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Multiple positive periodic solutions of a delayed predatory-prey system with Holling type II functional response

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Abstract—In this letter, we considers a delayed predatory-prey system with Holling type II functional response. Under some sufficient conditions, the existence of multiple positive periodic solutions is obtained by using Mawhin's continuation theorem of coincidence degree theory. An example is given to illustrate the effectiveness of our results.

Keywords—Multiple positive periodic solutions; Predatory-prey system; Coincidence degree; Holling type II functional response.

I. INTRODUCTION

IN population dynamics, the functional response of predator to prey density refers to the change in the density of prey attacked per unit time per predator as the prey density changes [1]. In [2], based on experiment, Holling suggested three kinds of functional response for different species to model the phenomena of predation, it seems more reasonable than the standard Lotka-Voltera type predator-prey system. Many scholars have deeply studied the dynamical behaviors of these system with constant coefficients (see [3-7]). However, realistic models require the inclusion of the effect of changing environment. This motivates us to consider the following nonautonomous model

$$\begin{cases} \dot{x} = a(t)x(t)(1-\frac{x(t)}{K}) - \frac{\alpha(t)x(t)y(t)}{1+d(t)x(t)}, \\ \dot{y} = y(t)(b(t) - c(t)y(t) + \frac{\beta(t)x(t-\tau(t))}{1+d(t)x(t-\tau(t))}), \end{cases} \tag{1}$$

where, $x(t)$ and $y(t)$ denote the densities of the prey and the predator, respectively. $a(t)$ and $b(t)$ denote the intrinsic growth rate of the prey and the predator. $c(t)$ stands for the intraspecific competition of the predator. $d(t)$ is interpreted as a handling time for each prey captured. $\alpha(t)$ and $\beta(t)$ are the conversion factor denoting the number of newly born predators for each captured prey. K represents the carrying capacity of the prey, which is a positive constant. $\tau(t)$ is a time lag. The term $\frac{\alpha(t)x(t)y(t)}{1+b(t)x(t)}$ denotes the functional response of the predator, which is termed as Holling type II response function (see Holling CS [2]). In addition, the effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Therefore, the assumptions of periodicity of the parameters are a way of incorporating the periodicity of the environment (e.g, seasonal effects of weather, food supplies, mating habits, etc), which leads us to assume that $a(t), b(t), c(t), d(t), \alpha(t), \beta(t)$ and $\tau(t)$ are all positive continuous ω-periodic functions.

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Since a very basic and important problem in the study of a population growth model with a periodic environment is the global existence and stability of a positive periodic solution, which plays a similar role as a globally stable equilibrium does in an autonomous model, also, on the existence of multiple positive periodic solutions to system (1), few results are found in literatures. This motivates us to investigate the existence of multiple positive periodic solutions for system (1). In fact, it is more likely for some biological species to take on multiple periodic change regulations and have multiple local stable periodic phenomena. Therefore it is essential for us to investigate the existence of multiple positive periodic solutions for population models. Our main purpose of this paper is by using Mawhin's continuation theorem of coincidence degree theory [8], to establish the existence of two positive periodic solutions for system (1). For the work concerning the multiple existence of periodic solutions of periodic population models which was done using coincidence degree theory, we refer to [9-14].

The organization of the rest of this paper is as follows. In section 2, by employing the continuation theorem of coincidence degree theory, we establish the existence of two positive periodic solutions of system (1). In section 3, an example is given to illustrate

II. EXISTENCE OF MULTIPLE POSITIVE PERIODIC SOLUTIONS

In this section, by using Mawhin's continuation theorem, we shall show the existence of positive periodic solutions of (1). To do so, we need to make some preparations.

