

# A Multi Task Scheme to Monitor Multivariate Environments Using Artificial Neural Network

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**Abstract**—When an assignable cause(s) manifests itself to a multivariate process and the process shifts to an out-of-control condition, a root-cause analysis should be initiated by quality engineers to identify and eliminate the assignable cause(s) affected the process. A root-cause analysis in a multivariate process is more complex compared to a univariate process. In the case of a process involved several correlated variables an effective root-cause analysis can be only experienced when it is possible to identify the required knowledge including the out-of-control condition, the change point, and the variable(s) responsible to the out-of-control condition, all simultaneously. Although literature addresses different schemes to monitor multivariate processes, one can find few scientific reports focused on all the required knowledge. To the best of the author's knowledge this is the first time that a multi task model based on artificial neural network (ANN) is reported to monitor all the required knowledge at the same time for a multivariate process with more than two correlated quality characteristics. The performance of the proposed scheme is evaluated numerically when different step shifts affect the mean vector. Average run length is used to investigate the performance of the proposed multi task model. The simulated results indicate the multi task scheme performs all the required knowledge effectively.

**Keywords**—Artificial neural network, Multivariate process, Statistical process control, Change point.

## I. INTRODUCTION

LITERATURE indicates statistical process control (SPC) approach could play an essential role to control the variability of processes. Among the SPC methods, control charts are known as an effective method to monitor a process behavior when SPC is approached (For more details the reader is directed to Montgomery [1]). Control charts first proposed by Shewhart [2] when he launched a new approach to monitor variability of a process. The importance of the process involved several correlated variables led researchers to develop the Shewhart control charts. Hotelling [3] considered multivariate processes and proposed T2 procedure. The major deficiency of T2 Hotelling method is relatively insensitive when a small or a moderate change(s) affects the process. To overcome the deficiency several authors contributed to develop multivariate cumulative sum (MCUSUM) and multivariate exponential weighted moving average (MEWMA) schemes. Several authors including Woodall and Ncube [4], Healy [5], Crosier [6], Pignatiello and Runger [7], Ngai and Zhang [8], Chan and Zhang [9], Qiu and Hawkins [10], [11], and Runger and Testik [12] focused on MCUSUM.

Many researcher such Lowry et al. [13], Rigdon [14], Yumin [15], Runger and Prabhu [16], Kramer and Schmid [17], Prabhu and Runger [18], Fasso [19], Borrer et al. [20], Runger et al. [21], Tseng et al. [22], Yeh et al. [23], Testik et al. [24], Testik and Borrer [25] and Chen et al. [26] contributed to MEWMA performance. The major capability of all the control charts introduced in literature is referred to as detecting the out-of-control condition when an assignable cause takes a place in the process. However when a process involved multivariable shifts to an out-of-control condition a quality engineer to an effective root-cause analysis needs to know the change point and the variable(s) contributed to the out-of-control condition. Change point is the time when the process really shifts to an out-of-control condition (For more details the reader is directed to Atashgar [27]). A control chart relative to its sensitivity signals with a delay after the process really shifts to an out-of-control condition. The delay is referred to as the out-of-control average run length (ARL). Literature involves several different schemes proposed to identify the required knowledge separately. Mason et al. [28], Apaarisi et al. [29] and Niaki and Abbasi [30] focused on to diagnose the variable responsible to the out-of-control condition, however, the authors including Nedumaran et al. [31] and Noorossana et al. [32] contributed to identify the change point in the mean vector of a multivariate process. Noorossana et al. [33] proposed an artificial neural network to identify all the important knowledge leading to an effective root-cause analysis. Although the scientific report addresses an effective performance, the proposed model does not allow one to use it in a process involved more than two variables. In this paper a multi task scheme based on a supervised ANN is proposed to provide all the required knowledge for multivariate environments. The multi task model is capable to identify the change point and diagnose the quality characteristic(s) responsible to the out-of-control condition at the same time that the model signals an out-of-control condition. The report addresses an effective performance for the model when the mean vector of a process involved three quality characteristics affecting different step shift magnitudes departs to an out-of-control condition.

In the next section the proposed model is introduced. The procedure used to train the ANN model and the results of the performance evaluation of the proposed multi task scheme are discussed in Section III. Finally, author's concluding remarks are provided in Section IV.

