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Multivariate High Order Fuzzy Time Series Forecasting for Car Road Accidents

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***Abstract*—**In this paper, we have presented a new multivariate fuzzy time series forecasting method. This method assumes m-factors with one main factor of interest. History of past three years is used for making new forecasts. This new method is applied in forecasting total number of car accidents in Belgium using four secondary factors. We also make comparison of our proposed method with existing methods of fuzzy time series forecasting. Experimentally, it is shown that our proposed method perform better than existing fuzzy time series forecasting methods. Practically, actuaries are interested in analysis of the patterns of causalities in road accidents. Thus using fuzzy time series, actuaries can define fuzzy premium and fuzzy underwriting of car insurance and life insurance for car insurance. National Institute of Statistics, Belgium provides region of risk classification for each road. Thus using this risk classification, we can predict premium rate and underwriting of insurance policy holders.

***Keywords*—**Average forecasting error rate (AFER), Fuzziness of fuzzy sets Fuzzy, If-Then rules, Multivariate fuzzy time series.

I. INTRODUCTION

IN our daily life, people often use forecasting techniques to model and predict economy, population growth, stocks, insurance/ re-insurance, portfolio analysis and etc. However, in the real world, an event can be affected by many factors. Therefore, if we consider more factors for prediction, with higher complexity then we can get better forecasting results.

During last few decades, various approaches have been developed for time series forecasting. Among them ARMA models and Box-Jenkins model building approaches are highly famous.

In recent years, many researchers used fuzzy time series to handle prediction problems. Song and Chissom [8] presented the concept of fuzzy time series based on the concepts of fuzzy set theory to forecast the historical enrollments of the University of Alabama. Huarng [4] presented the definition of two kinds of intervals in the universe of discourse to forecast the TAIFEX. Chen [10] presented a method for forecasting based on high-order fuzzy time series. Lee [6] presented a method for temperature prediction based on two-factor high-order fuzzy time series. Melike [7] proposed forecasting method using first order fuzzy time series for forecasting enrollments in University of Alabama. Lee [6] Presented handling of forecasting problems using two-factor high order fuzzy time series for TAIFEX and daily temperature in Taipei, Taiwan.

The rest of this paper is organized as follows. In section 2, brief review of fuzzy time series is given. In section 3, we present the new method for fuzzy time series modeling. Experimental results are performed in section 4. The conclusions are discussed in section 5.

In this paper, we present a new modified method to predict total number of annual car road accidents based on the m-factors high-order fuzzy time series. This method provides a general framework for forecasting that can be increased by increasing the stochastic fuzzy dependence [3]. For simplicity of computation, we have used triangular membership function. The proposed method constructs m-factor high-order fuzzy logical relationships based on the historical data to increase the forecasting accuracy rate. Our proposed forecasting method for fuzzy time series gives better results as compared to [4], [5] and [10].

II. FUZZY TIME SERIES

Time series analysis plays vital role in most of the actuarial related problems. As most of the actuarial issues are born with uncertainty, therefore, each observation of a fuzzy time series is assumed to be a fuzzy variable along with associated membership function. Based on fuzzy relation, and fuzzy inference rules, efficient modeling and forecasting of fuzzy time series is possible, see [1] and [9]. This field of fuzzy time series analysis is not very mature due to the time and space complexities in most of the actuarial related issue, thus we can extend this concept for many antecedents and single consequent. For example, in designing two-factor kth-order fuzzy time series model with X be the primary and Y be second fact. We assume that there are k antecedent $((X_1,Y_1),(X_2,Y_2),\ldots,(X_k,Y_k))$ and one consequent X_{k+1}.

$$\begin{aligned} &If\ (X_1=x_1,Y_1=y_1),(X_2=x_2,Y_2=y_2),\\ &\ldots,(X_k=x_k,Y_k=y_k)\rightarrow(X_{k+1}=x_{k+1}) \end{aligned} \tag{1}$$

In the similar way, we can define m-factor $i=1,2,\ldots,m$ and kth order fuzzy time series as

