

# Multiresolution Approach to Subpixel Registration by Linear Approximation of PSF

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**Abstract**—Linear approximation of point spread function (PSF) is a new method for determining subpixel translations between images. The problem with the actual algorithm is the inability of determining translations larger than 1 pixel. In this paper a multiresolution technique is proposed to deal with the problem. Its performance is evaluated by comparison with two other well known registration method. In the proposed technique the images are downsampled in order to have a wider view. Progressively decreasing the downsampling rate up to the initial resolution and using linear approximation technique at each step, the algorithm is able to determine translations of several pixels in subpixel levels.

**Keywords**—Point Spread Function, Subpixel translation, Superresolution, Multiresolution approach.

## I. INTRODUCTION

CALCULATION of spatial subpixel translations between multiple images of the same scene plays major role in many image processing algorithms and is fundamental for superresolution applications. Superresolution algorithms ([1]-[5]) make use of the different details recorded in multiple images and combining them, generate a single higher resolution/quality image. Presence of subpixel level translations and their accurate estimation are essential since overlapping regions of images with translations measured in integer numbers (of pixels) basically have the same info/detail. However, such stacks of images can still be used to create an image with reduced additive noise if present ([6], [7], [8]). [2] shows an example of superresolution using differently blurred images with no translation. In any case, just to confirm no-translation at least, images must be registered since it is very difficult/expensive, if not impossible, to obtain images with known translations.

Irani-Peleg in [1] presented a superresolution algorithm which uses a registration method based on the geometric affine transformations. Another superresolution technique based on *projections onto convex sets* (POCS) presented in [3] also uses the same registration technique. While advanced versions have been proposed ([9]), the method uses first two terms of the Taylor's series expansion of the

affine transforms.

Two other well known and closely related registration methods, normalized cross-correlation ([10]) and phase-correlation ([11]) use some type of interpolation in frequency domain, in order to determine subpixel translations. Although FFT can be used to obtain Fourier coefficients, these techniques usually recalled with their attached complexity. Other transforms, such as DCT ([12]) and Wavelets ([14]), have also been considered for the image registration. The use of mutual information ([15]) in images is another interesting approach to the subject. Zitova in [16] provided an historical (up to the publication date, 2003, of course) review of the image registration where interested readers may refer for references of image registration in general.

The noise in images is reported to be the single most important obstacle in front of the accurate subpixel registration. Since the accuracy of the registration is very crucial in the superresolution applications, most practical implementations of registration algorithms involve either one or both of pre-processing or regularization with presumed imaging parameters against the noise. The limits of superresolution applications are estimated by Baker and Kanade in [4] and recently by Lin and Shum in [5]. The limits of image registration are evaluated very recently in [17] where various cases in known registration methods have been subject to discussion, building a basic foundation for the further work.

Noise is generally accepted to be additive, white and with Gaussian pdf, as formulated in many imaging models. While various sources of noise in digital images exist, all are formulated under one AWGN signal as shown in Fig. 1. This is also the model used by Seke and Özkan in [12] which is summarized in the following since the technique proposed here is based on it.

The outline of the remaining sections of this paper is as follows. In section II the imaging model and piecewise linear approximation of *point spread function* (PSF) are presented upon which, in section III, the multiresolution method for the calculation of subpixel level translations is proposed. Section IV describes the practical work done to test the technique and compare the results with those of two other methods given in [1] and [11]. Commentary and the planned future work are in the last section.

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## II. THE IMAGING MODEL AND PIECEWISE LINEARIZATION OF PSF

The generally accepted imaging model contains sub-combination of blur, warp, rotation, skew, scaling, spatial translation, and decimation function blocks. A pure restoration or superresolution application tries to undo the effects of one or more of these function blocks under some assumptions. The assumptions usually get into the picture of the algorithm as constraints and/or regularizations. The model in [1] included shift invariant Gaussian blur, spatial translation, rotation and AWGN. The phase correlation in [11] and piecewise linearization in [12] do not handle the rotation in their original form. Therefore the model we used here, as a shortcoming in its present state, does not have rotation block either. In addition, all imaging parameters, except the translation, are expected to be time invariant and within the operational limits of the imaging device (i.e. CCD camera).

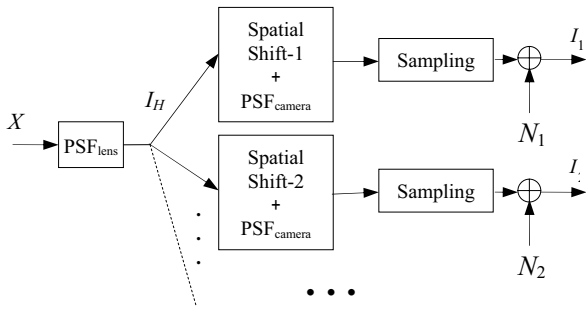


Figure 1. Imaging model used in this work.

