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Multivariate Assessment of Mathematics Test Scores of Students in Qatar

Ali Rashash Alzahrani, Elizabeth Stojanovski

***Abstract*—**Data on various aspects of education are collected at the institutional and government level regularly. In Australia, for example, students at various levels of schooling undertake examinations in numeracy and literacy as part of NAPLAN testing, enabling longitudinal assessment of such data as well as comparisons between schools and states within Australia. Another source of educational data collected internationally is via the PISA study which collects data from several countries when students are approximately 15 years of age and enables comparisons in the performance of science, mathematics and English between countries as well as ranking of countries based on performance in these standardised tests. As well as student and school outcomes based on the tests taken as part of the PISA study, there is a wealth of other data collected in the study including parental demographics data and data related to teaching strategies used by educators. Overall, an abundance of educational data is available which has the potential to be used to help improve educational attainment and teaching of content in order to improve learning outcomes. A multivariate assessment of such data enables multiple variables to be considered simultaneously and will be used in the present study to help develop profiles of students based on performance in mathematics using data obtained from the PISA study.

***Keywords*—**Cluster analysis, education, mathematics, profiles.

I. Introduction

MULTIVARIATE assessment of data enables the analysis and understanding of multiple variables simultaneously, thus enabling a more comprehensive understanding of the variables that may influence an outcome. Whilst there are many different multivariate approaches that can be used in different research contexts, cluster analysis will be the focal point of the present study.

Cluster analysis is a multivariate method which can be used in the educational setting to assess data at student level by forming groups of students that are homogenous within and heterogeneous between groups by simultaneous assessment of several variables. This technique is in contrast to other methods as it involves the analysis of correlations between students instead of among variables, which is used in standard techniques such as regression modelling.

It is intended with cluster analysis techniques, for the present study, to ascertain student attributes, for example, that better discriminate between different groups of students with varying levels of performance in mathematics in the education setting. Grouping students has the potential benefit of enabling educators to better develop strategies, for example, that are specific to student needs by identifying students with differing needs in terms of their learning requirements, thus enabling a better allocation of resources to students with different needs. Cluster analysis can assist with creating student profiles based on demographical information collected at the student level on entry into the schooling system which can then be used to better allocate learning resources to relevant groups of students based on their associated needs. In the context of this paper, information that has already been collected about students at entry level into the Qatar education system will be used to develop profiles that are then assessed as predictors of mathematics performance scores when students are aged 15 years old, which is used as an indicator of mathematics ability towards the end of secondary schooling years. This analysis is an attempt to identify the degree with which demographic information collected at entry level into the school system can be used to predict mathematics outcomes in later schooling years, to better identify where resources should be allocated in the early school years.

The multivariate analyses of data related to the education field is particularly important as while it has been demonstrated that student competency can strongly influence performance in mathematics, however, student competency alone is not the only important predictor of mathematics performance. It is important to consider additional variables that may also potentially influence student learning outcomes such as parental educational levels, the social and economic status of their families, the nature of teacher instruction and the nature of the instructional environment, among other factors. These variables have been shown to influence student outcomes in mathematics, reading, comprehension and interpretation; as well as the way in which students are able to apply the learnt concepts to solve mathematical problems and perform calculations. Use of the cluster analysis technique also allows for consideration of how teaching strategies and other instruction delivery approaches interact with parental and student variables and the resultant effect this has on particular subject outcomes. Cluster analysis is thus essential when assessing subject specific outcomes based on the assessment of multiple variables simultaneously, as it helps establish the extent to which each of the variables involved impacts on the subject scores.

The application of cluster analysis to the study of the relationships between different student profiles and both instructional approaches and educational outcomes will enable the establishment of the extent to which the current teaching

A. Alzahrani is with the University of Newcastle, NSW 2308 Australia (corresponding author, phone: +61-2-4921-5000; fax: +61-2-4921-6898; e-mail: Alirashashr.alzahrani@uon.edu.au).