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TABLE I
YEARLY CAR ACCIDENTS MORTALITIES AND VICTIMS FROM 1974 TO 2004

Year	Killed (X)	Mortally wounded (Y1)	Died 30 days (Y2)	Severely wounded (Y3)	Light casualties (Y4)
2004	953	141	1,094	5,949	41,627
2003	1,035	101	1,136	6,898	42,445
2002	1,145	118	1,263	6,834	39,522
2001	1,288	90	1,378	7,319	38,747
2000	1,253	103	1,356	7,990	39,719
1999	1,173	126	1,299	8,461	41,841
1998	1,224	121	1,345	8,784	41,038
1997	1,150	105	1,255	9,229	39,594
1996	1,122	115	1,237	9,123	38,390
1995	1,228	109	1,337	10,267	39,140
1994	1,415	149	1,564	11,160	40,294
1993	1,346	171	1,517	11,680	41,736
1992	1,380	173	1,553	12,113	41,772
1991	1,471	209	1,680	12,965	43,578
1990	1,574	190	1,764	13,864	46,818
1989	1,488	312	1,800	14,515	46,667
1988	1,432	339	1,771	14,029	45,956
1987	1,390	380	1,770	13,809	44,090
1986	1,456	330	1,786	13,764	42,965
1985	1,308	352	1,660	13,287	39,879
1984	1,369	363	1,732	14,471	42,456
1983	1,479	412	1,891	14,864	42,023
1982	1,464	406	1,870	14,601	40,936
1981	1,564	454	2,018	15,091	41,915
1980	1,616	557	2,173	15,915	42,670
1979	1,572	544	2,116	15,750	42,346
1978	1,644	728	2,372	16,645	44,797
1977	1,597	701	2,298	15,830	44,995
1976	1,536	728	2,264	16,057	44,227
1975	1,460	701	2,161	15,792	42,423
1974	1,574	819	2,393	16,506	44,640

$$\begin{aligned}&\textbf{If } (X_{11}=x_{11}, X_{12}=x_{12},...,X_{1k}=x_{1k}),\\&(X_{21}=x_{21}, X_{22}=x_{22},...,X_{2k}=x_{2k}),...,\\&(X_{m1}=x_{m1}, X_{m2}=x_{m2},...,X_{mk}=x_{mk})\\&\textbf{then } (X_{m+1,k+1}=x_{m+1k+1})\\&for\ \ i=1,2,...,m\ \ ,\ j=1,2,...,k\end{aligned} \tag{2}$$

Now, we formally give details of proposed method in section 3.

III. New Method of Forecasting using Fuzzy Time Series

Let $\mathrm{Y}(t),(t=...,0,1,2,...)$ be the universe of discourse and $Y(t)\subseteq R$. Assume that $f_i(t),\ i=1,2,...$ is defined in the universe of discourse $\mathrm{Y}(t)$ and $F(t)$ is a collection of $f(t_i),(i=...,0,1,2,...)$, then $F(t)$ is called a fuzzy time series of $\mathrm{Y}(t),\ i=1,2,...$ ... Using fuzzy relation, we define $F(t)=F(t-1)\circ R(t,t-1)$, where $R(t,t-1)$ is a fuzzy relation and "$\circ$" is the max–min composition operator, then $F(t)$ is caused by $F(t-1)$ where $F(t)$ and $F(t-1)$ are fuzzy sets. For forecasting purpose, we can define relationship among present and future state of a time series with the help of fuzzy sets. Assume the fuzzified data of the ith and $(i+1)th$ day are A_j and A_k, respectively, where $A_j, A_k\in U$, then $A_j\to A_k$ represented the fuzzy logical relationship between A_j and A_k. Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1)$, $F(t-2)$, ..., $F(t-n)$, then the fuzzy logical relationship is represented by

$$F(t-n),...,F(t-2),F(t-1)\to F(t) \tag{3}$$

is called the one-factor nth order fuzzy time series forecasting model. Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $(F_1(t-1),F_2(t-1))$, $(F_1(t-2),F_2(t-2))$, ..., $(F_1(t-n),F_2(t-n))$, then this fuzzy logical relationship is represented by

