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Multi-Objective Optimization of Combined System Reliability and Redundancy Allocation Problem

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***Abstract*—**This paper presents established 3^n enumeration procedure for mixed integer optimization problems for solving multi-objective reliability and redundancy allocation problem subject to design constraints. The formulated problem is to find the optimum level of unit reliability and the number of units for each subsystem. A number of illustrative examples are provided and compared to indicate the application of the superiority of the proposed method.

***Keywords*—**Integer programming, mixed integer programming, multi-objective optimization, reliability redundancy allocation.

I. Introduction

THE design of a multistage series system requires maximizing system reliability (or minimization of unreliability) while at the same time minimizing system cost and in some instances weight and volume as well. Thus minimization of multiple objective functions became an important aspect of system reliability in the design of engineering systems. The optimal system design, in some situation, involves several conflicting objectives such as to minimize system cost and weight while simultaneously maximizing the system reliability. Thus multiple objective function is an important aspect of reliability design of engineering systems where the system designer either requires a single solution where all design constraints are satisfied or identifies a Pareto optimal set. Misra et al. [1], [2] present a method for solving multiple criteria reliability design problems through a direct search technique in combination with a min-max approach. Here the series-parallel allocation problem has been solved using a fuzzy programming formulation of the problem by imposing a threshold on reliability. Ravi et al. [3] apply the same fuzzy optimization approach and solve a problem of choosing optimal redundancy for a series-parallel system with no alternative component choices. Dhingra [4] and Rao and Dhingra [5] studied a reliability and redundancy allocation problem for a four-stage and a five-stage over-speed protection system, using crisp and fuzzy multi-objective optimization approaches, respectively. Taboada et al. [6], [7] proposed a Genetic Algorithm approach for multi-objective reliability design problems maximizing the reliability and minimizing the total cost of the system and proposed a pruning scheme to reduce set of solutions. Particle swarm introduced by Kennedy and Eberhart [8] is a population based heuristic fast converging and robust technique to solve single objective optimization design problems which has been extended to solve multiple objective problems [9]-[12].

In earlier attempt to solve allocation problem, the component reliability was kept fixed and the optimal redundancy of each stage was found for maximum system reliability [1]. The general optimization problem deals with the situation where both optimal component reliability and the optimal redundancy of each stage are computed to maximize system reliability and simultaneously minimize other resources such cost, weight and volume. This problem is formulated as a multi-objective non-linear mixed integer programming problem. Single objective optimization cannot compromise among the mutually conflicting objectives. A Pareto optimal set is a set of solution that is all non-dominated with respect to each other. In the process of moving from one Pareto solution to another, one encounters sacrificing one objective to achieve gain in other. Thus, Pareto optimal solution provides a set from which one can select a best compromised solution among all non-dominated solutions [6]. Pareto optimal solutions set are generally preferred for multiple objective optimization problems; because it provides system designer a choice between crucial parameters.

Due to the non-convexity of the problems for the optimal reliability design, many optimization methods fail to attain solutions, and meta-heuristic algorithms have been proposed [1]-[12] including one proposed that utilizes two phases [13]-[15]. The proposed method to solve multi-objective problems captures possible solutions and permits several members of the Pareto optimal set in a single run of the algorithm. A host of various meta-heuristic and evolutionary algorithms used for finding solutions that are very near the global optimum of a complex optimization problem, is covered in [16].

II. Reliability Redundancy Allocation Multi-Objective Optimization Model

A reliability redundancy multi-objective problem can be formulated as a following the constrained mixed-integer programming problem:

$$\text{Minimize } f(x) = [f_1(x), f_2(x), \ldots\ldots\ldots, f_k(x)] \tag{1}$$

Subject to the m inequality constraints:

$$g_j(\mathbf{x}) \leq 0 \quad j = 1,2,\ldots\ldots,m \tag{2}$$

and the p equality constraints

$$h_j(\mathbf{x}) = 0 \quad j = 1,2,\ldots\ldots,p < n \tag{3}$$

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where $\boldsymbol{x} = [x_1, x_2, \ldots\ldots, x_n]$ is a vector of decision variables defined in n-dimensional *Euclidean* space of variables $\mathbf{E}^n$, $f(x) = [f_1(x), f_2(x), \ldots\ldots f_k(x)]$ is a vector defined in *Euclidean* space of objective functions $\mathbf{E}^k$, $f_p(\mathbf{x})$, $g_j(\mathbf{x})$, $h_j(\mathbf{x})$ are linear and/or non-linear functions of variables x_1, x_2, ……., x_n. The constraints (equality and inequality) define the feasible region and any point $\mathbf{x}$ in this region defines the feasible solution.

