

# Navigation and Self Alignment of Inertial Systems using Nonlinear $H_\infty$ Filters

Saman M. Siddiqui, Fang Jiancheng

**Abstract**—Micro electromechanical sensors (MEMS) play a vital role along with global positioning devices in navigation of autonomous vehicles. These sensors are low cost, easily available but depict colored noises and unpredictable discontinuities. Conventional filters like Kalman filters and Sigma point filters are not able to cope with nonwhite noises. This research has utilized  $H_\infty$  filter in nonlinear frame work both with Kalman filter and Unscented filter for navigation and self alignment of an airborne vehicle. The system is simulated for colored noises and discontinuities and results are compared with not robust nonlinear filters. The results are found 40%-70% more robust against colored noises and discontinuities.

**Keywords**—filtering, integrated navigation, MEMS, nonlinear filtering, self alignment

## I. INTRODUCTION

KALMAN filter (KF) accuracy highly depends on three factors first how much the random behavior of noises is known, second system model is accurate and thirdly all noise processes should have zero mean Gaussian distribution functions [1]. Industrial processes do not usually exhibit Gaussian behavior and sometimes worst case noises are to be dealt with. In the cases where tactical grade inertial sensors are not available and we have to rely on low cost Micro electromechanical sensors (MEMS) this problem get severe. MEMS error behavior cannot be approximated in all conditions as of Gaussian zero mean, these do exhibit certain levels of unpredictable noises and discontinuities. Moreover their nonlinear behavior changes drastically with environmental changes like temperature. MEMS are integrated with GPS which also depict uncertain behavior due to environment and weather conditions. To overcome these problems a more robust solution is required.

$H_\infty$  Filter is in use for decades as a robust filter. The main advantage of using  $H_\infty$  filter is that it does not assume anything about noise statistics which make it useful in certain applications. It only assumes that variance is bounded which means that power spectral density is integrable but unknown. It tries that the worst possible estimation of signal noises does not exceed a pre specified bound. Its performance depends upon the weighting matrices chosen by designers. The basic  $H_\infty$  filter is derived and defined in detail in [1,331-371] and [2].

Navigation and self alignment have nonlinear state equations, especially if large angle models are used. Small angle assumptions cannot work for MEMS alignment due to their inability to sense earth rate. Therefore robust filter should be designed in nonlinear framework.  $H_\infty$  filter are designed in nonlinear frame work of Extended Kalman filter(EKF) and Unscented Kalman filter (UKF) in [3] and [4] and used for self alignment of ships in [1],[5]and [6].This research explores their use in navigation and alignment of airborne vehicle. The navigation error model is small angle error models given in [7].The state vector comprises of 22 states i.e., three position, three velocities, four quaternion, and rest of the six are different error estimates of gyros and accelerometers. Whereas self alignment error model is large angles model described later in text. In this research MEMS IMU with gyro random drift of 5°/hr and accelerometer bias of 10mg is integrated with GPS in loosely coupled mode with six measurements i.e., three velocities and three positions. The data rate of GPS is 1Hz whereas that of IMU is 10 Hz.

## II. COMPARISON OF $H_\infty$ FILTER AND KALMAN FILTER IN THE PRESENCE OF UNMODELED NOISE IN NAVIGATION

A theoretical trajectory with S turns of bigger loops (low dynamic regions) and smaller loops (high dynamic region), was generated for simulation. All initial velocities, position and angles in local frame [6] are considered as zero. In one case sensor signals were corrupted with zero mean Gaussian noises of known variance and in the other case with high noises of unknown variance. In both cases KF and  $H_\infty$  Filter were applied and the results were compared. The results in Fig. 1 & Fig.2 clearly demonstrate that in normal conditions KF is better than  $H_\infty$  but during noisy measurement  $H_\infty$  performed consistently whereas KF estimates keep worsening as much as noise increased. The system and measurement equation of this set up are described in [5].

In the presence of frequent GPS outages of one minute and poorly modeled noise similar as in the case of Fig.2,  $H_\infty$  Filter performed well whereas KF errors kept increasing to a level of 200m approximately. Fig.3 shows that result. Another very serious problem of MEMS inertial measurement unit (IMU) is discontinuities in data. Fig. 4 demonstrates that result, that how well  $H_\infty$  Filter has coped up with this problem.