$$\begin{aligned}&(F_1(t-n),F_2(t-n)),...,(F_1(t-2),F_2(t-2)),\\&(F_1(t-1),F_2(t-1))\to F(t)\end{aligned} \tag{4}$$

is called the two-factors nth order fuzzy time series forecasting model, where $F_1(t)$ and $F_2(t)$ are called the main factor and the secondary factor fuzzy time series' respectively. In the similar way, we can define m-factor nth order fuzzy logical relationship as

$$\begin{aligned}&(F_1(t-n),F_2(t-n),...,F_m(t-n)),...,\\&(F_1(t-2),F_2(t-2),...,F_m(t-2)),\\&(F_1(t-1),F_2(t-1),...,F_m(t-1))\to F(t)\end{aligned} \tag{5}$$

Here $F_1(t)$ is called the main factor and $F_2(t),F_3(t),...,F_m(t)$ are called secondary factor fuzzy time series'. Here we can implement any of the fuzzy membership function to define the fuzzy time series in above equations. Comparative study by using different membership functions is also possible. We have used triangular membership function due to low computational cost. Using fuzzy composition rules, we establish a fuzzy inference system for fuzzy time series forecasting with higher accuracy. The accuracy of forecast can be improved by considering higher number of factors and higher dependence on history.

Now we present an extended method for handling forecasting problems based on m-factors high-order fuzzy time series. The proposed method is now presented as follows.

Step 1) Define the universe of discourse U of the main factor $U=[D_{\min}-D_1, D_{\max}-D_2]$, where $D_{\min}$ and $D_{\max}$ are the minimum and the maximum values of the main factor of the known historical data, respectively, and D_1, D_2 are two proper positive real numbers to divide the universe of discourse into n equal length intervals $u_1, u_2, ..., u_l$. Define the universes of discourse $\mathbf{V}_i$, $i=1,2,...,m-1$ of the secondary-factors $\mathbf{V}_i = \left[(\mathbf{E_i})_{\min} - \mathbf{E}_{i1}, (\mathbf{E_i})_{\max} - \mathbf{E}_{i2} \right]$, where $(\mathbf{E_i})_{\min} = \left[(E_1)_{\min}, (E_2)_{\min}, ..., (E_m)_{\min} \right]$ and $(\mathbf{E_i})_{\max} = \left[(E_1)_{\max}, (E_2)_{\max}, ..., (E_m)_{\max} \right]$ are the minimum and maximum values of the secondary-factors of the known historical data, respectively, and $\mathbf{E_{i1}}$, $\mathbf{E_{i2}}$ are vectors of proper positive numbers to divide each of the universe of discourse $\mathbf{V}_i$, $i=1,2,...,m-1$ into equal length intervals termed as $\mathbf{v}_{1,l}, \mathbf{v}_{2,l}, ..., \mathbf{v}_{m-1,l}$, $l=1,2,...,p$, where $\mathbf{v}_{1,l} = \left[\mathbf{v}_{1,1}, \mathbf{v}_{1,2}, ... \mathbf{v}_{1,p} \right]$ represents n intervals of equal length of universe of discourse $\mathbf{V_1}$ for first secondary-factor fuzzy time series. Thus we have $(m-1) \times l$ matrix of intervals for secondary-factors.

Step 2) Define the linguistic term A_i represented by fuzzy sets of the main factor shown as follows:

$$
\begin{aligned}
A_1 &= 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + ... + 0/u_{l-2} + 0/u_{l-1} + 0/u_l \\
A_2 &= 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + ... + 0/u_{l-2} + 0/u_{l-1} + 0/u_l \\
A_3 &= 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + ... + 0/u_{l-2} + 0/u_{l-1} + 0/u_l \\
&\;\;\vdots \\
A_n &= 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + ... + 0/u_{l-2} + 0.5/u_{l-1} + 1/u_l
\end{aligned}
\tag{6}
$$