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## II. THE PROPOSED MULTI TASK SCHEME

Assume  $X_1, X_2, \dots, X_\tau, X_{\tau+1}, \dots, X_T$  are independent vectors of a multivariate process observations which follow an identical normal distribution with mean vector  $\mu_0 = (\mu_{01}, \mu_{02}, \dots, \mu_{0p})$  and covariance matrix  $\Sigma$ . Assuming that after an unknown time  $\tau$  a disturbance of a step change type affects the mean vector, the process shifts to an out-of-control condition at time  $\tau$  but the shift is detected at time  $T$ . The out-of-control condition is detected when  $\chi^2$  statistic is computed as the following equation 1 and compared to a pre-specified control limit:

$$\chi^2 = n(\bar{X} - \mu_0)' \Sigma^{-1} (\bar{X} - \mu_0) \quad (1)$$

Furthermore, assume  $\mathbf{X} = (X_1, X_2, \dots, X_p)$  indicates a  $p \times 1$  random vector of the quality characteristics. In this case  $\tau$  is considered as the change point or the time when the disturbance first really has affected the multivariate normal process. However the control chart with a delay signals at time  $T$ . In this case also the knowledge of which of the quality characteristics has contributed to the out-of-control condition is known as a valuable factor for the quality engineers at the time when they want to start to identify the assignable cause.

The proposed multi task scheme follows modularity approach. The ANN model after training will be able to detect an out-of-control condition, identify the change point  $\tau$  and the variable(s) contributed to the out-of-control condition at the same time. In this research, supervised learning is approached to allow the ANN storing the knowledge to modify weights and biases. Multi layer perceptron (MLP) is used for the proposed model. Literature indicates MLP could provide an effective performance in which the pattern recognition is approached by researchers. The specification of ANN as shown in Table I contains two network modules with different layers. However after training Network A will be able to detect the shift in the mean vector along with diagnosing the variable(s) responsible to the shift and Network B will be able to identify the time when really the shift occurs in the process, i.e. the change point. In this research 24389 different combinations including one combination of in-control condition and 24388 combinations of out-of-control condition are used to train the proposed multi task model. The input layer of both networks contains 36 neurons, however Network A and Network B involve 7 and 1 neurons for the output layers, respectively. Table II shows the cases corresponding to the different conditions might be signaled by Network 1, where S indicates to the shift. For example when Network 1 signals 1 by the first neuron, it indicates that the process has shifted to the out-of-control and the first variable has contributed to the condition. Furthermore when number 1 appears in neuron 6 it indicates that the process works in an out-of-control condition and the quality specifications 2 and 3 are contributed to the unnatural condition.

## III. NETWORKS TRAINING AND PERFORMANCE EVALUATION

In this research, to perform the required training and data of performance evaluation Monte Carlo simulation is used for each ANN. The equation used here to generate the data sets is as follow:

$$\mathbf{X}_t = \boldsymbol{\mu} + \mathbf{n}_t + \mathbf{k}\boldsymbol{\sigma} \quad (2)$$

here  $t$  indicates the sampling time and  $\mathbf{X}_t$  represents an independent random vector corresponding to the quality characteristics measured at time  $t$ . When the process is in control,  $\mathbf{X}_t$  follows a normal distribution with mean vector  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}$ . In (2),  $\mathbf{n}_t$  indicates the variation corresponding to common cause at time  $t$  which follows  $N(\mathbf{0}, \boldsymbol{\Sigma})$ . In the equation vector  $\mathbf{k}$  represents the shift magnitude.

In this research four phases including standardization, zoning, permutation and scaling discussed by Atashgar and Noorossana [34] are used to improve the performance of each network prior to introducing data sets to the networks. Equation (2) is used to simulate the training data set to provide supervised learning approached in this research. Furthermore, to train the model the subinterval approach introduced first by Atashgar and Noorossana [34] is used here. Table III shows the breakdown of the intervals and the number of training iterations for each subinterval. For more details the reader is directed to Atashgar and Noorossana [34].

To evaluate the performance of the model using different shifts magnitude the moving window approach is considered here.