The method adopted to solve the multi-objective optimization is to combine all objective functions into a single aggregate function. A simple form of such function is a linear sum of weights of the form:

$$\text{Minimize } \sum_{i=1}^{k} \lambda_i f_i(x) \tag{4}$$

where $\lambda_i \geq 0$ are the weighting coefficients representing the relative importance of the k objective functions. It is usually assumed that:

$$\sum_{i=1}^{k} \lambda_i = 1 \tag{5}$$

A method for finding the optimum of a non-linear non-convex mixed discrete-function with mixed discrete-continuous constraints within the bounds of the real and discrete variables has been developed in [13], [14] for integer problems and in [15] for mixed-integer problems. This procedure has been used to find optimum of mixed objective function (4) with specified constraints along with restrictions imposed on the $\boldsymbol{x}$ i.e. $\mathbf{x_L} \leq \boldsymbol{x} \leq \mathbf{x_H}$. It should be noted that satisfying the design vector $\mathbf{x}$ upper bound $\mathbf{x_H}$ and lower bound $\mathbf{x_L}$ constraints does not imply satisfying the functional constraints $g_j(\boldsymbol{x})$. The search for the local optimum is started during Phase I of the gradient decent along the function gradient [13]-[15] where the search for the minimum point is carried by moving along the function gradient in a step and increasing the size of step if the value of the function decreases and reducing if it increases. However, if any of the constraints is violated then a move in a direction orthogonal to the constraint boundary is invoked to return the search to the feasible region. However, when two or more constraints are violated simultaneously, the return direction is given by $\ell_1\mathbf{n}_1 + \ell_2\mathbf{n}_2 + \ldots$, where $\mathbf{n}_1$, $\mathbf{n}_2$,… are unit normal to the violated constraints, and ℓ_1, ℓ_2,… are positive weighting factors which are proportional to the amounts by which the constraints are violated [13]. Now the search for the minimum again resumes in a direction that is a vector sum of the normalized gradient of the multi-objective function (4) and the constraints that had been violated [17]. The search for minimum in the feasible region now continues until convergence criteria established in [13] are met. The local minima found in the first phase, is now used in the second phase where the 3^n vector combinations neighboring this local are examined [13]-[15] and the local minimum obtained is now used as a seed for new iterative cycle. This cyclic search continues until there is no further improvement in objective function value and the best possible optimum solution vector X^* found is used to calculate value of individual objective functions. A search for set of Pareto vectors for multi-objective function is carried out during the second phase of the optimization run from the set of vectors provided by 3^n enumeration where $f(\mathbf{x}) \geq f(\mathbf{x}^*)$

III. Reliability Redundancy Multi-Objective Allocation Problems Solved

A number of integer and mixed-integer reliability redundancy problems have been solved using the algorithm described in Section II. These problems have been taken from [1], [2], [4] on reliability optimization techniques. These test problems are for system configuration composed of three/four stages in series that can be categorized into the following major groups

Problem P1 [1]

Maximize the reliability and minimize cost and weight of series-parallel systems subject to linear constraints [1].