Baker and Kanade in [4] split the Gaussian blur into two. First part represents the common blur caused by the optical imperfections and/or intentional defocusing in the lens system and is called  $PSF_{lens}$ . The second part combines the subpixel part of the spatial translation and the photon summation operation occurring in the cells of the CCD camera and is referred to as  $PSF_{camera}$ . Then, the objective of image registration algorithms is to determine the vector difference between the spatial translations of  $I_{ks}$  in Fig. 1, or equivalently the translation difference between spatial shift blocks.

Seke and Özkan in [12] modeled the spatial shift by calculating the weighted sum of hypothetical discrete light beams as illustrated in Fig. 2. White squares numbered 1-9 in the figure are the discrete beams. The larger square represents the pixel (value) generated with the weighted sum of these hypothetical pixels (values). Weighted sums given as

$$p_L = \sum_{n=1}^9 p_H(n)w(n) \quad (1)$$

for all pixels can be written in matrix form as

$$I_{Lk} = I'_H W_k \quad (2)$$

Here,  $W_k$  is the translation weight matrix whose elements,  $w(n)$ , are shown in Fig. 2 as  $w_n$  and  $I_{Lk}$  is the pixels of the  $k^{th}$  image. An image translated by  $W_1$  can again be translated by  $W_2$  to have a combined translation of  $W_{12}$ . It

$$W_{12} = W_{21} \quad (3)$$

Another way of stating (3) is

$$I_{L1} W_2 = I_{L2} W_1 \quad (4)$$

since  $I_{L1}$  and  $I_{L2}$  are the images which were already translated by  $W_1$  and  $W_2$  respectively. In [12]  $W_1$  is set to

$$W_0 = \left[ \frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{2} \quad 1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{4} \right]^T \quad (5)$$

which corresponds to zero translation, or reference, and (4) is solved for  $W_2$  by defining a set of constraints for the weights from the layout depicted in Fig. 2.

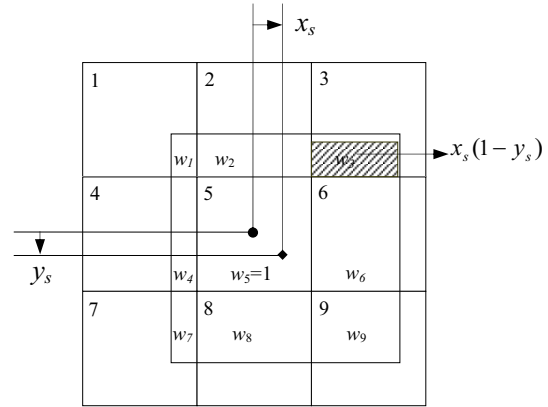


Figure 2. Pixel value is produced using hypothetical pixels.

The *least squares solution system with equality constraints* [18]

$$\begin{aligned} I_{L1} W_2 &= b \\ B W_2 &= d \end{aligned} \quad (6)$$

where  $b = I_{L2} W_0$ , and  $B$  and  $d$  are the constraint matrices derived from the layout, is strictly stable because of the constraints unless all pixels of the images have the same value. The translations in both directions, which are naturally within the  $(-\frac{1}{2}, +\frac{1}{2})$  range, are then

$$\begin{aligned} x_s &= W_2(6) - \frac{1}{2} \text{ and} \\ y_s &= W_2(8) - \frac{1}{2}. \end{aligned} \quad (7)$$

## III. MULTIREOLUTION APPROACH

Since the piecewise linearization of PSF algorithm requires the translations to be within 1 pixel range, one has to downsample images to reduce larger translations to that range. The system in (6) minimally requires 4 pixels (2 for each direction and all different), therefore the largest translation in one direction that can be calculated is theoretically limited to

$$x_{s \max} = \frac{N}{4} \quad (8)$$

where  $N$  is the size of the image in that direction. It should be the smaller of  $N$  and  $M$  for a uniform downsampling where  $N \times M$  is the number of pixels in the images. The requirement for all different pixels when the downsampled image size is as small as a couple pixels is necessitated by the linear equation system. Actually we require the rank of

$I_{Lk}$  to be at least 4, hence linearly independent 4 equations written for 4 pixels. In practice the largest translation that can be calculated is also limited by the pixel values, since downsampling with such a high rate reduces the differences between pixels, which actually are relied upon for the calculation.

In the algorithm, illustrated in Fig. 3, the downsampling rate is progressively reduced, finally to the 1:1 where the actual images are used. The calculated translations converted to actual image pixel terms are used to crop the corresponding overlapping image areas and the cropped images are downsampled with lesser rate again. The translations at each step are stored for the calculation of combined translations. The numbers  $D$  and  $r$  are selected according to the estimated translation range and image dynamics.