E. Stojanovski is with the University of Newcastle, NSW 2308 Australia (phone: +61-2-4921-5346; fax: +61-2-4921-6898; e-mail: Elizabeth.stojanovski@newcastle.edu.au).

strategies align with the needs of the students and the required improvements for various student groups. The technique also has the potential to help discover and categorise low and high achieving students in mathematics for the present study, but could potentially be used for any subject area, thus enabling educators to better direct educational resources at their disposal.

This study uses the cluster analysis method to explore educational data by utilising a number of student characteristics to create a set of student profiles from which potential learning and strategic benefits could be derived. It accomplishes this by dividing the data set into subgroups with similar traits but which vary substantially from traits outside the defined subgroups. The study will use cluster analysis to help develop profiles that will be integral in establishing the extent to which mathematics students are clustered based on a number of considered variables. In this study, therefore, cluster analysis will be used to assess different aspects of mathematics learning and performance which assumes within group or cluster homogeneity and between group heterogeneity.

II. Methods

The application and comparison of different cluster analytic techniques is assessed to develop a profile of school students using data from 10966 secondary school students in Qatar participating in the 2012 Programme for International Student Assessment (PISA) that measures the performance of 15 year olds in reading, mathematics, and science literacy at institutional and national levels [1]. The PISA study also provides vast student-level data including multiple measures of student demographics, measures such as parental education levels, parental occupation and parental wealth and socio-economic status as well as other factors, and those such as teaching strategies, the nature of math instruction and teaching, type of teacher directed instructions provided and classroom and teacher support, are among the measures that can impact on student capacity to learn [2].

The two-stage cluster analysis method will be applied to determine the clustering of students into distinct clusters, based on pre-identified parameters, which are consistent with those parameters that would be available at entry level into the schooling system, and that include the educational level of the father and of the mother; as well as factors of the school including assessment and maths teaching strategies commonly used, student orientation, teacher directed instructions, vignette teacher support and wealth. The average mathematics scores for each of these clusters was then obtained for each of the derived clusters.

The two-stage cluster analysis method combines both hierarchical and non-hierarchical approaches. In the first stage, the algorithm undertakes a procedure similar to the k-means algorithm, which is non-hierarchical, and hence can easily manage large data. Using the results of this approach, a modified hierarchical agglomerative clustering procedure is then conducted which combines cases sequentially to form homogenous clusters [3]. This is performed by building a cluster tree, the branches of the tree representing distinct features of the data. Students are assigned to locations on the tree, the locations determined according to students' distances from all other students based on a multinomial distribution that is imposed on all variables [4]. In the second step, the Akaike Information Criterion is used to select the most appropriate number of clusters into which to group students based on where they are located on the tree. Given that cluster analysis does not differentiate between explanatory and response variables, all the variables from the first phase of the analysis will be used in the second phase of the analysis with appropriate explanation of the data set and various variables provided in each phase. In this regard therefore, an additional variable examining mathematics performance scores will be incorporated using results of the second phase to further illuminate the underlying factors. An additional variable examining mathematics performance scores will be incorporated using the results of the second phase. More specifically, the profiles created from the clusters will be assessed in terms of mathematics performance as a post-hoc analysis to assess how each cluster rates according to its relationship with mathematics performance. However, at this stage, it is worth noting that the obtained ratings of the clusters will not be used at all to establish the original profiles.

The two-step clustering method will be compared to the k-means procedure. To prevent variables with larger variances contributing disproportionately to computations of distance, all measures were standardised prior to analyses [5]. Furthermore, evaluation of the selected clusters was based on an assessment of meaningfulness and parsimony.

III. Results

Applying the original two-stage cluster analysis resulted in three clusters being identified as illustrated in Table I. The three clusters were of sizes 2166, 1987 and 1430, respectively, as a result of list-wise deletion. Percentage sizes of the clusters were 38.8%, 35.4% and 25.6% for Cluster 1, Cluster 2 and Cluster 3, respectively. The three cluster groups differed significantly from each other on all variables included in the cluster analysis ($p<0.05$), except for wealth, which was intended as a measure of socio economic status (SES).