Let X and Z be real normed vector spaces. Let $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping and $N : X \times [0,1] \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \ker L$ = $\operatorname{codim} \operatorname{Im} L < \infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \ker L$ and $\ker Q = \operatorname{Im} L = \operatorname{Im}(I-Q)$, and $X = \ker L \bigoplus \ker P$, $Z = \operatorname{Im} L \bigoplus \operatorname{Im} Q$. It follows that $L|_{\operatorname{Dom} L \cap \ker P} : (I-P)X \to \operatorname{Im} L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\bar{\Omega}\times[0,1]$, if $QN(\bar{\Omega}\times[0,1])$ is bounded and $K_P(I-Q)N:\bar{\Omega}\times[0,1]\to X$ is compact. Because $\operatorname{Im}Q$ is isomorphic to $\ker L$, there exists an isomorphism $J:\operatorname{Im}Q\to\ker L$.

The Mawhin's continuous theorem [8, p.40] is given as follows:

Lemma 1. [8] *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}\times[0,1]$. Assume*

(a) *for each $\lambda\in(0,1)$, every solution x of $Lx=\lambda N(x,\lambda)$ is such that $x\notin\partial\Omega\cap\operatorname{Dom}L$;*
(b) *$QN(x,0)x\neq 0$ for each $x\in\partial\Omega\cap\operatorname{Ker}L$;*
(c) *$\deg(JQN(x,0),\Omega\cap\operatorname{Ker}L,0)\neq 0$.*

Then $Lx=N(x,1)$ has at least one solution in $\overline{\Omega}\cap\operatorname{Dom}L$.

For the sake of convenience, we denote by $f^l=\min_{t\in[0,\omega]}f(t),\ f^M=\max_{t\in[0,\omega]}f(t),\ \bar{f}=\frac{1}{\omega}\int_0^\omega f(t)\,\mathrm{d}t$, respectively, here $f(t)$ is a continuous ω-periodic function. For simplicity, we also introduce some positive numbers as follows.

$$A^-=\frac{b^l}{c^M},\qquad A^+=\frac{b^M}{c^l}+\frac{\beta^M K}{c^l(1+d^lK)},$$

$$\begin{aligned}l^\pm &= \frac{1}{2a^ld^l}\Big\{a^l(Kd^l-1)\\&\quad\pm\sqrt{[a^l(Kd^l-1)]^2-4a^ld^lK(\alpha^MA^+-a^l)}\Big\},\end{aligned}$$

$$\begin{aligned}L^\pm &= \frac{1}{2a^Md^M}\Big\{a^M(Kd^M-1)\\&\quad\pm\sqrt{[a^M(Kd^M-1)]^2-4a^Md^MK(\alpha^lA^--a^M)}\Big\}\end{aligned}$$

Throughout this paper, we need the following assumptions.

(H_1) $\alpha^lA^->a^M$;
(H_2) $a^l(Kd^l-1)>2\sqrt{a^ld^lK(\alpha^MA^+-a^l)}$.

Lemma 2. *Let $x>0,\ y>0$ and $x>2\sqrt{y}$, for the functions $f(x,y)=\dfrac{x+\sqrt{x^2-4y}}{2}$ and $g(x,y)=\dfrac{x-\sqrt{x^2-4y}}{2}$, the following assertions hold.*

(1) *$f(x,y)$ and $g(x,y)$ are monotonically increasing and monotonically decreasing on the variable $x\in(0,\infty)$, respectively.*
(2) *$f(x,y)$ and $g(x,y)$ are monotonically decreasing and monotonically increasing on the variable $y\in(0,\infty)$, respectively.*

Proof: In fact, for all $x>0,y>0$ we have

$$\frac{\partial f}{\partial x}=\frac{x+\sqrt{x^2-4y}}{2\sqrt{x^2-4y}}>0,\quad \frac{\partial f}{\partial y}=\frac{-1}{\sqrt{x^2-4y}}<0,$$

$$\frac{\partial g}{\partial x}=\frac{\sqrt{x^2-4y}-x}{2\sqrt{x^2-4y}}<0,\quad \frac{\partial g}{\partial y}=\frac{1}{\sqrt{x^2-4y}}>0.$$

