ denotes the neuron activation function ,and $I_0 = [I_1, I_2, \ldots, I_n]^T \in R^n$ is a constant input vector,$C = diag\{c_i\} \in R^n$ is a positive diagonal matrix , $A = (a_{ij})_{n\times n} \in R^n$ is the connection weight matrix,$B = (b_{ij})_{n\times n} \in R^n$ is the delayed connection weight matrix.

The following assumptions are adopted throughout the paper.

**Assumption 1**: The delay is time-varying continuous function and satisfies:

$$0 \leq \varsigma_0 \leq \varsigma(t) \leq \varsigma_m, \dot{\varsigma}(t) \leq \mu \leq 1 \tag{2}$$

where $\varsigma_0, \varsigma_m$,and $\mu$ are constants.

**Assumption 2**: Each neuron activation function $g_i(\cdot)$, in (1) satisfies the following condition:

$$\gamma_i^- \leq \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \leq \gamma_i^+, \forall \alpha, \beta \in R, \alpha \neq \beta \tag{3}$$

where $\gamma_i^-, \gamma_i^+, i = 1, 2, \ldots, n$ are constants,and assume that $\Sigma^- = diag\{\gamma_1^-, \gamma_2^-, \ldots, \gamma_n^-\}, \Sigma^+ = diag\{\gamma_1^+, \gamma_2^+, \ldots, \gamma_n^+\}$. Based on Assumption 1-2, it can be easily proven that there exists one equilibrium point for (1) by Brouwer's fixed-point theorem. Assuming that $z^* = [z_1^*, z_2^*, \ldots, z_n^*]^T$is the equilibrium point of (1) and using the transformation $y(\cdot) = z(\cdot) - z^*$,the system (1) can be converted to the following system :

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t - \varsigma(t))) \tag{4}$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T$,$f(y(t)) = [f_1(y_1(t)), f_2(y_2(t)), \ldots, f_n(y_n(t))]^T$, $f_i(y_i(\cdot)) = g_i(z_i(\cdot) + z_i^*) - g_i(z_i^*)$, $i = 1, 2, \ldots, n$.
From $Eq.(3)$,$f_i(\cdot)$ satisfies the following condition:

$$\gamma_i^- \le \frac{f_i(\alpha)}{\alpha} \le \gamma_i^+, \forall \alpha \neq 0, i = 1, 2, \ldots, n. \tag{5}$$

Due to the disturbance frequent occurs in many applications,so by translating $A, B$ and $C$ to function $A(t), B(t)$ and $C(t)$ respectively,we have

$$\dot{y}(t) = -C(t)y(t) + A(t)f(y(t)) + B(t)f(y(t - \varsigma(t))) \tag{6}$$

**Assumption 3**: Letting $A(t) = A + \Delta A(t), B(t) = B + \Delta B(t), C(t) = C + \Delta C(t)$,and $\Delta A(t), \Delta B(t), \Delta C(t)$ are unknown constant matrices representing time-varying parametric uncertainties, and are of linear fractional forms:

$$[\Delta C(t), \Delta A(t), \Delta B(t)] = G\Delta(t)[E_c, E_a, E_b] \tag{7}$$

with

$$\Delta(t) = \Lambda(t)(I - J\Lambda(t))^{-1}, \quad I - J^T J > 0 \tag{8}$$

where $G, J, E_a, E_b, E_c$,are known constant matrices of appropriate dimensions,$\Lambda(t)$ is an unknown time-varying matrix function satisfying $\Lambda^T(t)\Lambda(t) \le I$.

**Lemma 1** [10]. For any constant matrices $Q, S$ satisfy that $S = S^T, Q = Q^T > 0$, and $0 \le \varsigma_0 \le \varsigma_m$,the following inequality hold:

$$\begin{aligned} &-(\varsigma_m - \varsigma_0)\int_{t-\varsigma_m}^{t-\varsigma_0} y^T(s)Qy(s)ds \\ &\le -\begin{bmatrix} \int_{t-\varsigma(t)}^{t-\varsigma_0} y(s)ds \\ \int_{t-\varsigma_m}^{t-\varsigma(t)} y(s)ds \end{bmatrix}^T \begin{bmatrix} Q & S \\ * & Q \end{bmatrix} \begin{bmatrix} \int_{t-\varsigma(t)}^{t-\varsigma_0} y(s)ds \\ \int_{t-\varsigma_m}^{t-\varsigma(t)} y(s)ds \end{bmatrix} \end{aligned} \tag{9}$$

**Lemma 2** [20]. For any positive semi-definite matrices $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \ge 0$,the following integral integral inequality holds:

$$\begin{aligned} &-\int_{t-\varsigma(t)}^{t-\varsigma_0} y^T(s)X_{33}y(s)ds \\ &\le \int_{t-\varsigma(t)}^{t-\varsigma_0} \begin{bmatrix} y(t-\varsigma_0) \\ y(t-\varsigma(t)) \\ y(s) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & 0 \end{bmatrix} \begin{bmatrix} y(t-\varsigma_0) \\ y(t-\varsigma(t)) \\ y(s) \end{bmatrix} ds \end{aligned} \tag{10}$$

**Lemma 3** [29]. Let $I - G^T G > 0$ define the set $\Upsilon = \{\Delta(t) = \Sigma(t)[I - G\Sigma(t)]^{-1}, \Sigma^T(t)\Sigma(t) \le I\}$,for given matrices H,J and R of appropriate dimension and with H symmetrical,then $H + J\Delta(t)R + R^T\Delta^T(t)J^T < 0$,if and only if there exists a scalar $\rho > 0$ such that

$$H + \begin{bmatrix} \rho^{-1}R^T & \rho J \end{bmatrix} \begin{bmatrix} I & -G \\ -G^T & I \end{bmatrix} \begin{bmatrix} \rho^{-1}R \\ \rho J^T \end{bmatrix} < 0 \tag{11}$$