TABLE I  
SPECIFICATIONS OF THE NETWORKS

Network	No. of Hidden Layer	No. of Hidden Layer Neurons	No. of Output Layer Neurons	Training Algorithm
A	2	17	7	Trainbfg
B	2	14	1	Trainbfg

TABLE II  
THE CONCEPT OF THE SIGNALS IN OUTPUT LAYER

Quality specification			output						
$x_1$	$x_2$	$x_3$	1	2	3	4	5	6	7
S	-	-	1	0	0	0	0	0	0
-	S	-	0	1	0	0	0	0	0
-	-	S	0	0	1	0	0	0	0
S	S	-	0	0	0	1	0	0	0
S	-	S	0	0	0	0	1	0	0
-	S	S	0	0	0	0	0	1	0
S	S	S	0	0	0	0	0	0	1

TABLE III  
SUBINTERVALS OF THE NETWORKS

No.	Subinterval	No. of combinations	No. of iterations	Total
1		21952	2	43904
2		2352	45	105840
Network 1				
3		84	190	15960
4	In-control	1	50000	50000
Total		24389		215704
1		21952	3	65856
Network 2				
2		2352	47	110544
3		84	190	15960
4	In-control	1	30000	30000
Total		24389		222360

Moving window is discussed by Guh [35] and Hwang [36]. In this evaluation is assumed that the first 100 data set of observation are generated from an in-control condition. Beginning with time 101, a disturbance of step type occurs in the process and affects the mean vector. Average run length and correct classification criterions using 10000 iterations for each combination shown in Table IV which lead to an out-of-control condition is considered to evaluate the performance of the model. Table IV shows the results in term discussed before. Correct classification percentage is calculated using the following equation:

$$\text{Correct Classification \%} = \left(1 - \frac{ec}{n}\right) \times 100 \quad (3)$$

where,  $ec$  and  $n$  variables indicate to the number of error classifications and the number of inputs, respectively.