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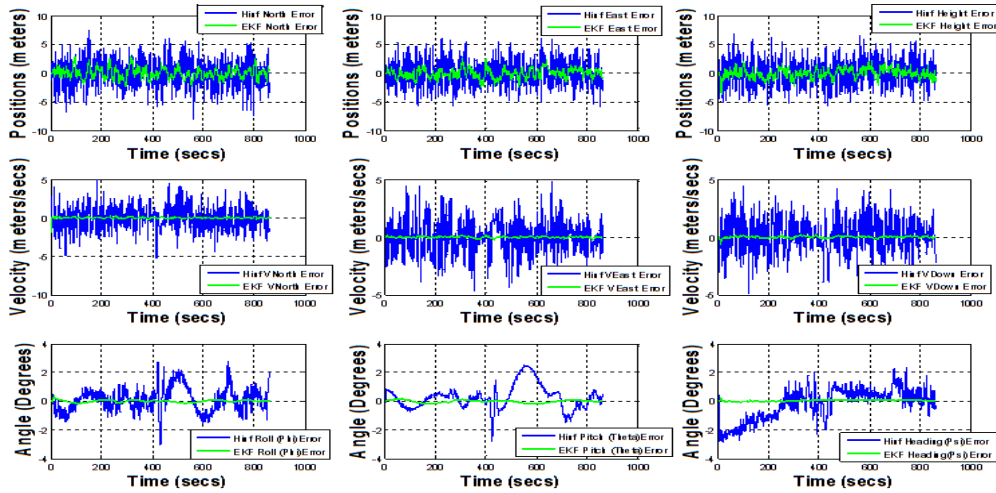


Fig. 1  $H^\infty/KF$  performance in presence of known errors

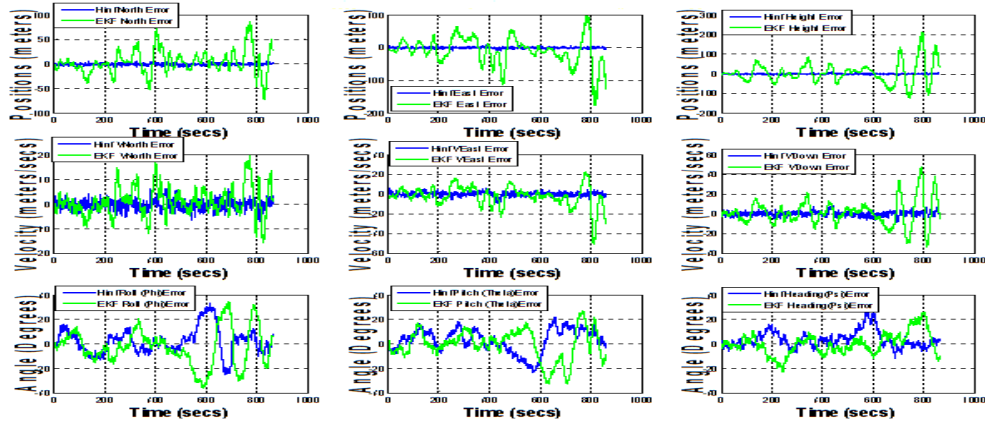


Fig. 2  $H^\infty/KF$  performance in presence of unmodeled noise of S/N ratio 1

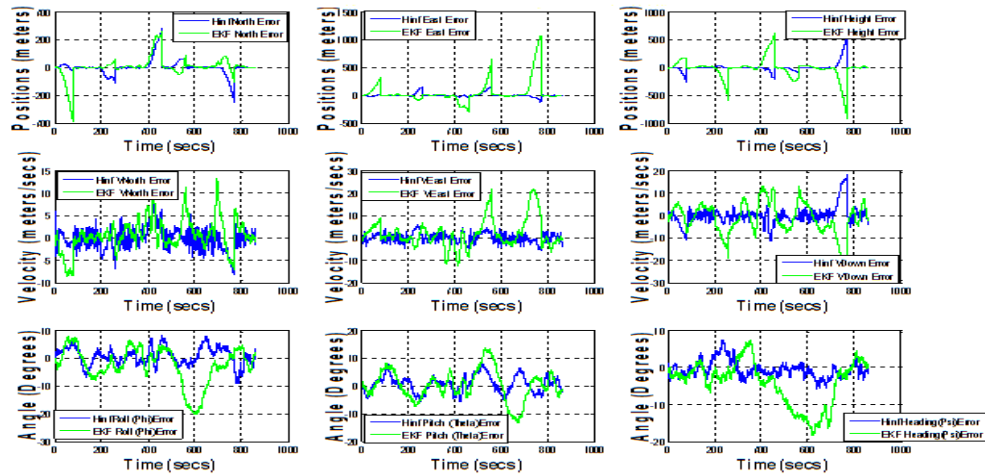


Fig. 3  $H^\infty/KF$  performance in presence of unmodeled noise of S/N ratio 10 and frequent GPS outages.