By the relationship of the derivative and the monotonicity, the above assertions obviously hold. The proof of Lemma 2 is complete. ■

Lemma 3. *Assume that (H_1) and (H_2) hold, then we have the following inequalities:*

$$0<L^-<l^-<l^+<L^+.$$

Proof: In fact, according to Lemma 2, we have

$$\frac{a^M(Kd^M-1)}{a^Md^M}=K-\frac{1}{d^M}>K-\frac{1}{d^l}=\frac{a^l(Kd^l-1)}{a^ld^l},$$
$$\frac{(\alpha^MA^+-a^l)K}{a^ld^l}>\frac{(\alpha^lA^--a^M)K}{a^Md^M},$$

which imply that

$$\begin{aligned}0 &< L^-=g\Big(\frac{a^M(Kd^M-1)}{a^Md^M},\ \frac{(\alpha^lA^--a^M)K}{a^Md^M}\Big)\\&< g\Big(\frac{a^l(Kd^l-1)}{a^ld^l},\ \frac{(\alpha^MA^+-a^l)K}{a^ld^l}\Big)=l^-\\&< l^+=f\Big(\frac{a^l(Kd^l-1)}{a^ld^l},\ \frac{(\alpha^MA^+-a^l)K}{a^ld^l}\Big)\\&< f\Big(\frac{a^M(Kd^M-1)}{a^Md^M},\ \frac{(\alpha^lA^--a^M)K}{a^Md^M}\Big)=L^+.\end{aligned}$$

This completes the proof. ■

Theorem 1. *Assume that (H_1) and (H_2) hold. Then system (1) has at least two positive ω-periodic solutions.*

Proof: By making the substitution

$$x(t)=\exp\{u_1(t)\},\quad y(t)=\exp\{u_2(t)\},\tag{2}$$

system (1) can be reformulated as

$$\begin{cases}\dot{u}_1(t)=a(t)(1-\frac{e^{u_1(t)}}{K})-\frac{\alpha(t)e^{u_2(t)}}{1+d(t)e^{u_1(t)}},\\ \dot{u}_2(t)=b(t)-c(t)e^{u_2(t)}+\frac{\beta(t)e^{u_1(t-\tau(t))}}{1+d(t)e^{u_1(t-\tau(t))}}.\end{cases}\tag{3}$$

Let

$$X=Z=\Big\{u=(u_1,u_2)^T\in C(R,R^2):u(t+\omega)=u(t)\Big\}$$

and define

$$\|u\|=\sum_{i=1}^2\max_{t\in[0,\omega]}|u_i(t)|,\quad u\in X\text{ or }Z.$$

Equipped with the above norm $\|\cdot\|$, X and Z are Banach spaces. Let

$$N(u,\lambda)=\begin{pmatrix}a(t)(1-\frac{e^{u_1(t)}}{K})-\frac{\alpha(t)e^{u_2(t)}}{1+d(t)e^{u_1(t)}}\\ b(t)-c(t)e^{u_2(t)}+\lambda\frac{\beta(t)e^{u_1(t-\tau(t))}}{1+d(t)e^{u_1(t-\tau(t))}}\end{pmatrix},\quad u\in X,$$

$Lu=\dot{u}=\frac{du(t)}{dt}$. We put $Pu=\frac{1}{\omega}\int_0^\omega u(t)dt,\ u\in X;\ Qz=\frac{1}{\omega}\int_0^\omega z(t)dt,\ z\in Z$. Thus it follows that $\ker\ L=R^2$, $\operatorname{Im}L=\{z\in Z:\int_0^\omega z(t)dt=0\}$ is closed in Z, $\dim\ker L=2=\operatorname{codim}\operatorname{Im}L$, and P,Q are continuous projectors such that

$$\operatorname{Im}P=\ker\ L,\quad \ker\ Q=\operatorname{Im}L=\operatorname{Im}(I-Q).$$