## III. Main results

In this section,a new Lyapunov functional is constructed and a less conservative delay-dependent stability criterion is obtained. First, we take up the case where $\Delta A(t) = 0$,$\Delta B(t) = 0$,$\Delta C(t) = 0$ in system (6).
Denote

$$\begin{aligned} \xi_i^T(t) = [&y^T(t)\ y^T(t - \frac{\varsigma_0 + \varsigma_m}{2})\ y^T(t - \varsigma_0)\ y^T(t - \varsigma_m) \\ &y^T(t - \varsigma(t))\ f^T(y(t))\ f^T(y(t - \varsigma(t)))\ \eta_i^T(t)] \end{aligned}$$

where

$$\eta_1^T(t) = \begin{bmatrix} \int_{t-\varsigma(t)}^{t-\varsigma_0} y^T(s)ds & \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma(t)} y^T(s)ds \end{bmatrix}$$
$$\eta_2^T(t) = \begin{bmatrix} \int_{t-\varsigma(t)}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y^T(s)ds & \int_{t-\varsigma_m}^{t-\varsigma(t)} y^T(s)ds \end{bmatrix}$$

**Theorem 1** Given that the Assumption 1-2 hold, the system (6) is globally asymptotic stability if there exist symmetric positive definite matrices $S_1, S_2, Q_i, i = 1, 2, \ldots, 8, R_i, i = 1, \ldots, 6, P, H, \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}$,symmetric positive semi-definite $\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & Q_5 \end{bmatrix}$, $\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Q_5 \end{bmatrix}$, $\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ * & U_{22} & U_{23} \\ * & * & Q_6 \end{bmatrix}$, $\begin{bmatrix} V_{11} & V_{12} & V_{13} \\ * & V_{22} & V_{23} \\ * & * & Q_6 \end{bmatrix}$, positive diagonal matrices $W_1, W_2, K = diag\{k_1, k_2, \ldots, k_n\}, L = diag\{l_1, l_2, \ldots, l_n\}$,and any symmetric matrix $S_3, S_4, S_5, S_6$ such that the following LMIs hold:

$$\begin{bmatrix} Q_5 & S_6 \\ * & S_1 \end{bmatrix} > 0 \tag{12}$$

$$\begin{bmatrix} Q_6 & S_5 \\ * & S_2 \end{bmatrix} > 0 \tag{13}$$

$$\begin{bmatrix} Q_7 & S_1 \\ * & Q_8 \end{bmatrix} > 0 \tag{14}$$

$$\begin{bmatrix} R_5 & S_2 \\ * & R_6 \end{bmatrix} > 0 \tag{15}$$

$$\begin{bmatrix} E & \aleph^T Z \\ * & -Z \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} F & \aleph^T Z \\ * & -Z \end{bmatrix} < 0 \tag{17}$$

where

$$E=\begin{bmatrix} E_{11} & 0 & 0 & 0 & 0 & E_{16} & E_{17} & 0 & 0 \\ * & E_{22} & E_{23} & E_{24} & E_{25} & 0 & 0 & 0 & E_{29} \\ * & * & E_{33} & 0 & E_{35} & 0 & 0 & E_{38} & 0 \\ * & * & * & E_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & E_{55} & 0 & E_{57} & E_{58} & E_{59} \\ * & * & * & * & * & E_{66} & E_{67} & 0 & 0 \\ * & * & * & * & * & * & E_{77} & 0 & 0 \\ * & * & * & * & * & * & * & E_{88} & E_{89} \\ * & * & * & * & * & * & * & * & E_{99} \end{bmatrix}$$

$$F=\begin{bmatrix} F_{11} & 0 & 0 & 0 & 0 & F_{16} & F_{17} & 0 & 0 \\ * & F_{22} & F_{23} & F_{24} & F_{25} & 0 & 0 & F_{28} & 0 \\ * & * & F_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & F_{44} & F_{45} & 0 & 0 & F_{48} & F_{49} \\ * & * & * & * & F_{55} & 0 & F_{57} & F_{58} & F_{59} \\ * & * & * & * & * & F_{66} & F_{67} & 0 & 0 \\ * & * & * & * & * & * & F_{77} & 0 & 0 \\ * & * & * & * & * & * & * & F_{88} & F_{89} \\ * & * & * & * & * & * & * & * & F_{99} \end{bmatrix}$$

$$\aleph = \begin{bmatrix} C & 0 & 0 & 0 & 0 & -A & -B & 0 & 0 \end{bmatrix}$$

$$\varsigma = \frac{\varsigma_m - \varsigma_0}{2}$$

$$Z = \frac{\varsigma}{4}[(\varsigma_m + 3\varsigma_0)Q_8 + (3\varsigma_m + \varsigma_0)R_6]$$

$$\begin{aligned} E_{11} =& -C^T P - PC - 2(\Sigma^+ L - \Sigma^- K)C - 2\Sigma^- W_1 \Sigma^+ \\ &+ H + \frac{\varsigma}{4}[(\varsigma_m + 3\varsigma_0)Q_7 + (3\varsigma_m + \varsigma_0)R_5] \\ &+ \varsigma(S_1 + S_2) \end{aligned}$$

$$\begin{aligned} E_{16} =& PA + (\Sigma^+ L - \Sigma^- K)A - (K - L)C \\ &+ W_1(\Sigma^+ + \Sigma^-) \end{aligned}$$