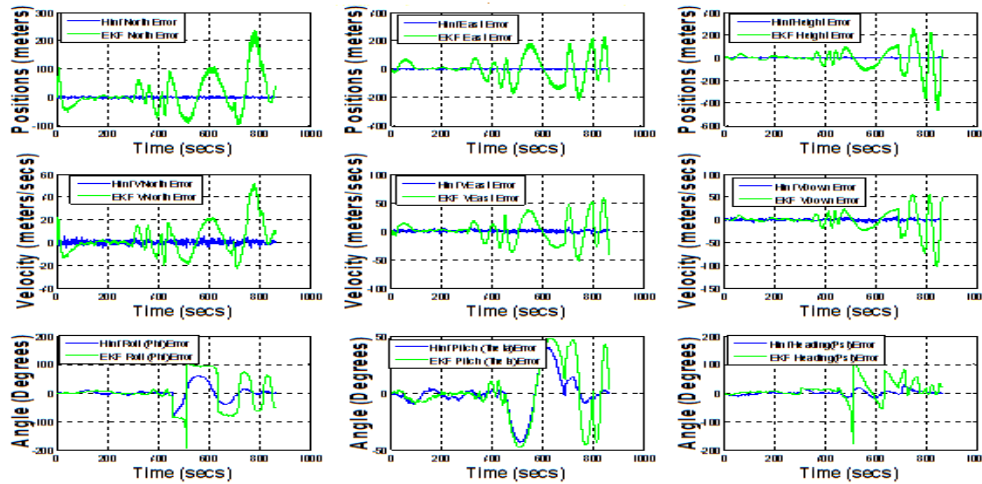


Fig. 4  $H^\infty/KF$  performance in presence of unmodeled noise of S/N ratio 10 and discontinuities in INS data.

### III. COMPARISON OF $H^\infty$ UKF FILTER UKF FILTER IN THE PRESENCE OF COLORED NOISE IN NAVIGATION

As the extended  $H^\infty$  filter adopts the idea of the EKF, the inherent disadvantages associated with the EKF, such as smoothing and lower nonlinearity requirements of the nonlinear functions and the computation errors of Jacobian matrices remains a challenge to overcome [6]. So Nonlinear  $H^\infty$  Filter was derived in UKF setup in [6]. This research has

utilized  $H^\infty UKF$  both for navigation and self alignment .Fig 5 compares UKF with  $H^\infty UKF$  in presence of colored noises and Fig.6 compares both filters in the presence of initial attitude error of  $30^\circ$ .As UKF is prone towards initial conditions error same is the case with  $H^\infty UKF$ .Both results show that in the presence of colored noise  $H^\infty UKF$  did not diverge and overall errors remain within bound.