Hence, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_P:\operatorname{Im}L\to\ker\ P\bigcap\operatorname{Dom}L$ is given by

$$K_P(z)=\int_0^t z(s)\mathrm{d}s-\frac{1}{\omega}\int_0^\omega\int_0^s z(s)\mathrm{d}s.$$

Then

$$QN(u,\lambda)=\begin{pmatrix}\frac{1}{\omega}\int_0^\omega F_1(s,\lambda)\mathrm{d}s\\ \frac{1}{\omega}\int_0^\omega F_2(s,\lambda)\mathrm{d}s\end{pmatrix}$$

and

$$\begin{aligned}&K_P(I-Q)N(u,\lambda)\\ =&\begin{pmatrix}\int_0^t F_1(s,\lambda)\mathrm{d}s-\frac{1}{\omega}\int_0^\omega\int_0^t F_1(s,\lambda)\mathrm{d}s\mathrm{d}t\\ +(\frac{1}{2}-\frac{t}{\omega})\int_0^\omega F_1(s,\lambda)\mathrm{d}s\\ \int_0^t F_2(s,\lambda)\mathrm{d}s-\frac{1}{\omega}\int_0^\omega\int_0^t F_2(s,\lambda)\mathrm{d}s\mathrm{d}t\\ +(\frac{1}{2}-\frac{t}{\omega})\int_0^\omega F_2(s,\lambda)\mathrm{d}s\end{pmatrix},\end{aligned}$$

where

$$F_1(s,\lambda)=a(s)(1-\frac{e^{u_1(s)}}{K})-\frac{\alpha(s)e^{u_2(s)}}{1+d(s)e^{u_1(s)}},$$

$$F_2(s,\lambda)=b(s)-c(s)e^{u_2(s)}+\lambda\frac{\beta(s)e^{u_1(s-\tau(s))}}{1+d(s)e^{u_1(s-\tau(s))}}.$$

Obviously, QN and $K_P(I-Q)N$ are continuous. Similar to the proof of Theorem 2.1 in [15], it is not difficult to show that $K_P(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega\subset X$ by using the Arzela-Ascoli theorem. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega\subset X$.

In order to use Lemma 1, We have to find at least two appropriate open bounded subsets in X. Considering the operator equation $Lu=\lambda N(u,\lambda),\lambda\in(0,1)$, we have

$$\begin{cases}\dot{u}_1(t)=\lambda\big(a(t)(1-\frac{e^{u_1(t)}}{K})-\frac{\alpha(t)e^{u_2(t)}}{1+d(t)e^{u_1(t)}}\big),\\ \dot{u}_2(t)=\lambda\big(b(t)-c(t)e^{u_2(t)}+\lambda\frac{\beta(t)e^{u_1(t-\tau(t))}}{1+d(t)e^{u_1(t-\tau(t))}}\big).\end{cases}\tag{4}$$

Assume that $u\in X$ is an ω-periodic solution of system (4) for some $\lambda\in(0,1)$. Then there exist $\xi_i,\eta_i\in[0,\omega]$ such that

$$u_i(\xi_i)=\max_{t\in[0,\omega]}u_i(t),\quad u_i(\eta_i)=\min_{t\in[0,\omega]}u_i(t),i=1,2.$$

It is clear that $\dot{u}_i(\xi_i)=0,\dot{u}_i(\eta_i)=0,i=1,2$. From this and (4), we have

$$\begin{cases}a(\xi_1)(1-\frac{e^{u_1(\xi_1)}}{K})-\frac{\alpha(\xi_1)e^{u_2(\xi_1)}}{1+d(\xi_1)e^{u_1(\xi_1)}}=0,\\ b(\xi_2)-c(\xi_2)e^{u_2(\xi_2)}+\lambda\frac{\beta(\xi_2)e^{u_1(\xi_2-\tau(\xi_2))}}{1+d(\xi_2)e^{u_1(\xi_2-\tau(\xi_2))}}=0,\end{cases}\tag{5}$$