$$E_{17} = PB + (\Sigma^+ L - \Sigma^- K)B$$

$$E_{22} = G_{22} - G_{11} + \varsigma Q_6 + \varsigma Y_{22} + S_5$$

$$E_{23} = G_{12}^T, E_{24} = -G_{12}, E_{25} = \varsigma Y_{12}^T, E_{29} = Y_{23}$$

$$E_{33} = G_{11} + \varsigma Q_5 + \varsigma X_{11} - H + Q_1 + Q_2 + Q_3 + Q_4$$

$$E_{35} = \varsigma X_{12}, E_{38} = X_{13}, E_{44} = -G_{22} - Q_4 - S_5$$

$$E_{55} = -(1 - \mu)Q_2 + \varsigma(X_{22} + Y_{11}) - 2\Sigma^- W_2 \Sigma^+$$

$$E_{57} = W_2(\Sigma^+ + \Sigma^-), E_{58} = X_{23}, E_{59} = Y_{13}$$

$$E_{66} = 2(K - L)A + R_1 + R_2 + R_3 + R_4 - 2W_1$$

$$E_{67} = (K - L)B, E_{77} = -(1 - \mu)R_2 - 2W_2$$

$$E_{88} = -\frac{1}{\varsigma}S_1, E_{89} = -\frac{1}{\varsigma}S_3$$

$$E_{99} = -\frac{1}{\varsigma}S_1$$

$$F_{22} = G_{22} - G_{11} + \varsigma Q_6 + \varsigma V_{11} - S_6$$

$$F_{25} = \varsigma V_{12}, F_{28} = V_{13}$$

$$F_{33} = G_{11} + \varsigma Q_5 - H + S_6$$

$$F_{44} = -G_{22} - Q_4 + \varsigma G_{22}$$

$$F_{55} = -(1 - \mu)Q_2 + \varsigma(V_{22} + U_{11}) - 2\Sigma^- W_2 \Sigma^+$$

$$F_{58} = V_{23}, F_{59} = U_{13}, F_{88} = -\frac{1}{\varsigma}S_2$$

$$F_{89} = -\frac{1}{\varsigma}S_4, F_{99} = -\frac{1}{\varsigma}S_2$$

All the other items in matrix F satisfies $F_{ij} \neq 0$,we can get $F_{ij} = E_{ij}, i, j = 1, 2, \ldots, 9$.

*Proof:* Construct a new class of Lyapunov functional candidate as follow:

$$V(y_t) = \sum_{i=1}^{7} V_i(y_t)$$

with

$$V_1(y_t) = y^T(t)Py(t)$$

$$\begin{aligned} V_2(y_t) =& 2\sum_{i=1}^{n}[\int_0^{y_i(t)} k_i(f_i(s) - \gamma_i^- s)ds \\ &+ \int_0^{y_i(t)} l_i(\gamma_i^+ s - f_i(s))ds] \end{aligned}$$

$$V_3(y_t) = \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma_0} \begin{bmatrix} y(s) \\ y(s - \frac{\varsigma_m - \varsigma_0}{2}) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} y(s) \\ y(s - \frac{\varsigma_m - \varsigma_0}{2}) \end{bmatrix} ds$$

$$\begin{aligned} V_4(y_t) =& \int_{t-\varsigma_0}^{t} y^T(s)Hy(s)ds + \int_{t-\frac{\varsigma(t)+\varsigma_0}{2}}^{t-\varsigma_0} y^T(s)Q_1 y(s)ds \\ &+ \int_{t-\varsigma(t)}^{t-\varsigma_0} y^T(s)Q_2 y(s)ds + \int_{t-\frac{\varsigma(t)+\varsigma_m}{2}}^{t-\varsigma_0} y^T(s)Q_3 y(s)ds \\ &+ \int_{t-\varsigma_m}^{t-\varsigma_0} y^T(s)Q_4 y(s)ds \end{aligned}$$

$$\begin{aligned} V_5(y_t) =& \int_{t-\frac{\varsigma(t)+\varsigma_0}{2}}^{t} f^T(y(s))R_1 f(y(s))ds \\ &+ \int_{t-\varsigma(t)}^{t} f^T(y(s))R_2 f(y(s))ds \\ &+ \int_{t-\frac{\varsigma(t)+\varsigma_m}{2}}^{t} f^T(y(s))R_3 f(y(s))ds \\ &+ \int_{t-\varsigma_m}^{t} f^T(y(s))R_4 f(y(s))ds \end{aligned}$$

$$V_6(y_t) = \int_{-\frac{\varsigma_m+\varsigma_0}{2}}^{-\varsigma_0} \int_{t+\theta}^{t-\varsigma_0} y^T(s)Q_5 y(s) ds d\theta + \int_{-\varsigma_m}^{-\frac{\varsigma_m+\varsigma_0}{2}} \int_{t+\theta}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y^T(s)Q_6 y(s) ds d\theta$$

$$V_7(y_t) = \int_{-\frac{\varsigma_m+\varsigma_0}{2}}^{-\varsigma_0} \int_{\theta}^{0} \int_{t+\lambda}^{t} (y^T(s)Q_7 y(s) + \dot{y}^T(s)Q_8 \dot{y}(s)) ds d\lambda d\theta + \int_{-\varsigma_m}^{-\frac{\varsigma_m+\varsigma_0}{2}} \int_{\theta}^{0} \int_{t+\lambda}^{t} (y^T(s)R_5 y(s) + \dot{y}^T(s)R_6 \dot{y}(s)) ds d\lambda d\theta$$