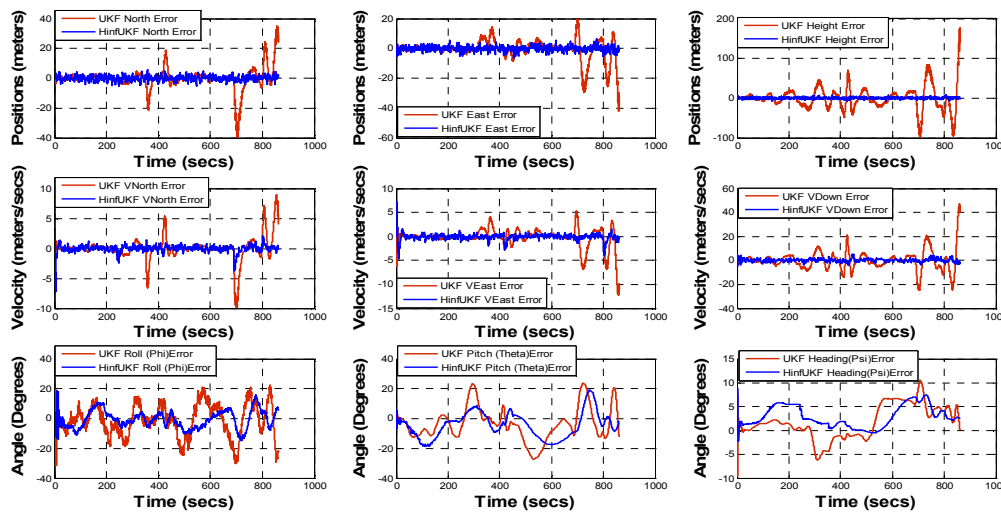
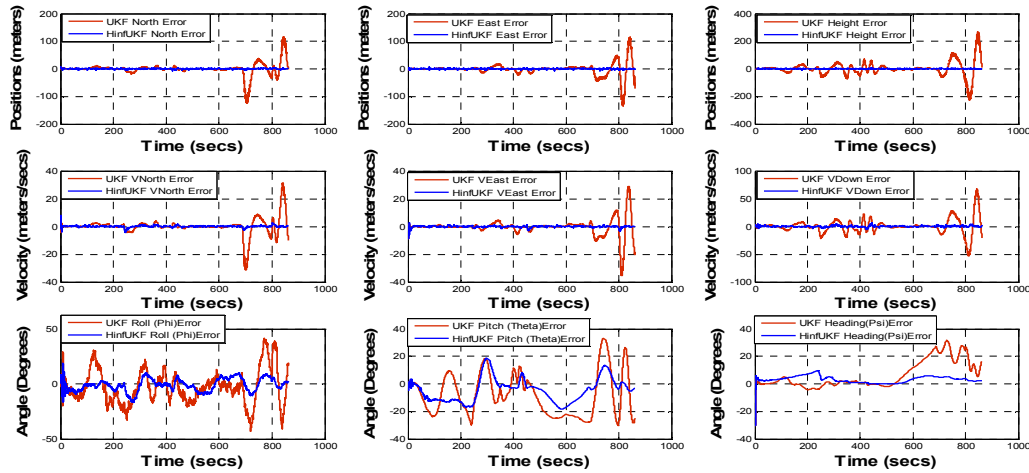


Fig. 5  $H^\infty UKF/UKF$  performance in presence of colored noise


 Fig. 6  $H^\infty$ UKF/UKF performance with initial attitude error of  $30^\circ$ 

#### IV. COMPARISON OF $H^\infty$ UKF FILTER UKF FILTER IN THE PRESENCE OF COLORED NOISE IN SELF ALIGNMENT

A fine self alignment model with large angle error assumption, described in section A was utilized with initial heading of  $45^\circ$  and Beijing's latitude, longitude. Fig. 8 shows result of self alignment when colored noises are present in the data with initial attitude error of  $10^\circ$  in heading only. The figure demonstrates that the presence of colored noises has corrupted the estimation of pitch and roll very severely but heading estimate is accurate. We are not using any height and vertical velocity measurement that is why pitch estimate is not good, but in this particular problem the main task was to estimate heading which was fulfilled with in a tolerance not more than half to one degree in 400 seconds which is 20% improvement as compared to same simulation with UKF shown in Fig. 7 with same parameters.

##### A. INS Process Model with Large Angle Errors

Assuming all Psi angles (psi angle may refer to term defined in [8]) are large, system is stationary and all position and vertical velocity errors are negligible. Velocity, attitude error models can be written as (1)-(7) [9], [10].

$$\delta \dot{V}_N = -g(\sin \psi_N \sin \psi_D + \cos \psi_N \sin \psi_E \cos \psi_D) - 2\omega_{ie} \sin \phi \delta V_E + \nabla_N \quad (1)$$

$$\delta \dot{V}_E = -g(-\sin \psi_N \cos \psi_D + \cos \psi_N \sin \psi_E \sin \psi_D) - 2\omega_{ie} \sin \phi \delta V_N + \nabla_E \quad (2)$$

$$\dot{\psi}_N = (1 - \cos \psi_D \cos \psi_E) \omega_{ie} \cos \phi - (\sin \psi_N \sin \psi_D + \cos \psi_N \sin \psi_E \cos \psi_D) \omega_{ie} \sin \phi - \epsilon_N \quad (3)$$

$$\dot{\psi}_E = (\cos \psi_E \sin \psi_D) \omega_{ie} \cos \phi - (-\sin \psi_N \cos \psi_D + \cos \psi_N \sin \psi_E \sin \psi_D) \omega_{ie} \sin \phi - \epsilon_E \quad (4)$$

$$\dot{\psi}_D = -\sin \psi_E \omega_{ie} \cos \phi - (1 - \cos \psi_N \cos \psi_E) \omega_{ie} \sin \phi - \epsilon_D \quad (5)$$

$$\dot{\nabla}_b = 0 \quad (6)$$

$$\dot{\epsilon}_b = 0 \quad (7)$$

Here  $\delta V_N$ ,  $\delta V_E$  are north and east velocities.  $\psi_N$ ,  $\psi_E$ ,  $\psi_D$  are error angles between computer frames (INS computed frame) and local geographic frame.  $\omega_{ie}$  is earth rate.  $\phi$  is site latitude.  $\epsilon_N$ ,  $\epsilon_E$  and  $\epsilon_D$  are gyro drifts. In (6) and (7) bias of accelerometers and gyro drifts are modeled in body frame respectively.