and

$$\begin{cases}a(\eta_1)(1-\frac{e^{u_1(\eta_1)}}{K})-\frac{\alpha(\eta_1)e^{u_2(\eta_1)}}{1+d(\eta_1)e^{u_1(\eta_1)}}=0,\\ b(\eta_2)-c(\eta_2)e^{u_2(\eta_2)}+\lambda\frac{\beta(\eta_2)e^{u_1(\eta_2-\tau(\eta_2))}}{1+d(\eta_2)e^{u_1(\eta_2-\tau(\eta_2))}}=0.\end{cases}\tag{6}$$

Noting that $0<e^{u_1(t)}\le K$, according to the second equation of (5), we have

$$\begin{aligned}c^le^{u_2(\xi_2)}&\le c(\xi_2)e^{u_2(\xi_2)}=b(\xi_2)\\ &+\lambda\frac{\beta(\xi_2)e^{u_1(\xi_2-\tau(\xi_2))}}{1+d(\xi_2)e^{u_1(\xi_2-\tau(\xi_2))}}<b^M+\frac{\beta^MK}{1+d^lK}\end{aligned}$$

and

$$\begin{aligned}c^Me^{u_2(\xi_2)}&\ge c(\xi_2)e^{u_2(\xi_2)}=b(\xi_2)\\ &+\lambda\frac{\beta(\xi_2)e^{u_1(\xi_2-\tau(\xi_2))}}{1+d(\xi_2)e^{u_1(\xi_2-\tau(\xi_2))}}>b^l,\end{aligned}$$

which imply that

$$\begin{aligned}\ln A^-&=\ln\frac{b^l}{c^M}<u_2(\xi_2)\\ &<\ln(\frac{b^M}{c^l}+\frac{\beta^MK}{c^l(1+d^lK)})=\ln A^+.\end{aligned}\tag{7}$$

Similarly, by the second equation of (6), we obtain

$$\begin{aligned}\ln A^-&=\ln\frac{b^l}{c^M}<u_2(\eta_2)\\ &<\ln(\frac{b^M}{c^l}+\frac{\beta^MK}{c^l(1+d^lK)})=\ln A^+.\end{aligned}\tag{8}$$

The first equation of (5) give

$$\begin{aligned}\alpha^MA^+&>\alpha^Me^{u_2(\xi_2)}\ge\alpha(\xi_1)e^{u_2(\xi_1)}\\ &=a(\xi_1)(1-\frac{e^{u_1(\xi_1)}}{K})(1+d(\xi_1)e^{u_1(\xi_1)})\\ &\ge a^l(1-\frac{e^{u_1(\xi_1)}}{K})(1+d^le^{u_1(\xi_1)})\end{aligned}$$

and

$$\begin{aligned}\alpha^lA^-&<\alpha^le^{u_2(\eta_2)}\le\alpha(\xi_1)e^{u_2(\xi_1)}\\ &=a(\xi_1)(1-\frac{e^{u_1(\xi_1)}}{K})(1+d(\xi_1)e^{u_1(\xi_1)})\\ &\le a^M(1-\frac{e^{u_1(\xi_1)}}{K})(1+d^Me^{u_1(\xi_1)}),\end{aligned}$$

that is

$$a^ld^le^{2u_1(\xi_1)}-a^l(Kd^l-1)e^{u_1(\xi_1)}+(\alpha^MA^+-a^l)K>0,$$

and

$$\begin{aligned}0&>a^Md^Me^{2u_1(\xi_1)}-a^M(Kd^M-1)e^{u_1(\xi_1)}\\ &+(\alpha^lA^--a^M)K,\end{aligned}$$