Then, taking the time derivative of V(t) with respect to t along the system (6) yield

$$\dot{V}(y_t) = \sum_{i=1}^{7} \dot{V}_i(y_t)$$

where

$$\dot{V}_1(y_t) = 2y^T(t)P\dot{y}(t) \tag{18}$$

$$\dot{V}_2(y_t) = 2[f^T(y(t))(K-L) + y^T(t)(\Sigma^+ L - \Sigma^- K)]\dot{y}(t) \tag{19}$$

$$\dot{V}_3(y_t) = \begin{bmatrix} y(t-\varsigma_0) \\ y(t-\frac{\varsigma_m+\varsigma_0}{2}) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} y(t-\varsigma_0) \\ y(t-\frac{\varsigma_m+\varsigma_0}{2}) \end{bmatrix} - \begin{bmatrix} y(t-\frac{\varsigma_m+\varsigma_0}{2}) \\ y(t-\varsigma_m) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix} \begin{bmatrix} y(t-\frac{\varsigma_m+\varsigma_0}{2}) \\ y(t-\varsigma_m) \end{bmatrix} \tag{20}$$

$$\begin{aligned} \dot{V}_4(y_t) &= y^T(t-\varsigma_0)(Q_1+Q_2+Q_3+Q_4-H)y(t-\varsigma_0) \\ &+ y^T(t)Hy(t) - y^T(t-\varsigma_m)Q_4 y(t-\varsigma_m) \\ &- (1-\frac{\mu}{2})y^T(t-\frac{\varsigma(t)+\varsigma_m}{2})Q_3 y(t-\frac{\varsigma(t)+\varsigma_m}{2}) \\ &- (1-\frac{\mu}{2})y^T(t-\frac{\varsigma(t)+\varsigma_0}{2})Q_1 y(t-\frac{\varsigma(t)+\varsigma_0}{2}) \\ &- (1-\mu)(y^T(t-\varsigma(t))Q_2 y(t-\varsigma(t)) \end{aligned} \tag{21}$$

$$\begin{aligned} \dot{V}_5(y_t) &= f^T(y(t))(R_1+R_2+R_3+R_4)f(y(t)) \\ &- f^T(y(t-\varsigma_m))R_4 f(y(t-\varsigma_m)) \\ &- (1-\mu)f^T(y(t-\varsigma(t)))R_2 f(y(t-\varsigma(t))) \\ &- (1-\frac{\mu}{2})[f^T(y(t-\frac{\varsigma(t)+\varsigma_0}{2}))R_1 f(y(t-\frac{\varsigma(t)+\varsigma_0}{2})) \\ &- f^T(y(t-\frac{\varsigma(t)+\varsigma_m}{2}))R_3 f(y(t-\frac{\varsigma(t)+\varsigma_m}{2}))] \end{aligned} \tag{22}$$

$$\begin{aligned} \dot{V}_6(y_t) &= \varsigma y^T(t-\varsigma_0))Q_5 y(t-\varsigma_0) \\ &+ \varsigma y^T(t-\frac{\varsigma_m+\varsigma_0}{2})Q_6 y(t-\frac{\varsigma_m+\varsigma_0}{2}) \\ &- \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma_0} y^T(s)Q_5 y(s) ds \\ &- \int_{t-\varsigma_m}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y^T(s)Q_6 y(s) ds \end{aligned} \tag{23}$$

$$\begin{aligned} \dot{V}_7(y_t) &= \frac{\varsigma(\varsigma_m+3\varsigma_0)}{4}(y^T(t)Q_7 y(t) + \dot{y}^T(t)Q_8 \dot{y}(t)) \\ &+ \frac{\varsigma(3\varsigma_m+\varsigma_0)}{4}(y^T(t)R_5 y(t) + \dot{y}^T(t)R_6 \dot{y}(t)) \\ &- \int_{-\frac{\varsigma_m+\varsigma_0}{2}}^{-\varsigma_0} \int_{t+\theta}^{t} (y^T(s)Q_7 y(s) + \dot{y}^T(s)Q_8 \dot{y}(s)) ds d\theta \\ &- \int_{-\varsigma_m}^{-\frac{\varsigma_m+\varsigma_0}{2}} \int_{t+\theta}^{t} (y^T(s)R_5 y(s) + \dot{y}^T(s)R_6 \dot{y}(s)) ds d\theta \end{aligned} \tag{24}$$

The following four zero equalities with symmetric positive definite matrices $S_1$ ,$S_2$,and any symmetric matrix $S_5, S_6$ are considered:

$$y^T(t-\frac{\varsigma_m+\varsigma_0}{2})S_5 y(t-\frac{\varsigma_m+\varsigma_0}{2}) - y^T(t-\varsigma_m)S_5 y(t-\varsigma_m) - 2\int_{t-\varsigma_m}^{t-\frac{\varsigma_0+\varsigma_m}{2}} y^T(s)S_5 \dot{y}(s) ds = 0 \tag{25}$$

$$y^T(t-\varsigma_0)S_6 y(t-\varsigma_0) - y^T(t-\frac{\varsigma_m+\varsigma_0}{2})S_6 y(t-\frac{\varsigma_m+\varsigma_0}{2}) - 2\int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma_0} y^T(s)S_6 \dot{y}(s) ds = 0 \tag{26}$$

$$\varsigma y^T(t)S_1 y(t) - \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma_0} y^T(s)S_1 y(s) ds - 2\int_{-\frac{\varsigma_m+\varsigma_0}{2}}^{-\varsigma_0} \int_{t+\theta}^{t} y^T(s)S_1 \dot{y}(s) ds d\theta = 0 \tag{27}$$

$$\varsigma y^T(t)S_2 y(t) - \int_{t-\varsigma_m}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y^T(s)S_2 y(s) ds - 2\int_{-\varsigma_m}^{-\frac{\varsigma_m+\varsigma_0}{2}} \int_{t+\theta}^{t} y^T(s)S_2 \dot{y}(s) ds d\theta = 0 \tag{28}$$