TABLE IV  
PERFORMANCE REPORT OF THE PROPOSED MODEL UNDER DIFFERENT SHIFTS

Shift Combination	(-3,-3,-3)	(-2,-3,-3)	(-1,-3,-3)	(0,-3,-3)	(1,-3,-3)	(2,-3,-3)	(3,-3,-3)	(-3,-3,-2)	(-2,-3,-2)
Out-of-Control ARL	2.7795	3.2421	3.8447	4.4187	4.2829	3.2214	2.7907	3.2085	3.7561
Correct Classification %	94.70	89.65	62.64	85.05	58.67	86.58	92.48	90.24	92.24
Change Point	99.9891	100.0554	100.1638	100.2211	100.2059	100.1117	100.0407	100.1920	100.3396
Standard Error	0.0043	0.0053	0.0067	0.0072	0.0070	0.0061	0.0052	0.0068	0.0089
Shift Combination	(-1,-3,-2)	(0,-3,-2)	(1,-3,-2)	(2,-3,-2)	(3,-3,-2)	(-3,-3,-1)	(-2,-3,-1)	(-1,-3,-1)	(0,-3,-1)
Out-of-Control ARL	4.9042	5.2640	5.3019	3.7393	3.1644	4.0796	5.0997	8.7030	11.7287
Correct Classification %	73.73	89.10	69.54	89.52	87.27	64.31	73.95	80.74	91.58
Change Point	100.6238	100.8058	100.7503	100.5798	100.4585	100.5577	101.0802	102.1909	103.2777
Standard Error	0.0121	0.0141	0.0141	0.0122	0.0112	0.0106	0.0167	0.0296	0.0412
Shift Combination	(1,-3,-1)	(2,-3,-1)	(3,-3,-1)	(-3,-3,0)	(-2,-3,0)	(-1,-3,0)	(1,-3,0)	(2,-3,0)	(3,-3,0)
Out-of-Control ARL	9.5546	4.9334	3.9116	4.2779	5.3158	12.2505	11.4519	5.2042	4.2185
Correct Classification %	77.68	69.82	59.09	88.96	89.64	90.75	92.72	91.47	90.22
Change Point	103.1379	102.3401	101.9630	100.7416	101.5805	104.3854	107.4700	104.5669	103.5188
Standard Error	0.0404	0.0326	0.0352	0.0124	0.0214	0.0541	0.0863	0.0580	0.0604
Shift Combination	(-3,-3,1)	(-2,-3,1)	(-1,-3,1)	(0,-3,1)	(1,-3,1)	(2,-3,1)	(3,-3,1)	(-3,-3,2)	(-2,-3,2)
Out-of-Control ARL	4.2815	5.2504	8.9664	12.2681	9.2380	5.4125	4.3717	3.2651	3.7689
Correct Classification %	63.27	73.66	82.99	90.77	79.30	72.48	64.81	89.50	92.32
Change Point	100.5536	101.0191	102.0209	102.7315	102.5966	101.9981	101.6383	100.1738	100.2930
Standard Error	0.0106	0.0162	0.0283	0.0356	0.0348	0.0270	0.0227	0.0066	0.0084
Shift Combination	(-1,-3,2)	(0,-3,2)	(1,-3,2)	(2,-3,2)	(3,-3,2)	(-3,-3,3)	(-2,-3,3)	(-1,-3,3)	(0,-3,3)
Out-of-Control ARL	5.1677	5.2527	5.3861	3.6636	3.1700	2.8054	3.2846	4.3620	4.1373
Correct Classification %	73.80	91.52	71.26	91.48	90.21	94.51	90.14	61.37	92.86
Change Point	100.4322	100.5110	100.5047	100.4534	100.4011	99.9812	99.9990	100.0091	100.0304
Standard Error	0.0102	0.0111	0.0108	0.0106	0.0096	0.0040	0.0044	0.0048	0.0050
Shift Combination	(1,-3,3)	(2,-3,3)	(3,-3,3)	(-3,-2,3)	(-2,-2,3)	(-1,-2,3)	(0,-2,3)	(1,-2,3)	(2,-2,3)
Out-of-Control ARL	4.8552	3.2115	2.7494	2.7830	2.2582	3.8207	4.4350	4.3003	3.2470
Correct Classification %	60.54	91.65	94.70	94.92	90.03	62.08	85.10	58.88	87.20
Change Point	100.0239	100.0093	100.0000	99.9844	100.0543	100.1675	100.2110	100.2022	100.1024
Standard Error	0.0049	0.0047	0.0045	0.0042	0.0053	0.0067	0.0072	0.0071	0.0062
Shift Combination	(3,-2,-3)	(-3,-2,-2)	(-2,-2,-2)	(-1,-2,-2)	(0,-2,-2)	(1,-2,-2)	(2,-2,-2)	(3,-2,-2)	(-3,-2,-1)
Out-of-Control ARL	2.7797	3.2272	3.7620	4.8934	5.2302	5.3214	3.7143	3.1863	4.1342