which imply that

$$u_1(\xi_1)>\ln l^+\quad\text{or}\quad u_1(\xi_1)<\ln l^-\tag{9}$$

and

$$\ln L^-<u_1(\xi_1)<\ln L^+.\tag{10}$$

Similarly, by the first equation of (6), we get

$$u_1(\eta_1)>\ln l^+\quad\text{or}\quad u_1(\eta_1)<\ln l^-\tag{11}$$

and

$$\ln L^-<u_1(\eta_1)<\ln L^+.\tag{12}$$

From (7)-(12) and Lemma 3, we obtain for all $t\in$ R,

$$\ln L^-<u_1(t)<\ln l^-\quad\text{or}\quad\ln l^+<u_1(t)<\ln L^+\tag{13}$$

and

$$\ln A^-<u_2(t)<\ln A^+.\tag{14}$$

Clearly, $\ln l^\pm$, $\ln L^\pm$ and $\ln A^\pm$ are independent of λ. Now let

$$\begin{aligned}\Omega_1=&\Big\{u=(u_1,u_2)^T\in X:\ln L^-<u_1(t)<\ln l^-,\\ &\ln A^-<u_2(t)<\ln A^+\Big\},\end{aligned}$$

and

$$\Omega_2 = \Big\{u = (u_1, u_2)^T \in X : \ln l^+ < u_1(t) < \ln L^+, \ln A^- < u_2(t) < \ln A^+\Big\}.$$

Then Ω_1 and Ω_2 are bounded open subsets of X, $\Omega_1 \cap \Omega_2 = \phi$. Thus $\Omega_i (i = 1, 2)$ satisfies the requirement (a) in Lemma 1.

Now we show that (b) of Lemma 1 holds, i.e., we prove when $u \in \partial\Omega_i \cap \ker L = \partial\Omega_i \cap R^2, QN(u, 0) \neq (0, 0)^T, i = 1, 2$. If it is not true, then when $u \in \partial\Omega_i \cap \ker L = \partial\Omega_i \cap R^2, i = 1, 2$, constant vector $u = (u_1, u_2)^T$ with $u \in \partial\Omega_i, i = 1, 2$ satisfies

$$\begin{cases} \int_0^\omega a(t)(1 - \frac{e^{u_1}}{K})\mathrm{d}t - \int_0^\omega \frac{\alpha(t)e^{u_2}}{1+d(t)e^{u_1}}\mathrm{d}t = 0, \\ \int_0^\omega b(t)\mathrm{d}t - \int_0^\omega c(t)e^{u_2}\mathrm{d}t = 0. \end{cases}$$

In terms of differential mean value theorem, there exist two points $t_i (i = 1, 2)$ such that

$$a(t_1)(1 - \frac{e^{u_1}}{K}) - \frac{\alpha(t_1)e^{u_2}}{1 + d(t_1)e^{u_1}} = 0, \tag{15}$$

$$b(t_2) - c(t_2)e^{u_2} = 0. \tag{16}$$

Following the arguments of (7)-(12), we have

$$\ln L^- < u_1 < \ln l^- \quad \text{or} \quad \ln l^+ < u_1 < \ln L^+ \tag{17}$$

and

$$\ln A^- < u_2 < \ln A^+. \tag{18}$$

Then $u \in \Omega_1 \cap R^2$ or $u \in \Omega_2 \cap R^2$. This contradicts the fact that $u \in \partial\Omega_i \cap R^2, i = 1, 2$. This proves (b) in Lemma 1 holds.

Finally, we show that (c) in Lemma 1 holds. Note that the system of algebraic equations:

$$\begin{cases} a(t_1)(1 - \frac{e^x}{K}) - \frac{\alpha(t_1)e^y}{1+d(t_1)e^x} = 0, \\ b(t_2) - c(t_2)e^y = 0 \end{cases}$$

has two distinct solutions since (H_1) and (H_2) hold,

$$(x_1^*, y_1^*) = (\ln x_-, \ln \bar{y}), \quad (x_2^*, y_2^*) = (\ln x_+, \ln \bar{y}),$$

where

$$\bar{y} = \frac{b(t_2)}{c(t_2)},$$

$$x_\pm = \frac{1}{2a(t_1)d(t_1)}\Big\{a(t_1)(Kd(t_1) - 1) \pm \Big([a(t_1)(Kd(t_1) - 1)]^2 - 4a(t_1)d(t_1)K(\alpha(t_1)\bar{y} - a(t_1))\Big)^{\frac{1}{2}}\Big\}.$$