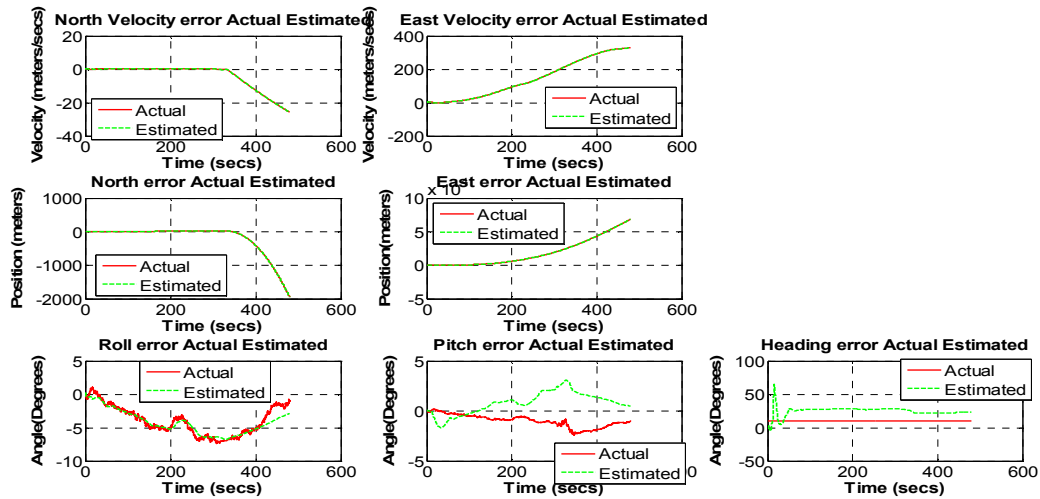
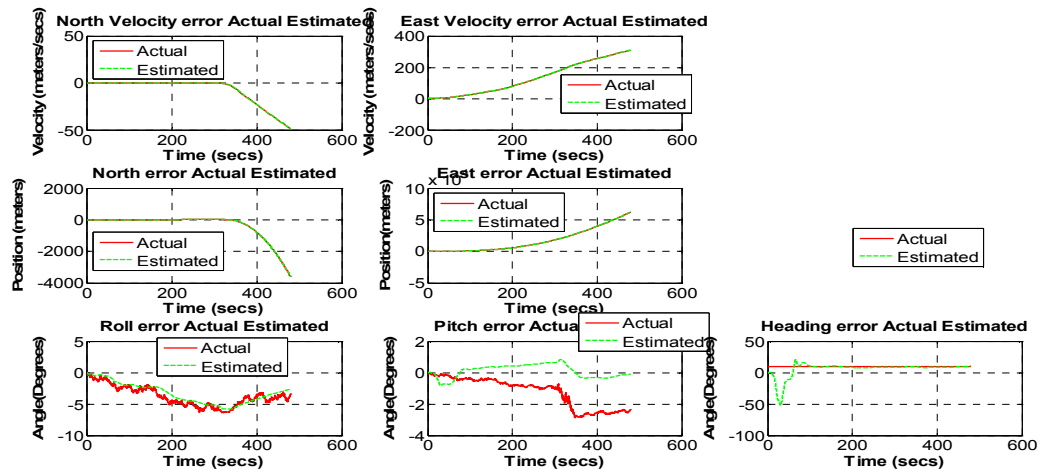


Fig. 7 Self alignment with colored noise and UKF

Fig. 8 Self alignment with colored noise and  $H_{\infty}$ UKF

## V. CONCLUSION

The present research focused on the advantages of robust filters in navigation and self alignment and proved that they performed 40-70% better in presence of colored, poorly modeled noises. Future work has to be done in testing these filters in real time environment with some hybrid approaches to avoid computational burden. The tuning parameters of robust filters are more than conventional filters and so as the computation time, so in case of Gaussian white noises with known variances Kalman filter is better choice than any robust filter.

## ACKNOWLEDGMENT

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