Correct Classification %	92.15	90.81	91.93	73.62	89.23	70.42	89.29	87.97	64.94
Change Point	100.0397	100.2057	100.3871	100.6157	100.7987	100.7345	100.5777	100.4353	100.5576
Standard Error	0.0051	0.0068	0.0093	0.0122	0.0141	0.0136	0.0120	0.0107	0.0107
Shift Combination	(-2,-2,-1)	(-1,-2,-1)	(0,-2,-1)	(1,-2,-1)	(2,-2,-1)	(3,-2,-1)	(-3,-2,0)	(-2,-2,0)	(-1,-2,0)
Out-of-Control ARL	5.0903	8.6491	11.6994	9.5067	4.9269	3.8793	4.2950	5.3485	12.1269
Correct Classification %	74.03	80.68	92.03	78.04	69.97	59.12	88.72	90.19	90.59
Change Point	101.0637	102.2911	103.1901	103.0740	102.3006	102.0233	100.7418	101.6163	104.3824
Standard Error	0.0165	0.0307	0.0394	0.0391	0.0323	0.0365	0.0124	0.0213	0.0541
Shift Combination	(1,-2,0)	(2,-2,0)	(3,-2,0)	(-3,-2,1)	(-2,-2,1)	(-1,-2,1)	(0,-2,1)	(1,-2,1)	(2,-2,1)
Out-of-Control ARL	11.5195	5.1740	4.2370	4.2813	5.2293	9.0930	12.3464	9.3376	5.4249
Correct Classification %	92.55	91.44	90.10	62.43	72.89	83.13	90.79	80.26	72.79
Change Point	107.5967	104.5857	103.4334	100.5471	101.0193	102.0006	102.7529	102.6865	102.0562
Standard Error	0.0892	0.0585	0.0592	0.0106	0.0163	0.0284	0.0361	0.0361	0.0278
Shift Combination	(3,-2,1)	(-3,-2,2)	(-2,-2,2)	(-1,-2,2)	(0,-2,2)	(1,-2,2)	(2,-2,2)	(3,-2,2)	(-3,-2,3)
Out-of-Control ARL	4.3927	3.2693	3.7615	5.1369	5.2310	5.4206	3.6625	3.1863	2.8148
Correct Classification %	64.01	89.70	92.17	73.00	91.84	71.71	91.38	90.14	95.10
Change Point	101.6681	100.1808	100.3059	100.4533	100.5116	100.4977	100.4502	100.3834	99.9661
Standard Error	0.0229	0.0068	0.0084	0.0103	0.0112	0.0110	0.0104	0.0096	0.0040
Shift Combination	(-2,-2,3)	(-1,-2,3)	(0,-2,3)	(1,-2,3)	(2,-2,3)	(3,-2,3)	(-3,-1,-3)	(-2,-1,-3)	(-1,-1,-3)
Out-of-Control ARL	3.2976	4.4345	4.1614	4.8143	3.1986	2.7455	2.8013	1.2598	3.8601
Correct Classification %	90.61	61.44	92.57	58.98	90.48	94.64	94.83	90.57	63.05
Change Point	99.9877	100.0263	100.0343	100.278	100.0155	99.9973	99.9865	100.0635	100.1680
Standard Error	0.0044	0.0048	0.0050	0.0049	0.0047	0.0045	0.0042	0.0054	0.0065
Shift Combination	(0,-1,-3)	(1,-1,-3)	(2,-1,-3)	(3,-1,-3)	(-3,-1,-2)	(-2,-1,-2)	(-1,-1,-2)	(0,-1,-2)	(1,-1,-2)
Out-of-Control ARL	4.4288	4.2816	3.2319	2.7754	3.2135	3.7355	4.8846	5.2090	5.3270
Correct Classification %	85.82	58.30	86.52	92.60	90.60	91.99	72.57	89.49	69.85
Change Point	100.2160	100.1892	100.1005	100.0430	100.1958	100.4332	100.6163	100.6163	100.7375
Standard Error	0.0073	0.0070	0.0061	0.0051	0.0068	0.0088	0.0121	0.0137	0.0138
Shift Combination	(2,-1,-2)	(3,-1,-2)	(-3,-1,-1)	(-2,-1,-1)	(-1,-1,-1)	(0,-1,-1)	(1,-1,-1)	(2,-1,-1)	(3,-1,-1)
Out-of-Control ARL	3.7328	3.1893	4.0791	5.1150	8.5512	11.9294	9.5243	4.9688	3.8686
Correct Classification %	89.09	88.10	64.17	74.14	80.76	92.21	77.94	69.99	58.90
Change Point	100.5788	100.4548	100.5596	101.0822	102.2248	103.3391	103.1184	102.2371	102.0065
Standard Error	0.0120	0.0112	0.0104	0.0168	0.0294	0.0410	0.0400	0.0326	0.0352