Similarly, for ith secondary fuzzy time series, we define the linguistic term $B_{i,j}$, $i=1,2,...,m-1$, $j=1,2,...,n$ represented by fuzzy sets of the secondary-factors,

$$
\begin{aligned}
B_{i,1} &= 1/\mathbf{v}_{i,1} + 0.5/\mathbf{v}_{i,2} + 0/\mathbf{v}_{i,3} + 0/\mathbf{v}_{i,4} + ... + 0/\mathbf{v}_{i,l-2} + 0/\mathbf{v}_{i,l-1} + 0/\mathbf{v}_{i,l} \\
B_{i,2} &= 0.5/\mathbf{v}_{i,1} + 1/\mathbf{v}_{i,2} + 0.5/\mathbf{v}_{i,3} + 0/\mathbf{v}_{i,4} + ... + 0/\mathbf{v}_{i,l-2} + 0/\mathbf{v}_{i,l-1} + 0/\mathbf{v}_{i,l} \\
B_{i,3} &= 0/\mathbf{v}_{i,1} + 0.5/\mathbf{v}_{i,2} + 1/\mathbf{v}_{i,3} + 0.5/\mathbf{v}_{i,4} + ... + 0/\mathbf{v}_{i,l-2} + 0/\mathbf{v}_{i,l-1} + 0/\mathbf{v}_{i,l} \\
&\;\;\vdots \\
B_{i,n} &= 0/\mathbf{v}_{i,1} + 0/\mathbf{v}_{i,2} + 0/\mathbf{v}_{i,3} + 0/\mathbf{v}_{i,4} + ... + 0/\mathbf{v}_{i,l-2} + 0.5/\mathbf{v}_{i,l-1} + 1/\mathbf{v}_{i,l}
\end{aligned}
\tag{7}
$$

Step 3) Fuzzify the historical data described as follows. Find out the interval u_l, $l=1,2,...,p$ to which the value of the main factor belongs

Case 1) If the value of the main factor belongs to u_1, then the value of the main factor is fuzzified into $1/A_1 + 0.5/A_2 + 0.0/A_3$, denoted by X_1.

TABLE II
FUZZIFIED YEARLY DATA FOR MORTALITY ACCIDENTS FROM 1974 TO 2004

Year	Mortality Accidents	Fuzzified Mortality Accidents
2004	953	$0.5/A_1 + 1.0/A_2 + 0.5/A_3$ (X_2)
2003	1,035	$0.5/A_1 + 1.0/A_2 + 0.5/A_3$ (X_2)
2002	1,145	$0.5/A_2 + 1.0/A_3 + 0.5/A_4$ (X_3)
2001	1,288	$0.5/A_4 + 1.0/A_5 + 0.5/A_6$ (X_5)
2000	1,253	$0.5/A_4 + 1.0/A_5 + 0.5/A_6$ (X_5)
1999	1,173	$0.5/A_3 + 1.0/A_4 + 0.5/A_5$ (X_4)
1998	1,224	$0.5/A_3 + 1.0/A_4 + 0.5/A_5$ (X_4)
1997	1,150	$0.5/A_3 + 1.0/A_4 + 0.5/A_5$ (X_4)
1996	1,122	$0.5/A_2 + 1.0/A_3 + 0.5/A_4$ (X_3)
1995	1,228	$0.5/A_3 + 1.0/A_4 + 0.5/A_5$ (X_4)
1994	1,415	$0.5/A_5 + 1.0/A_6 + 0.5/A_7$ (X_6)
1993	1,346	$0.5/A_4 + 1.0/A_5 + 0.5/A_6$ (X_5)
1992	1,380	$0.5/A_5 + 1.0/A_6 + 0.5/A_7$ (X_6)
1991	1,471	$0.5/A_6 + 1.0/A_7 + 0.5/A_8$ (X_7)
1990	1,574	$0.0/A_6 + 0.5/A_7 + 1.0/A_8$ (X_8)
1989	1,488	$0.5/A_6 + 1.0/A_7 + 0.5/A_8$ (X_7)
1988	1,432	$0.5/A_5 + 1.0/A_6 + 0.5/A_7$ (X_6)
1987	1,390	$0.5/A_5 + 1.0/A_6 + 0.5/A_7$ (X_6)
1986	1,456	$0.5/A_6 + 1.0/A_7 + 0.5/A_8$ (X_7)
1985	1,308	$0.5/A_4 + 1.0/A_5 + 0.5/A_6$ (X_5)
1984	1,369	$0.5/A_5 + 1.0/A_6 + 0.5/A_7$ (X_6)
1983	1,479	$0.5/A_6 + 1.0/A_7 + 0.5/A_8$ (X_7)
1982	1,464	$0.5/A_6 + 1.0/A_7 + 0.5/A_8$ (X_7)
1981	1,564	$0.0/A_6 + 0.5/A_7 + 1.0/A_8$ (X_8)
1980	1,616	$0.0/A_6 + 0.5/A_7 + 1.0/A_8$ (X_8)
1979	1,572	$0.0/A_6 + 0.5/A_7 + 1.0/A_8$ (X_8)
1978	1,644	$0.0/A_6 + 0.5/A_7 + 1.0/A_8$ (X_8)
1977	1,597	$0.0/A_6 + 0.5/A_7 + 1.0/A_8$ (X_8)
1976	1,536	$0.5/A_6 + 1.0/A_7 + 0.5/A_8$ (X_7)
1975	1,460	$0.5/A_6 + 1.0/A_7 + 0.5/A_8$ (X_7)
1974	1,574	$0.0/A_6 + 0.5/A_7 + 1.0/A_8$ (X_8)