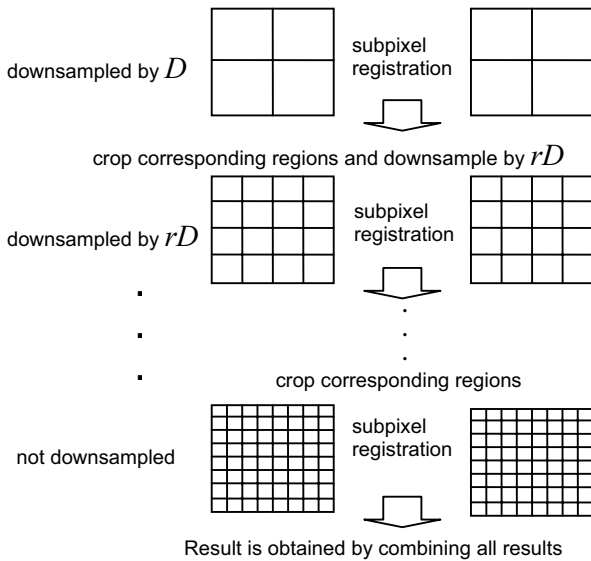


Figure 3. Flow of multiresolution approach.  $r \geq 1$ .

#### IV. TESTS

We created subpixel translated test images using the method similar to one described in [11]. To do that, we first created subpixel translated Gaussian kernel matrices. Multiplying the kernels with the high resolution gray level image matrices, we obtained downsampled and subpixel translated lower resolution images. Random numbers representing uncorrelated noise are added to images and the results are truncated to integers within 0-255. Parts of these images are cropped to include big translations. The downsampling operations in the algorithm are done by calculating the Gaussian distance weighted averaging for each pixel in the downsampled image. Some overlapping of the Gaussian blobs is allowed not to allow aliasing. Several subpixel translations and noise levels are used in the tests. The test image pairs are fed to three algorithms; affine transformations technique used in [1], phase correlation method given in [11], and the linear approximation method developed in this work. No pre/post filtering is employed for a fair comparison. The results of three techniques are then tabulated and compared. Sections of original noise free and translated and noise added Lena and Pentagon images with SNR=20 and SNR=10 respectively are given in Fig. 5

Vol.1, No.1, 2007. In some regions the noise overwhelms details.



Figure 4. a) Original, b) Shifted by (8.43, 5.65) SNR=20dB



Figure 5. a) Original, b) Shifted by (8.43, 5.65) SNR=10dB

TABLE I. Lena,  $x_s=8.43, y_s=5.65$

SNR	Linear Apx.		Phase Corr.		Affine Trans.	
45	8.4138	5.6779	8.3433	5.6723	8.5255	5.6848
40	8.4151	5.6777	8.3646	5.6539	8.5284	5.6742
35	8.4153	5.6812	8.3967	5.6335	8.5264	5.6569
30	8.4102	5.6783	8.4248	5.6125	8.5133	5.7064
25	8.4202	5.6785	8.4431	5.5962	8.5602	5.6317
20	8.4196	5.6795	8.4517	5.5703	8.6097	5.6766
15	8.3231	5.6721	8.4447	5.5554	8.3370	5.2830

TABLE II. Pentagon,  $x_s=8.43, y_s=5.65$

SNR	Linear Apx.		Phase Corr.		Affine Trans.	
40	8.4138	5.6817	8.3480	5.6725	8.4819	5.6918
35	8.4147	5.6817	8.3695	5.6583	8.4807	5.6916
30	8.4146	5.6821	8.3947	5.6413	8.4809	5.6919
25	8.4151	5.6819	8.4187	5.6173	8.4815	5.6933
20	8.4136	5.6823	8.4361	5.6013	8.4817	5.6962
15	8.4128	5.6811	8.4553	5.5902	8.4764	5.6838
10	8.4181	5.6804	8.4615	5.5747	7.2257	3.8213

Translations given in both samples are  $x_s=8.43$  and  $y_s=5.65$ . With these translations, the algorithm only needed two steps; one for the images downsampled by 1:16 and one for the final subpixel part on the original resolution. Another note about the tests is that in all tests the linear approximation method was superior to others, as indicated by the numbers in the tables. Although given for only two test sets, the tables are representative of all other tests.

## V. CONCLUSION

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The proposed algorithm performed better than two popular methods used for comparisons within the reasonably wide range of noise and translations. It is known that intensity or gradient based algorithms are affected most from the noise. However, by using constraints as inherent regularization and employing downsampling, which is believed to ease the effects of noise by averaging, we were able to achieve successful results. A thorough error analysis is still required, however, since empirical study would not entirely be counted as a proof. Error analysis and complexity formulation are currently being worked on. Three other tentative research directions which we believe to be fruitful are;

- Performing linear approximation on gradient data in which the edges gets special attention by the algorithm
- Performing linear approximation in Fourier domain where again intensity variations between images have little effect on the performance.
- Solving the problem of rotation by somehow combining the algorithm with the affine transformations.

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