From (27)-(28),we can obtain the following equality:

$$\begin{aligned} \dot{V}_7(y_t) &= \frac{\varsigma(\varsigma_m+3\varsigma_0)}{4}(y^T(t)Q_7 y(t) + \dot{y}^T(t)Q_8 \dot{y}(t)) \\ &+ \frac{\varsigma(3\varsigma_m+\varsigma_0)}{4}(y^T(t)R_5 y(t) + \dot{y}^T(t)R_6 \dot{y}(t)) \\ &+ \varsigma y^T(t)(S_1+S_2)y(t) - \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma_0} y^T(s)S_1 y(s) ds \\ &- \int_{t-\varsigma_m}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y^T(s)S_2 y(s) ds \\ &- \int_{-\frac{\varsigma_m+\varsigma_0}{2}}^{-\varsigma_0} \int_{t+\theta}^{t} \begin{bmatrix} y(s) \\ \dot{y}(s) \end{bmatrix}^T \begin{bmatrix} Q_7 & S_1 \\ * & Q_8 \end{bmatrix} \begin{bmatrix} y(s) \\ \dot{y}(s) \end{bmatrix} ds d\theta \\ &- \int_{-\varsigma_m}^{-\frac{\varsigma_m+\varsigma_0}{2}} \int_{t+\theta}^{t} \begin{bmatrix} y(s) \\ \dot{y}(s) \end{bmatrix}^T \begin{bmatrix} R_5 & S_2 \\ * & R_6 \end{bmatrix} \begin{bmatrix} y(s) \\ \dot{y}(s) \end{bmatrix} ds d\theta \end{aligned}$$

From (5), we can get that there exist positive diagonal matrices $W_1, W_2$ such that the following inequalities holds:

$$-2f^T(y(t))W_1 f(y(t)) + 2y^T(t)W_1(\Sigma^- + \Sigma^+)f(y(t)) - 2y^T(t)\Sigma^- W_1 \Sigma^+ y(t) \geq 0 \tag{29}$$

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$$-2f^T(y(t-\varsigma(t)))W_2 f(y(t-\varsigma(t))) + 2y^T(t-\varsigma(t))W_2(\Sigma^- +\Sigma^+)f(y(t-\varsigma(t))) - 2y^T(t-\varsigma(t))\Sigma^- W_2\Sigma^+ y(t-\varsigma(t)) \geq 0 \quad (30)$$

Using Lemma 1,one can obtain

$$-\int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma_0} y^T(s)S_1 y(s)ds \leq -\frac{1}{\varsigma}\begin{bmatrix}\int_{t-\varsigma(t)}^{t-\varsigma_0} y(s)ds \\ \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma(t)} y(s)ds\end{bmatrix}^T \begin{bmatrix} S_1 & S_3 \\ * & S_1\end{bmatrix}\begin{bmatrix}\int_{t-\varsigma(t)}^{t-\varsigma_0} y(s)ds \\ \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma(t)} y(s)ds\end{bmatrix} \quad (31)$$

$$-\int_{t-\varsigma_m}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y^T(s)S_2 y(s)ds \leq -\frac{1}{\varsigma}\begin{bmatrix}\int_{t-\varsigma(t)}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y(s)ds \\ \int_{t-\varsigma_m}^{t-\varsigma(t)} y(s)ds\end{bmatrix}^T \begin{bmatrix} S_2 & S_4 \\ * & S_2\end{bmatrix}\begin{bmatrix}\int_{t-\varsigma(t)}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y(s)ds \\ \int_{t-\varsigma_m}^{t-\varsigma(t)} y(s)ds\end{bmatrix} \quad (32)$$

(1) when $\varsigma_0 \leq \varsigma(t) \leq \frac{\varsigma_m+\varsigma_0}{2}$,one can obtain

$$-\int_{t-\varsigma(t)}^{t-\varsigma_0} y^T(s)Q_5 y(s)ds \leq \int_{t-\varsigma(t)}^{t-\varsigma_0}\begin{bmatrix} y(t-\varsigma_0) \\ y(t-\varsigma(t)) \\ y(s)\end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & 0\end{bmatrix}\begin{bmatrix} y(t-\varsigma_0) \\ y(t-\varsigma(t)) \\ y(s)\end{bmatrix} ds \quad (33)$$

$$-\int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma(t)} y^T(s)Q_5 y(s)ds \leq \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma(t)}\begin{bmatrix} y(t-\varsigma(t)) \\ y(t-\frac{\varsigma_m+\varsigma_0}{2}) \\ y(s)\end{bmatrix}^T \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & 0\end{bmatrix}\begin{bmatrix} y(t-\varsigma(t)) \\ y(t-\frac{\varsigma_m+\varsigma_0}{2}) \\ y(s)\end{bmatrix} ds \quad (34)$$

From (18)-(25),(29)-(32),and (34)-(35) one can obtain

$$\begin{aligned}
\dot{V}(y_t) \leq\ & \xi_1^T(t)\bar{E}\xi_1(t) - f^T(y(t-\varsigma_m))R_4 f(y(t-\varsigma_m)) \\
& - (1-\frac{\mu}{2})y^T(t-\frac{\varsigma(t)+\varsigma_m}{2})Q_3 y(t-\frac{\varsigma(t)+\varsigma_m}{2}) \\
& - (1-\frac{\mu}{2})y^T(t-\frac{\varsigma(t)+\varsigma_0}{2})Q_1 y(t-\frac{\varsigma(t)+\varsigma_0}{2}) \\
& - (1-\frac{\mu}{2})f^T(y(t-\frac{\varsigma(t)+\varsigma_0}{2}))R_1 f(y(t-\frac{\varsigma(t)+\varsigma_0}{2})) \\
& - (1-\frac{\mu}{2})f^T(y(t-\frac{\varsigma(t)+\varsigma_m}{2}))R_3 f(y(t-\frac{\varsigma(t)+\varsigma_m}{2})) \\
& - \int_{t-\varsigma_m}^{t-\frac{\varsigma_m+\varsigma_0}{2}} \begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} Q_6 & S_5 \\ * & S_2\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} ds \\
& - \int_{-\frac{\varsigma_m+\varsigma_0}{2}}^{-\varsigma_0}\int_{t+\theta}^{t} \begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} Q_7 & S_1 \\ * & Q_8\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} dsd\theta \\
& - \int_{-\varsigma_m}^{-\frac{\varsigma_m+\varsigma_0}{2}}\int_{t+\theta}^{t} \begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} R_5 & S_2 \\ * & R_6\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} dsd\theta
\end{aligned}$$