It is easy to verify that

$$\ln L_1^- < \ln x_- < \ln l^- < \ln l^+ < \ln x_+ < \ln L^+$$

and

$$\ln A^- < \ln \bar{y} < \ln A^+.$$

Therefore, $(x_1^*, y_1^*) \in \Omega_1$, $(x_2^*, y_2^*) \in \Omega_2$. Since $\ker L = \operatorname{Im} Q$, we can take $J = I$. In the light of the definition of the Leray-Schauder degree, a direct computation gives, for $i = 1, 2$,

$$\begin{aligned} &\deg\Big\{JQN(u, 0), \Omega_i \cap \ker L, (0, 0)^T\Big\} \\ = \ & sign\begin{vmatrix} -\frac{a(t_1)}{K} + \frac{\alpha(t_1)d(t_1)y^*}{(1+d(t_1)x^*)^2} & \frac{\alpha(t_1)}{1+d(t_1)x^*} \\ 0 & -c(t_2) \end{vmatrix} \\ = \ & -sign\Big[-\frac{a(t_1)}{K} + \frac{\alpha(t_1)d(t_1)y^*}{(1 + d(t_1)x^*)^2}\Big]. \end{aligned}$$

Since

$$a(t_1)(1 - \frac{x^*}{K}) - \frac{\alpha(t_1)y^*}{1 + d(t_1)x^*} = 0,$$

then

$$\begin{aligned} &\deg\Big\{JQN(u, 0), \Omega_i \cap \ker L, (0, 0)^T\Big\} \\ = \ & -sign\Big[d(t_1)K - 2d(t_1)x^* - 1\Big], \ \ i = 1, 2. \end{aligned}$$

Thus

$$\deg\Big\{JQN(u, 0), \Omega_1 \cap \ker L, (0, 0)^T\Big\} = -1$$

$$\deg\Big\{JQN(u, 0), \Omega_2 \cap \ker L, (0, 0)^T\Big\} = 1$$

So far, we have prove that $\Omega_i (i = 1, 2)$ satisfies all the assumptions in Lemma 1. Hence, system (3) has at least two different ω-periodic solutions. Thus by (2) system (1) has at least two different positive ω-periodic solutions. This completes the proof of Theorem 1. ■

III. An Example

Consider the following prey-predator system:

$$\begin{cases} \dot{x}(t) = a(t)x(t)(1 - \frac{x(t)}{K}) - \frac{\alpha(t)x(t)y(t)}{1+d(t)x(t)}, \\ \dot{y}(t) = y(t)(b(t) - c(t)y(t) + \frac{\beta(t)x(t-\tau(t))}{1+d(t)x(t-\tau(t))}), \end{cases} \tag{19}$$

where, $a(t) = 2 + \sin t, b(t) = 2 + \cos t, c(t) = \frac{7+\cos t}{30}$, $d(t) = 2 + \sin 2t$, $\alpha(t) = 2 + \sin 3t$, $\beta(t) = 2 + \cos 3t$ and $K = 200$. By the simple calculation, we have

$$a^l A^- = \frac{15}{4} > 3 = a^M,$$

$$\begin{aligned} &a^l(Kd^l - 1) = 199 > 2\sqrt{200 \times 90} \\ > \ & 2\sqrt{200[15(1 + \frac{200}{201}) - 1]} = 2\sqrt{a^l d^l K(\alpha^M A^+ - a^l)}. \end{aligned}$$

Hence, all conditions of Theorem 1 are satisfied. By Theorem 1, system (19) has at least two positive 2π-periodic solutions.

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