Shift Combination	(-3,-1,0)	(-2,-1,0)	(-1,-1,0)	(1,-1,0)	(2,-1,0)	(3,-1,0)	(-3,-1,1)	(-2,-1,1)	(-1,-1,1)
Out-of-Control ARL	4.2947	5.3346	12.3297	11.5599	5.1869	4.2345	4.2938	5.2138	8.9909
Correct Classification %	89.16	90.46	90.79	92.66	91.28	89.57	63.14	73.79	83.11
Change Point	100.7481	101.6111	104.3996	107.5090	104.6273	103.5005	100.5566	101.0301	101.9612
Standard Error	0.0124	0.0214	0.0543	0.0895	0.0567	0.0618	0.0106	0.0164	0.0279
Shift Combination	(0,-1,1)	(1,-1,1)	(2,-1,1)	(3,-1,1)	(-3,-1,2)	(-2,-1,2)	(-1,-1,2)	(0,-1,2)	(1,-1,2)
Out-of-Control ARL	12.2183	9.3191	5.3998	4.4103	3.2768	3.7710	5.1327	5.2888	5.4527
Correct Classification %	90.95	79.25	73.10	64.49	90.02	92.06	73.73	90.97	71.35
Change Point	102.7549	102.5284	101.9697	101.6194	100.1883	100.2909	100.4583	100.5091	100.5001
Standard Error	0.0364	0.0339	0.0268	0.0227	0.0068	0.0082	0.0106	0.0111	0.0110
Shift Combination	(2,-1,2)	(3,-1,2)	(-3,-1,3)	(-2,-1,3)	(-1,-1,3)	(0,-1,3)	(1,-1,3)	(2,-1,3)	(3,-1,3)
Out-of-Control ARL	3.6584	3.1747	2.8122	3.2938	4.3311	4.1715	4.9215	3.2003	2.7399
Correct Classification %	92.07	90.00	94.81	90.22	62.07	92.92	60.03	91.04	94.41
Change Point	100.4285	100.3881	99.9770	99.9979	100.0112	100.0401	100.0252	10.0152	100.0026
Standard Error	0.0101	0.0095	0.0040	0.0044	0.0048	0.0051	0.0049	0.0048	0.0044
Shift Combination	(-2,1,-3)	(-1,1,-3)	(0,1,-3)	(1,1,-3)	(2,1,-3)	(3,1,-3)	(-3,1,-2)	(-2,1,-2)	(-1,1,-2)
Out-of-Control ARL	3.2482	3.8613	4.3827	4.3073	3.2388	2.7960	3.2044	3.7421	4.8728
Correct Classification %	90.54	63.59	85.11	59.10	86.16	92.83	90.34	92.04	73.07
Change Point	100.0594	100.1735	10.2327	100.1871	100.1050	100.0386	100.1935	100.3709	100.6383
Standard Error	0.0053	0.0067	0.0072	0.0070	0.0060	0.0051	0.0069	0.0092	0.0124
Shift Combination	(0,1,-2)	(1,1,-2)	(2,1,-2)	(3,1,-2)	(-3,1,-1)	(-2,1,-1)	(-1,1,-1)	(0,1,-1)	(1,1,-1)
Out-of-Control ARL	5.2358	5.3319	3.7110	3.1631	4.1101	5.1082	8.6579	11.6192	9.5112
Correct Classification %	89.56	70.19	89.35	87.48	64.67	73.82	80.22	91.77	77.25
Change Point	100.7759	100.7505	100.5520	100.4460	100.5374	101.0586	102.2101	103.2265	103.1098
Standard Error	0.0141	0.0139	0.0118	0.0113	0.0105	0.0165	0.0298	0.0403	0.0404
Shift Combination	(2,1,-1)	(3,1,-1)	(-3,1,0)	(-2,1,0)	(-1,1,0)	(1,1,0)	(2,1,0)	(3,1,0)	(-3,1,1)
Out-of-Control ARL	4.9692	3.8664	4.3169	5.3132	12.3960	11.4385	5.1660	4.2320	4.2734
Correct Classification %	70.39	59.48	88.70	89.51	90.18	92.81	91.03	89.30	62.55
Change Point	102.3802	102.0469	100.7505	101.5603	104.4455	107.5763	104.5797	103.5209	100.5401
Standard Error	0.0329	0.0371	0.0126	0.0212	0.0546	0.0889	0.0577	0.0595	0.0106
Shift Combination	(-2,1,1)	(-1,1,1)	(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)	(-3,1,2)	(-2,1,2)	(-1,1,2)
Out-of-Control ARL	5.2605	9.0381	12.3460	9.2576	5.3976	4.4596	3.2603	3.7642	5.1685
Correct Classification %	73.64	83.01	90.65	79.34	73.44	63.93	89.66	91.85	73.53
Change Point	101.0464	102.0128	102.7313	102.5801	102.0209	101.6733	100.1721	100.2842	100.4450
Standard Error	0.0168	0.0282	0.0358	0.0341	0.0273	0.0229	0.0066	0.0084	0.0106
Shift Combination	(0,1,2)	(1,1,2)	(2,1,2)	(3,1,2)	(-3,1,3)	(-2,1,3)	(-1,1,3)	(0,1,3)	(1,1,3)