where

$$\begin{aligned}
\bar{E}_{11} =\ & -C^T P - PC - 2(\Sigma^+ L - \Sigma^- K)C - 2\Sigma^- W_1\Sigma^+ + H \\
& + \varsigma(S_1+S_2) + \frac{\varsigma}{4}[(\varsigma_m+3\varsigma_0)Q_7 + (3\varsigma_m+\varsigma_0)R_5] \\
& + C^T ZC
\end{aligned}$$

$$\begin{aligned}
\bar{E}_{16} =\ & PA + (\Sigma^+ L - \Sigma^- K)A - (K-L)C + W_1(\Sigma^+ + \Sigma^-) \\
& - C^T ZA
\end{aligned}$$

$$\bar{E}_{17} = PB + (\Sigma^+ L - \Sigma^- K)B - C^T ZB$$

$$\bar{E}_{66} = 2(K-L)A + R_1 + R_2 + R_3 + R_4 - 2W_1 + A^T ZA$$

$$\bar{E}_{67} = (K-L)B + A^T ZB$$

$$\bar{E}_{77} = -(1-\mu)R_2 - 2W_2 + B^T ZB$$

All the other items in matrix $\bar{E}$ ,we can get $\bar{E}_{ij} = E_{ij}, i,j = 1,2,\ldots,9$.

(2) when $\frac{\varsigma_m+\varsigma_0}{2} \leq \varsigma(t) \leq \varsigma_m$,one can obtain

$$-\int_{t-\varsigma(t)}^{t-\frac{\varsigma_m+\varsigma_0}{2}} y^T(s)Q_6 y(s)ds \leq \int_{t-\varsigma(t)}^{t-\frac{\varsigma_m+\varsigma_0}{2}}\begin{bmatrix} y(t-\frac{\varsigma_m+\varsigma_0}{2}) \\ y(t-\varsigma(t)) \\ y(s)\end{bmatrix}^T \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ * & U_{22} & U_{23} \\ * & * & 0\end{bmatrix}\begin{bmatrix} y(t-\frac{\varsigma_m+\varsigma_0}{2}) \\ y(t-\varsigma(t)) \\ y(s)\end{bmatrix} ds \quad (35)$$

$$-\int_{t-\varsigma_m}^{t-\varsigma(t)} y^T(s)Q_6 y(s)ds \leq \int_{t-\varsigma_m}^{t-\varsigma(t)}\begin{bmatrix} y(t-\varsigma(t)) \\ y(t-\varsigma_m) \\ y(s)\end{bmatrix} \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ * & V_{22} & V_{23} \\ * & * & 0\end{bmatrix}\begin{bmatrix} y(t-\varsigma(t)) \\ y(t-\varsigma_m) \\ y(s)\end{bmatrix} ds \quad (36)$$

From (18)-(24),(26),(29)-(31),(33) and (37)-(38) one can obtain

$$\begin{aligned}
\dot{V}(y_t) \leq\ & \xi_2^T(t)\bar{F}\xi_2(t) - f^T(y(t-\varsigma_m))R_4 f(y(t-\varsigma_m)) \\
& - (1-\frac{\mu}{2})y^T(t-\frac{\varsigma(t)+\varsigma_m}{2})Q_3 y(t-\frac{\varsigma(t)+\varsigma_m}{2}) \\
& - (1-\frac{\mu}{2})y^T(t-\frac{\varsigma(t)+\varsigma_0}{2})Q_1 y(t-\frac{\varsigma(t)+\varsigma_0}{2}) \\
& - (1-\frac{\mu}{2})f^T(y(t-\frac{\varsigma(t)+\varsigma_0}{2}))R_1 f(y(t-\frac{\varsigma(t)+\varsigma_0}{2})) \\
& - (1-\frac{\mu}{2})f^T(y(t-\frac{\varsigma(t)+\varsigma_m}{2}))R_3 f(y(t-\frac{\varsigma(t)+\varsigma_m}{2})) \\
& - \int_{t-\frac{\varsigma_m+\varsigma_0}{2}}^{t-\varsigma_0} \begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} Q_5 & S_6 \\ * & S_1\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} ds \\
& - \int_{-\frac{\varsigma_m+\varsigma_0}{2}}^{-\varsigma_0}\int_{t+\theta}^{t} \begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} Q_7 & S_1 \\ * & Q_8\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} dsd\theta \\
& - \int_{-\varsigma_m}^{-\frac{\varsigma_m+\varsigma_0}{2}}\int_{t+\theta}^{t} \begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix}^T \begin{bmatrix} R_5 & S_2 \\ * & R_6\end{bmatrix}\begin{bmatrix} y(s) \\ \dot{y}(s)\end{bmatrix} dsd\theta
\end{aligned}$$

where

$$\bar{F}_{11} = \bar{E}_{11}, \bar{F}_{16} = \bar{E}_{16}, \bar{F}_{17} = \bar{E}_{17}$$

$$\bar{F}_{66} = \bar{E}_{66}, \bar{F}_{67} = \bar{E}_{67}, \bar{F}_{77} = \bar{E}_{77}$$

All the other items in matrix $\bar{F}$ ,we can get $\bar{F}_{ij} = F_{ij}, i,j = 1,2,\ldots,9$.