Case 2) If the value of the main factor belongs to u_l, $l=2,3,...,p-1$ then the value of the main factor is fuzzified into $0.5/A_{i-1} + 1/A_i + 0.5/A_{i+1}$, denoted by X_i .

Case 3) If the value of the main factor belongs to u_p , then the value of the main factor is fuzzified into $0/A_{n-2} + 0.5/A_{n-1} + 1/A_n$, denoted by X_n .

Now, for i[th] secondary-factor, find out the interval $\mathbf{V}_{i,l}$ to which the value of the secondary-factor belongs.

Case 1) If the value of the ith secondary-factor belongs to $\mathbf{v}_{i,1}$, then the value of the secondary-factor is fuzzified into $1/B_{i,1} + 0.5/B_{i,2} + 0/B_{i,3}$, denoted by $\mathbf{Y}_{i,1} = \left[Y_{1,1}, Y_{2,1}, ..., Y_{m-1,1} \right]$.

Case 2) If the value of the ith secondary-factor belongs to $\mathbf{v}_{i,l}$, $l=2,3,..,p-1$, then the value of the ith secondary-factor is fuzzified into $0.5/B_{i,j-1} + 1/B_{i,j} + 0.5/B_{i,j+1}$, $j=i=2,3,...,n-1$ denoted by $Y_{i,j}$, where $j=2,3,....,n-1$.

Case 3) If the value of the ith secondary-factor belongs to $\mathbf{v}_{i,p}$, then the value of the secondary-factor is fuzzified into $0/B_{i,n-2}+0.5/B_{i,n-1}+1/B_{i,n}$, denoted by $Y_{i,n}$.

Step 4) Get the m-factors kth-order fuzzy logical relationships based on the fuzzified main and secondary factors from the fuzzified historical data obtained in Step 3**).** If the fuzzified historical data of the main-factor of ith day is X_i, then construct the m-factors kth-order fuzzy logical relationships,

$$\begin{aligned}&\left(X_{j-k};Y_{2,j-k},...,Y_{m-1,j-k}\right),...,\left(X_{j-2};Y_{2,j-2},...,Y_{m-1,j-2}\right),\\&\left(X_{j-1};Y_{1,j-1},Y_{2,j-1},...,Y_{m-1,j-1}\right),\rightarrow X_j\end{aligned} \tag{8}$$

where $j>k$. X_{j-k} shows the k-step dependence of jth value of main factor X_j, $Y_{i,j-k}$, $i=1,...,m-1,\ j=1,...,k$.Then, divide the derived fuzzy logical relationships into fuzzy logical relationship groups based on the current states of the fuzzy logical relationships. The secondary factors acts like a secondary component to the m-dimensional state vector and is used in Step 5).