With an average score of 0.21, students in Cluster 1 scored the lowest on standardised mean mathematics score. The students in this cluster also had parents with the lowest mean levels of education and the lowest measure of SES, but scored highest on formative assessment, student orientation, teacher directed learning and mathematics teaching components. The teaching aspects contributed less to the clusters than did the parent levels of education. In contrast, students in Cluster 2 were associated with the highest mean mathematics score (mean=0.31) and had parents with the highest mean SES scores. Mean scores on the parental education aspects were higher than the scores for the other two clusters on those components. The scores on the teaching aspects for the cluster were also significantly higher than those of the third cluster and lower than those of the first cluster. As noted earlier, the parental education levels contributed more to the cluster formations than did the teaching aspects. Further investigation showed similar relationships when examined at the school level, when assessed among schools comprising of more than 200 students in an effort to demonstrate the sensitivity of the results.

With an average math score of 0.25, Cluster 3 had the second highest math score of the three clusters. This is despite having the lowest score for the formative assessment component, conducted when students are joining school. Of significance also, is the fact that the cluster had dismal scores with respect to the teaching aspects under consideration and substantially good average scores with respect to parental education components. The higher score for the cluster as shown by the total math average is thus attributable to the relatively higher parental education levels for the group in comparison to other clusters.

TABLE I
RESULTS OF APPLYING THE TWO-STAGE CLUSTER ANALYSIS

Clusters:	1	2	3
Percent (Sample Size)	38.8% (2166)	35.4% (1987)	25.6% (1430)
Parameters	Educational level of father	Educational level of father	Educational level of father
	3.92	5.66	4.70
	Educational level of mother	Educational level of mother	Educational level of mother
	3.52	5.45	4.29
	Formative Assessment	Formative Assessment	Formative Assessment
	3.43	2.84	1.96
	Maths Teaching	Maths Teaching	Maths Teaching
	3.66	3.40	2.38
	Student Orientation	Student Orientation	Student Orientation
	3.31	2.39	1.84
	Teacher Directed Instructions	Teacher Directed Instructions	Teacher Directed Instructions
	3.59	3.20	2.26
	Vignette Teacher Support	Vignette Teacher Support	Vignette Teacher Support
	Wealth	Wealth	Wealth
	1.17	1.30	1.20
Evaluation Fields	Math Average Total	Math Average Total	Math Average Total
	0.21	0.31	0.25

An additional analysis was conducted comprising only parameters that are likely known at the commencement date of schooling of a student in an attempt to build a profile of students according to the demographics of the parents. Unlike the other variables considered in the two-stage cluster analysis, these parameters, that is the education level of the mother, the education level of the father and wealth, are typically fixed. Results for the analysis of the parental parameters that are known at the commencement of schooling are presented in Table II. As can be discerned from the results presented, Cluster 1, despite having the largest number of students of the two groups, had the highest math average score and corresponding higher means for the fixed parental demographics under analysis. On the other hand, Cluster 2 despite being half the size of Cluster 1, featured the least math average and the lowest mean scores for the fixed parental demographics (SES and education) parameters under consideration.

The profiles created from the clusters are also assessed in terms of mathematics performance as a post-hoc analysis to see how each cluster rates according to mathematics performance whereby mathematics performance was not used to determine the original profiles. Similar to the findings presented in Table I, a higher mathematics performance score was associated with a significantly higher mean education levels for both parents relative to students in Cluster 2, who were associated with lower mean mathematics performance scores and significantly lower mean SES scores relative to students in Cluster 1.