Hence,combined with the Schur Complement and (12)-(15), we can obtain

$$\dot{V}(y_t) \le 0$$

This means that the system (6) is asymptotically stable, which complete the proof. ■

Based on Theorem 1,we have the following result for neural networks with time-varying.

**Theorem 2** Given that the Assumption 1-3 hold, the system (6) is globally asymptotic stability if there exist symmetric positive definite matrices $Q_i, i = 1, \dots, 8, R_i, i = 1, \dots, 6$, $\begin{bmatrix} G_{11} & G_{12} \\ * & G_{22} \end{bmatrix}, P, H, S_1, S_2$, symmetric positive semi-definite $\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & Q_5 \end{bmatrix}, \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Q_5 \end{bmatrix}, \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ * & U_{22} & U_{23} \\ * & * & Q_6 \end{bmatrix}$, $\begin{bmatrix} V_{11} & V_{12} & V_{13} \\ * & V_{22} & V_{23} \\ * & * & Q_6 \end{bmatrix}$,positive diagonal matrices $W_1, W_2, K = diag\{k_1, k_2, \dots, k_n\}, L = diag\{l_1, l_2, \dots, l_n\}$,any symmetric matrix $S_3, S_4, S_5, S_6$ and $\rho_i > 0, i = 1, 2$,such that the following LMIs hold:

$$\begin{bmatrix} Q_5 & S_6 \\ * & S_1 \end{bmatrix} > 0 \tag{37}$$

$$\begin{bmatrix} Q_6 & S_5 \\ * & S_2 \end{bmatrix} > 0 \tag{38}$$

$$\begin{bmatrix} Q_7 & S_1 \\ * & Q_8 \end{bmatrix} > 0 \tag{39}$$

$$\begin{bmatrix} R_5 & S_2 \\ * & R_6 \end{bmatrix} > 0 \tag{40}$$

$$\begin{bmatrix} E & \aleph^T Z & \rho_1^{-1}\Phi_1^T & \rho_1\theta_1^T \\ * & -Z & 0 & \rho_1 ZG \\ * & * & -I & J \\ * & * & * & -I \end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix} F & \aleph^T Z & \rho_2^{-1}\Phi_1^T & \rho_2\theta_1^T \\ * & -Z & 0 & \rho_2 ZG \\ * & * & -I & J \\ * & * & * & -I \end{bmatrix} < 0 \tag{42}$$

where

$$\Phi_1 = \begin{bmatrix} E_c & 0 & 0 & 0 & 0 & -E_a & -E_b & 0 & 0 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \Phi_1 & 0 \end{bmatrix}$$

$$\theta_1 = \begin{bmatrix} -G^T P - \Sigma^+ L + \Sigma^- K & 0 & 0 & 0 & 0 & L - K & 0 & 0 & 0 \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 & G^T Z \end{bmatrix}^T$$

*Proof:* Replacing $C, A$ and $B$ in (14),(15) with $C(t) = C + G\Delta(t)E_c, A(t) = A + G\Delta(t)E_a$,and $B(t) = B + G\Delta(t)E_b$,respectively,it follows that,the LMIs in (14),(15) are equivalent to

$$\begin{bmatrix} E & \aleph^T Z \\ * & -Z \end{bmatrix} + \theta\Delta(t)\Phi + \Phi^T\Delta^T(t)\theta^T < 0$$

$$\begin{bmatrix} F & \aleph^T Z \\ * & -Z \end{bmatrix} + \theta\Delta(t)\Phi + \Phi^T\Delta^T(t)\theta^T < 0$$

By Lemma 3,there exists two positive scalars $\rho_i, i = 1, 2$,such that

$$\begin{bmatrix} E & \aleph^T Z \\ * & -Z \end{bmatrix} + \begin{bmatrix} \rho_1^{-1}\Phi^T & \rho_1\theta \end{bmatrix} \begin{bmatrix} I & -J \\ -J^T & I \end{bmatrix}^{-1} \begin{bmatrix} \rho_1^{-1}\Phi \\ \rho_1\theta^T \end{bmatrix} < 0$$

$$\begin{bmatrix} F & \aleph^T Z \\ * & -Z \end{bmatrix} + \begin{bmatrix} \rho_2^{-1}\Phi^T & \rho_2\theta \end{bmatrix} \begin{bmatrix} I & -J \\ -J^T & I \end{bmatrix}^{-1} \begin{bmatrix} \rho_2^{-1}\Phi \\ \rho_2\theta^T \end{bmatrix} < 0$$

By Schur Complement,the inequalities (46),(47) are equivalent to the LMIs in (44),(45) .This completes the proof. ■

**Remark 1** Theorem 1 and Theorem 2 proposes an improved global asymptotic stability for delayed neural networks.This paper not only divide the delay interval $[\varsigma_0, \varsigma_m]$ into $[\varsigma_0, \frac{\varsigma_0+\varsigma_m}{2}]$,$[\frac{\varsigma_0+\varsigma_m}{2}, \varsigma_m]$,but divides the interval $[\varsigma_0, \varsigma_m]$ into $[\varsigma_0, \frac{\varsigma_0+\varsigma(t)}{2}]$,$[\frac{\varsigma_0+\varsigma(t)}{2}, \varsigma(t)]$,$[\varsigma(t), \frac{\varsigma_m+\varsigma(t)}{2}]$,$[\frac{\varsigma_m+\varsigma(t)}{2}, \varsigma_m]$, Each segments has a different Lyapunov matrix in function $V$,which have potential to yield less conservative results.