Step 5) For m-factor kth order fuzzy logical relationship, the forecasted value of day j based on history of third order is calculated as follows,

$$t_j=\frac{\sum_{j-1}^{j+1} w}{\frac{w_{j-1}}{a_{j-1}}+\frac{w_j}{a_j}+\frac{w_{j+1}}{a_{j+1}}} \tag{9}$$

Where a_{l-1}, a_l and a_{l+1} are the midpoints of the intervals $u_{l-1,}u_l$ and u_{l+1} respectively. Above forecasting formula fulfills the axioms of fuzzy sets like monotonicity, boundary conditions, continuity and idempotency. For measurement of accuracy of forecasting for fuzzy time series forecasting, we use average forecasting error rate (AFER) as the performance criteria, defined as

$$AFER=\frac{\sum_{i=1}^{n}\left|\left(Forecasted\ value\ of\ Day\ j-Actual\ Value\ of\ Day\ j\right)\right|}{n}\times 100\% \tag{10}$$

IV. Experiment

In this experiment, our goal is to extend the work of [6]. We have applied this new technique on car road accident data taken from National Institute of Statistics, Belgium for the period of 1974-2005. In this data, the main factor of interest is the yearly road accident causalities and secondary factors are mortally wounded, died within one month, severely wounded and light casualties.

TABLE III
FORECASTED YEARLY CAR ACCIDENT CAUALITIES FROM 1974-2004

Year	Actual Killed A_i	Forecasted Kills F_i	$F_i - A_i$	$\left\lvert\frac{F_i - A_i}{A_i}\right\rvert$
2004	953	995	-42	0.04404
2003	1,035	995	40	0.038676
2002	1,145	1095	50	0.043319
2001	1,288	1296	-8	0.006289
2000	1,253	1296	-43	0.034397
1999	1,173	1196	-23	0.019437
1998	1,224	1196	28	0.023039
1997	1,150	1196	-46	0.039826
1996	1,122	1095	27	0.023708
1995	1,228	1396	-168	0.137134
1994	1,415	1296	119	0.084028
1993	1,346	1396	-50	0.037444
1992	1,380	1497	-117	0.084565
1991	1,471	1497	-26	0.017471
1990	1,574	1497	77	0.049111
1989	1,488	1396	92	0.061559
1988	1,432	1396	36	0.02486
1987	1,390	1497	-107	0.076763
1986	1,456	1296	160	0.109821
1985	1,308	1396	-88	0.067584
1984	1,369	1497	-128	0.09328
1983	1,479	1497	-18	0.011968
1982	1,464	1497	-33	0.022336
1981	1,564	1497	67	0.043031
1980	1,616	1497	119	0.073824
1979	1,572	1497	75	0.047901
1978	1,644	1497	147	0.089599
1977	1,597	1497	100	0.062805
1976	1,536	1497	39	0.025586
1975	1,460	1497	-37	0.025137
1974	1,574	1497	77	0.0489199

We assumed eight intervals of equal length for the main and secondary fuzzy time series'. For main factor, we assume $D_{\min}$ =953 and $D_{\max}$ =1644, thus for main factor time series we get $U=[850,1650]$. Similarly for secondary factors Y_1, Y_2, Y_3 *and* Y_4, we assumed that $\mathbf{E}_{\min}=[90,1094,5949,38390]$ and $\mathbf{E}_{\max}=[819,2393,16645,46818]$ to determine $\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4$. Selection of $D_{\min}$, $D_{\max}$, $\mathbf{E}_{\min}$ and $\mathbf{E}_{\max}$ have significant effects on the accuracy of this new method. We can introduce learning to stabilize the heuristic selection of these constants.