Out-of-Control ARL	5.3253	5.4238	3.6517	3.1646	2.8267	3.2936	4.3343	4.1414	4.8536
Correct Classification %	91.51	71.31	91.03	89.73	95.20	90.63	61.98	92.98	61.02
Change Point	100.5312	100.4932	100.4525	100.3770	99.9743	99.9908	100.0192	100.0352	100.0235
Standard Error	0.0116	0.0108	0.0104	0.0096	0.0041	0.0044	0.0048	0.0050	0.0048
Shift Combination	(2,1,3)	(3,1,3)	(-3,2,-3)	(-2,2,-3)	(-1,2,-3)	(0,2,-3)	(1,2,-3)	(2,2,-3)	(3,2,-3)
Out-of-Control ARL	3.2138	2.7403	2.7865	3.2562	3.8521	4.4224	4.3059	3.2550	2.7957
Correct Classification %	90.64	94.41	94.92	89.92	62.62	85.75	59.06	87.27	92.87
Change Point	100.0167	100.0053	99.9870	100.0550	100.1709	100.2332	100.2027	100.0994	100.0383
Standard Error	0.0047	0.0046	0.0042	0.0052	0.0066	0.0074	0.0071	0.0061	0.0052
Shift Combination	(-3,2,-2)	(-2,2,-2)	(-1,2,-2)	(0,2,-2)	(1,2,-2)	(2,2,-2)	(3,2,-2)	(-3,2,-1)	(-2,2,-1)
Out-of-Control ARL	3.2119	3.7385	4.9076	5.2112	5.3378	3.7301	3.1879	4.0737	5.1026
Correct Classification %	90.54	91.97	73.1	89.80	70.41	89.36	87.54	63.80	73.7
Change Point	100.1980	100.3580	100.6492	100.7959	100.7544	100.5825	100.4371	100.5565	101.0720
Standard Error	0.0068	0.0090	0.0124	0.0144	0.0137	0.0120	0.0107	0.0106	0.0168
Shift Combination	(-1,2,-1)	(0,2,-1)	(1,2,-1)	(2,2,-1)	(3,2,-1)	(-3,2,0)	(-2,2,0)	(-1,2,0)	(1,2,0)
Out-of-Control ARL	8.5634	11.6575	9.5186	5.0006	3.8882	4.2910	5.3134	12.4540	11.4138
Correct Classification %	80.00	91.64	78.61	71.01	59.77	88.70	90.95	90.70	92.17
Change Point	102.2600	103.2664	103.1395	102.3949	101.9942	100.7501	101.5931	104.4138	107.5986
Standard Error	0.0300	0.0413	0.0401	0.0333	0.0363	0.0123	0.0215	0.0545	0.0881
Shift Combination	(2,2,0)	(3,2,0)	(-3,2,1)	(-2,2,1)	(-1,2,1)	(0,2,1)	(1,2,1)	(2,2,1)	(3,2,1)
Out-of-Control ARL	5.2078	4.2245	4.2959	5.2223	9.0037	12.2175	9.2177	5.3535	4.4140
Correct Classification %	91.23	89.46	63.81	73.68	82.63	91.22	78.99	73.01	63.56
Change Point	104.5731	103.6618	100.5525	101.0294	102.0193	102.7048	102.5810	102.0341	101.6662
Standard Error	0.0572	0.0632	0.0105	0.0164	0.0282	0.0356	0.0345	0.0273	0.0228
Shift Combination	(-3,2,2)	(-2,2,2)	(-1,2,2)	(0,2,2)	(1,2,2)	(2,2,2)	(3,2,2)	(-3,2,3)	(-2,2,3)
Out-of-Control ARL	3.2618	3.7624	5.1528	5.3041	5.4895	3.6669	3.1553	2.8286	3.2971
Correct Classification %	90.02	92.51	74.11	91.59	71.6	92.09	89.54	94.93	90.35
Change Point	100.1966	100.2828	100.4461	100.5077	100.5027	100.4529	100.3984	99.9698	99.9924
Standard Error	0.0068	0.0083	0.0103	0.0110	0.0110	0.0103	0.0096	0.0040	0.0044
Shift Combination	(-1,2,3)	(0,2,3)	(1,2,3)	(2,2,3)	(3,2,3)	(-3,3,-3)	(-2,3,-3)	(-1,3,-3)	(0,3,-3)
Out-of-Control ARL	4.3046	4.1805	4.8674	3.2182	2.7663	2.7919	3.2445	3.8507	4.4214
Correct Classification %	61.38	92.69	59.88	91.01	95.08	94.99	89.89	62.67	85.23
Change Point	100.0183	100.0200	100.0234	100.0127	100.0026	99.9849	100.0659	100.1764	100.2278
Standard Error	0.0047	0.0049	0.0049	0.0048	0.0047	0.0043	0.0053	0.0067	0.0073
Shift Combination	(1,3,-3)	(2,3,-3)	(3,3,-3)	(-3,3,-2)	(-2,3,-2)	(-1,3,-2)	(0,3,-2)	(1,3,-2)	(2,3,-2)
Out-of-Control ARL	4.3127	3.2452	2.7687	3.1086	3.7395	4.8999	5.2439	5.2697	3.7195