**Remark 2** In this paper,Theorem 1 and Theorem 2 require the upper bound $\mu$ of the time-varying delay $\varsigma(t)$ to be known.However,in many cases $\mu$ is unknown,considering this situation,we can set $Q_i = R_i = 0, i = 1, 2, 3$ in $V(y_t)$,and employ the similar methods in Theorem 1 and Theorem 2,we can obtain that satisfy delay-dependent and delay-derivative-independent stability criteria.

**Remark 3** When $J = 0$,the Assumption 3 can be reduced to the popular expression such as $G\Delta(t)E_c = G\Lambda(t)E_c$,in which $\Delta^T(t)\Delta(t) = \Lambda^T(t)\Lambda(t) \le I$.Thus,the form includes the norm-bounded uncertainty as its special case.

## IV. Numerical examples

In this section,we provide three numerical examples to demonstrate the effectiveness and less conservatism of our delay-dependent stability criteria.

**Example 1** Consider a delayed recurrent neural networks with the following parameters:

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t - \varsigma(t)))$$

where

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, B = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}$$

The neuron activation functions are assumed to satisfy Assumption 2 with $\Sigma^- = diag\{0, 0\}, \Sigma^+ = diag\{0.4, 0.8\}$. For the case of $\varsigma_0 = 0$,the upper bounds of $\varsigma_m$ for different $\mu$ is derived by Theorem 1,According to Table I,this example shows that the stability condition in this paper gives much less conservative results than those in the literature.

**Example 2** Consider a delayed recurrent neural networks

TABLE I
ALLOWABLE UPPER BOUND OF $\varsigma_m$ FOR EXAMPLE 1.

| Method | $\mu = 0.8$ | $\mu = 0.9$ | Unknown $\mu$ |
|---|---|---|---|
| [5] | 2.3534 | 1.6050 | 1.5103 |
| [6] | 2.8854 | 1.9631 | 1.7810 |
| [7] | 3.0604 | 1.9956 | 1.7860 |
| [8] | 4.1626 | 3.9766 | 3.1690 |
| Theorem 1 | 5.0145 | 4.3630 | 3.7893 |

TABLE II
COMPARIONS THE UPPER BOUND OF $\varsigma_m$ FOR VARIOUS $\mu$ IN EXAMPLE 2.

| Method | $\mu = 0.4$ | $\mu = 0.45$ | $\mu = 0.5$ | $\mu = 0.55$ |
|---|---|---|---|---|
| [7] | 5.2420 | 4.4301 | 4.1055 | 3.9231 |
| [8] | 7.9626 | 7.6766 | 7.1690 | 6.9895 |
| Theorem 1 | 8.4870 | 8.2450 | 7.6975 | 7.0875 |

with the following parameters:

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t - \varsigma(t)))$$

where

$$C=\begin{bmatrix}1.5 & 0\\ 0 & 0.7\end{bmatrix},A=\begin{bmatrix}0.0503 & 0.0454\\ 0.0987 & 0.2075\end{bmatrix},B=\begin{bmatrix}0.2381 & 0.9320\\ 0.0388 & 0.5062\end{bmatrix}$$

The neuron activation functions are assumed to satisfy Assumption 2 with $\Sigma^- = diag\{0, 0\}, \Sigma^+ = diag\{0.3, 0.8\}$. According to Table II ,we can see the comparison results on the maximum delay bound allowed via the method in recent papers [7,8] and our new study,and this example shows that the stability criterion in the paper can lead to less conservative results than [7,8].

**Example 3** Consider a delayed recurrent neural networks with the following parameters:

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t - \varsigma(t)))$$

where

$$C = \begin{bmatrix}0.6321 & 0 & 0\\ 0 & 0.9230 & 0\\ 0 & 0 & 0.4480\end{bmatrix}$$

$$A = \begin{bmatrix}0.5988 & -0.3224 & 1.2352\\ -0.0860 & -0.3824 & -0.5785\\ 0.3253 & -0.9534 & -0.5015\end{bmatrix}$$

$$B = \begin{bmatrix}-0.9164 & 0.0360 & 0.9816\\ 2.6117 & -0.3788 & 0.8428\\ 0.5179 & 1.1734 & -0.2775\end{bmatrix}$$

The neuron activation functions are assumed to satisfy Assumption 2 with $\Sigma^- = diag\{-0.1279, -0.7994, -0.2368\}$, $\Sigma^+ = diag\{0.1279, 0.7994, 0.2368\}$.
Table III provides the maximum allowable delay bounds with the variables $\varsigma_0$,and $\mu$.

TABLE III
ALLOW UPPER BOUND OF $\varsigma_m$ IN EXAMPLE 3.

| $\varsigma_0$ | $\mu = 0.6$ | $\mu = 0.7$ | $\mu = 0.8$ | $\mu = 0.9$ |
|---|---|---|---|---|
| $\varsigma_0 = 0.2$ | 6.5216 | 6.1651 | 5.5750 | 4.9461 |
| $\varsigma_0 = 0.4$ | 6.8570 | 6.4083 | 6.1695 | 5.1135 |
| $\varsigma_0 = 0.6$ | 7.0145 | 6.9649 | 6.3215 | 5.3438 |

## V. CONCLUSION

In this paper, a new delay-dependent asymptotic stability criterion for neural networks with time-delaying has been investigated.By dividing the delay interval and constructing new Lyapunov-Krasovskii functional which contains some new integral terms and triple-integral terms ,and fully uses the information about the bounding technique of integral terms with different free-weighting matrices in different delay intervals to reduce the conservatism of stability criteria. Finally, numerical examples have presented to illustrate the benefits and less conservativeness of the proposed method.

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