TABLE II
RESULTS OF APPLYING THE TWO-STAGE CLUSTER ANALYSIS

Clusters:	1	2
Percent (size)	63.2% (6121)	36.8% (3558)
Parameters	Educational level of father	Educational level of father
	5.77	2.95
	Educational level of mother	Educational level of mother
	5.43	2.59
	Wealth	Wealth
	1.23	1.21
Evaluation Fields	Math Average Total	Math Average Total
	0.28	0.19

The two-stage analytical approach for the clusters was compared to the non-hierarchical k-means procedure. Results are presented in Table III. On the standardized scale, Cluster 3 had the highest formative assessments core compared to Cluster 1 and Cluster 2. Cluster 3 is also associated with a higher: mean mathematics teaching score, teacher directed instructions, educational level of mother and father and the middle mean mathematics score, compared with Cluster 1 and Cluster 2.

This result is somewhat contradictory to the two-stage process, whereby the original clusters created were done independently of the mathematics score and was later evaluated in terms of mathematics performance. Furthermore, students grouped under Cluster 1 were associated with the lowest total mathematics mean score, the lowest SES (parental/family wealth levels) score, and substantially lower mother and father education levels relative to the other two clusters. More intriguing is the fact that despite the cluster recording the lowest mean math score, it had the highest score in formative assessment, student orientation, teacher directed learning and mathematics teaching components, thus giving the impression that the teaching aspects contributed less to the clusters than did the parent levels of education. Also worth noting is that those scoring highest on the mathematics scores fell in Cluster 2, with a total mean math score of 0.31, which was also associated with moderately high mother and father education levels and the lowest mathematics teaching, student orientation and formative assessment scores and the highest parental SES scores compared with Cluster 1 and Cluster 3. In addition, mean scores for the teaching components were between the scores for Cluster 1 and Cluster 3. Of significance however is the fact that, as established with the stage-two cluster analysis approach, use of the non-hierarchical k-means procedures demonstrated that parental education levels contributed more to the formations of the clusters than the teaching aspects did.

As well as the underlying method not being the optimal for the underlying data, the items measuring the teaching components is also questionable in this instance. Nonetheless, with a p-value of more than 0.05, as can be discerned from the associated analysis of variance (ANOVA) tables, all variables had a significant impact on determining to which cluster subjects were allocated.

TABLE III
FINAL CLUSTER RELATIVE CENTRES FROM THE K MEANS PROCEDURE

Cluster (size)	1 (n=1165)	2 (n=1778)	3 (n=2604)
Formative Assessment	0.38	-0.98	0.52
Mathematics Teaching	0.33	-0.87	0.49
Student Orientation	0.50	-0.91	0.37
Teacher Directed Instructions	0.38	-0.99	0.54
Educational level of mother	-1.27	0.24	0.45
Educational level of father	-1.37	0.26	0.48
Vignette Classroom Management	0.30	-0.43	0.12
SES	-0.05	0.03	-0.01
Mathematics Score	-0.42	0.33	0.24

IV. DISCUSSION

Cluster analysis is a useful way to create profiles of subjects to see if students are naturally clustered around a number of variables. Thus by employing the two-stage and k-means clustering analytical approaches, the study established groups or clusters in which the observations in each group or cluster had the same properties with one another and the observations of different clusters had different properties, thus indicating the distinctiveness of the clusters. The approach showed that students in each distinct cluster had similar scores vis-à-vis the teaching, parental education and SES aspects and the scores for these parameters varied from cluster to cluster. It was demonstrated that students who perform better in mathematics are more likely to have parents who are better educated and with higher measures of SES and the vice versa. This can help guide policy as parental information is available at the beginning of schooling for every student and can consequently help with the allocation of resources at early stages of schooling. This was demonstrated using a combination of cluster methods, both on the entire dataset and in separate schools to account for this potential multilevel aspect. The two-stage analysis was better suited to the current data structure and objectives. This is because of its ability to combine both hierarchical and non-hierarchical approaches, thus allowing for easy management of such large data as the one generated by the PISA study used here. Two-step clustering technique also helped address issues associated with the assumption that the distributions are normal and that the variables being analysed are independent. The approach also allowed for sequential combination of varied data to produce homogenous clusters. In terms of parsimony, reducing a dataset of several thousand students to two or three homogenous groups does summarise the data to a large degree. Further research will focus on Bayesian clustering of multilevel data.

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