Using (6) and (7), we formed fuzzy times series from main and secondary factors. Therefore, each observation of a time series is now represented by a combination of fuzzy sets. Using (10), we calculated the forecasted values corresponding to each actual value of the main factor time series in III. Using equation (11) for AFER, we formed table IV, showing a comparison of actual and forecasted values. Finally, we have compared proposed method with [6].

TABLE IV
COMPARISON OF PROPOSED METHOD AND LEE L. W. (2006). METHOD FOR YEALRY CAR ACCIDENT CAUALITIES IN BELGIUM FROM 1974-2004

Year	Actual Killed (A_i)	Forecasted causalities (F_i)	$\left\|\frac{F_i - A_i}{A_i}\right\|$	Forecasted causalities (F_i)	$\left\|\frac{F_i - A_i}{A_i}\right\|$
2004	953	995	0.044040	1000	0.049318
2003	1035	995	0.038676	1000	0.033816
2002	1145	1095	0.043319	1100	0.039301
2001	1288	1296	0.006289	1300	0.009317
2000	1253	1296	0.034397	1300	0.037510
1999	1173	1196	0.019437	1200	0.023018
1998	1224	1196	0.023039	1200	0.019608
1997	1150	1196	0.039826	1200	0.043478
1996	1122	1095	0.023708	1100	0.019608
1995	1228	1396	0.137134	1400	0.140065
1994	1415	1296	0.084028	1300	0.081272
1993	1346	1396	0.037444	1400	0.040119
1992	1380	1497	0.084565	1500	0.086957
1991	1471	1497	0.017471	1500	0.019714
1990	1574	1497	0.049111	1500	0.047014
1989	1488	1396	0.061559	1400	0.059140
1988	1432	1396	0.024860	1400	0.022346
1987	1390	1497	0.076763	1500	0.079137
1986	1456	1296	0.109821	1300	0.107143
1985	1308	1396	0.067584	1400	0.070336
1984	1369	1497	0.093280	1500	0.095690
1983	1479	1497	0.011968	1500	0.014199
1982	1464	1497	0.022336	1500	0.024590
1981	1564	1497	0.043031	1500	0.040921
1980	1616	1497	0.073824	1500	0.071782
1979	1572	1497	0.047901	1500	0.045802
1978	1644	1497	0.089599	1500	0.087591
1977	1597	1497	0.062805	1500	0.060739
1976	1536	1497	0.025586	1500	0.023438
1975	1460	1497	0.025137	1500	0.027397
1974	1574	1497	0.048920	1500	0.025413
$AFER = \frac{\sum_{i=1}^{n} \frac{\|F_i - A_i\|}{A_i}}{31} \times 100\%$			= 5.061793%		= 5.067887%

V. CONCLUSION

From Table IV, we can see that our proposed method is better that [6]. As the work of Lee et. al. [6] outperformed the work of [4], [5] and [10], so, indirectly we can conclude that our general class of methods for fuzzy time series modeling and forecasting.

Furthermore, we have shown fuzziness of fuzzy observations by presenting each datum of the main series as composed of many fuzzy sets. Thus, fuzzy time series modeling extends to type-II fuzzy time series modeling. The type-II defuzzified forecasted values (t_j) may also be calculated using some other method, e.g. learning rules from fuzzy time series.

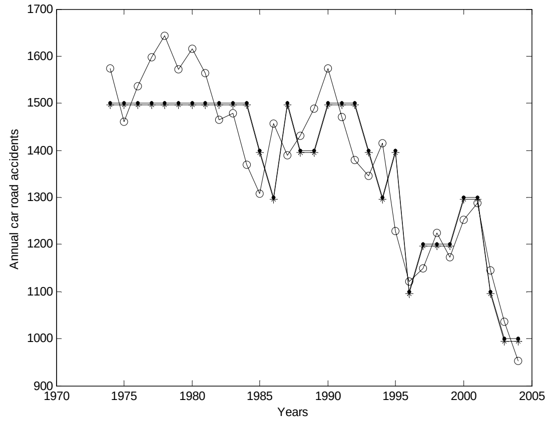

Fig. 1 A Comparison of proposed and Lee L. W. et. al. (2006) [6] Methods

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