Correct Classification %	58.30	86.80	92.35	90.70	91.83	72.81	89.25	69.74	89.46
Change Point	100.1914	100.1049	100.0271	100.1996	100.3589	100.6305	100.7811	100.7604	100.6040
Standard Error	0.0070	0.0061	0.0051	0.0069	0.0092	0.0122	0.0139	0.0139	0.0122
Shift Combination	(3,3,-2)	(-3,3,-1)	(-2,3,-1)	(-1,3,-1)	(0,3,-1)	(1,3,-1)	(2,3,-1)	(3,3,-1)	(-3,3,0)
Out-of-Control ARL	3.1791	4.1293	5.1030	8.5891	11.7086	9.5472	4.9842	3.8922	4.2817
Correct Classification %	87.60	65.28	74.41	80.40	91.88	77.56	70.48	59.36	88.82
Change Point	100.4555	100.5546	101.0987	102.2632	103.2990	103.1824	102.3946	102.0207	100.7718
Standard Error	0.0107	0.0106	0.0167	0.0304	0.0409	0.0410	0.0335	0.0351	0.0127
Shift Combination	(-2,3,0)	(-1,3,0)	(1,3,0)	(2,3,0)	(3,3,0)	(-3,3,1)	(-2,3,1)	(-1,3,1)	(0,3,3)
Out-of-Control ARL	5.3276	12.2106	11.4751	5.1728	4.2090	4.2929	5.1941	8.9355	12.0587
Correct Classification %	90.01	90.96	92.64	91.56	89.76	63.68	73.63	82.59	91.17
Change Point	101.6087	104.3907	107.5159	104.5898	103.4984	100.5474	101.0108	102.0734	102.6929
Standard Error	0.0214	0.0542	0.0881	0.0575	0.0614	0.0106	0.0159	0.0288	0.0351
Shift Combination	(1,3,1)	(2,3,1)	(3,3,1)	(-3,3,2)	(-2,3,2)	(-1,3,2)	(0,3,2)	(1,3,2)	(2,3,2)
Out-of-Control ARL	9.2506	5.3887	4.3996	3.2424	3.7790	5.1532	5.2658	5.3745	3.6578
Correct Classification %	79.53	72.81	63.55	90.51	92.17	73.75	91.71	71.56	91.91
Change Point	102.5587	102.0309	101.6013	100.1776	100.2810	100.4357	100.5141	100.5098	100.4502
Standard Error	0.0338	0.0271	0.0222	0.0067	0.0084	0.0105	0.0113	0.0112	0.0103
Shift Combination	(3,3,2)	(-3,3,3)	(-2,3,3)	(-1,3,3)	(0,3,3)	(1,3,3)	(2,3,3)	(3,3,3)	
Out-of-Control ARL	3.1823	2.8212	3.2982	4.2904	4.1435	4.8685	3.2056	2.7630	
Correct Classification %	90.15	94.93	90.60	60.88	93.49	59.68	91.09	94.73	
Change Point	100.3983	99.9688	99.9777	100.0100	100.0273	100.0205	100.0261	99.9989	
Standard Error	0.0097	0.0040	0.0045	0.0047	0.0049	0.0050	0.0048	0.0046	

#### IV. CONCLUSIONS

When a process involved multi related quality characteristics is controlled statistically, an out-of-control signal itself could not lead the practitioners to an effective root-cause analysis. In this case a multi task scheme which is able to estimate the change point and simultaneously performs effectively a diagnostic analysis to identify the quality characteristic contributing to the out-of-control condition is required. In this paper a multi task scheme based on supervised learning was proposed which could help practitioners not only detect an out-of-control condition, but also the scheme helps to identify the change point and diagnose the variable(s) responsible to the new condition, all at the same time. Performance of the multi task scheme was evaluated via 287 scenarios of mean step change and the results indicated the high capabilities of the model.

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