Maximize: $R_s(\mathbf{x}) = \prod_{i=1}^{3} R_i(x_i)$ where each $R_i(x_i) = 1-(1-r_i)^{x_i}$

Minimize: $C_s = 4x_1 + 8x_2 + 6x_3$

Minimize: $W_s = 6x_1 + 6x_2 + 10x_3$

Subject to:

$$g_1(x) = 50.0 - (4x_1 + 8x_2 + 6x_3\} \geq 0$$

$$g_2(x) = 52.0 - (6x_1 + 6x_2 + 10x_3\} \geq 0$$

$$g_3(x) = 65.0 - (10x_1 + 5x_2 + 10x_3\} \geq 0$$

$$g_4(x) = \prod_{i=1}^{3} R_i(x_i) - 0.94 \geq 0$$

Component reliability: (r_1, r_2, r_3) = (0.86, 0.91, 0.96)

Results

- Multi-objective optimal redundancy allocation $\mathbf{X}^*$= (3,2,1); Objective function values are: $R_s(\mathbf{X}^*)$ = 0.949611, $C_s(\mathbf{X}^*)$ = 34.0, $W_s(\mathbf{X}^*)$ = 40.0 units.
- The Pareto optimal solution is (2, 2, 2); Objective function values are: $R_s(\mathbf{X}^*)$ = 0.970903, $C_s(\mathbf{X}^*)$ = 36.0, $W_s(\mathbf{X}^*)$ = 44.0 units.
- Single objective optimal redundancy allocation $\mathbf{X}^*$= (3,2,2); Objective function values are: $R_s(\mathbf{X}^*)$ = 0.98759, $C_s(\mathbf{X}^*)$ = 40.0, $W_s(\mathbf{X}^*)$ = 50.0 units.

The results shown in [1] for multi-objective problem is in fact for one obtained for maximization of single objective system reliability subject to four constraints shown in the problem description.

Problem P2 [1]

Maximize the reliability and minimize cost of series-parallel systems subject to non-linear constraints. This example describes system configuration [1] composed of three stages where the subsystem reliability of the first stage is selected out of four possible candidates for component reliability. The reliability of subsystem of the second stage is increased by adding additional units in parallel and for third subsystem assumes 2-out-of-x_3: G configuration.

Statement of the Problem

Maximize: $R_s(\mathbf{x}) = \prod_{i=1}^{3} R_i(x_i)$ where

$$R_1(x_i) = \begin{bmatrix} 0.88 \\ 0.92 \\ 0.98 \\ 0.99 \end{bmatrix}, \text{ for } x_i = 1,2,3,4$$

$$R_2(x_2) = 1 - (1 - 0.81)^{x_2}$$

$$R_3(x_3) = \sum_{i=2}^{x_3} \binom{x_3}{i} (0.77)^i (1-0.77)^{x_3-i}$$

Minimize: $C_s(\mathbf{x}) = 4 \exp\{0.02/(1-R_1(x_i)\} + 5x_2 + 2x_3$

Subject to:

$$g_1(\mathbf{x}) = 45.0 - [4 \exp\{0.02/(1-R_1(x_i)\} + 5x_2 + 2x_3] \ge 0$$

$$g_2(\mathbf{x}) = 65.0 - \{e^{x_1/8} + 3(x_2 + e^{x_2/4} + 5(x_3 + e^{(x_3-1)/4})\} \ge 0$$

$$g_3(\mathbf{x}) = 230 - \{8x_2 e^{x_2/4} + 6(x_3 - 1)e^{(x_3-1)/4}\} \ge 0$$

$$g_4(X) = \prod_{i=1}^{3} R_i(x_i) - 0.90 \ge 0$$

Also the reliability of each subsystem is constrained to have a minimum reliability of 0.95, i.e. $R_j(\mathbf{x}) \ge 0.95$ for j=1, 2, 3. The above constraint establishes the lower limit on the number units on each subsystem as (3, 2, 4)

Results

- Multi-objective Optimal redundancy allocation $\mathbf{X}^*$= (3,2,4): Objective function values are: $R_s(\mathbf{X}^*) = 0.90658$, $C_s(\mathbf{X}^*) = 28.8731$. The inequality constraints values are: $g_1(\mathbf{x}) = 28.8731$, $g_2(\mathbf{x}) = 42.9861$, $g_3(\mathbf{x}) = 64.4855$
- Single objective Optimal redundancy allocation $\mathbf{X}^*$= (3,3,6): Objective function values are: $R_s(\mathbf{X}^*) = 0.970234$; The inequality constraints values are: $C_s(\mathbf{X}^*) = 37.8731$, $W_s(\mathbf{X}^*) = 64.2577$, $V_s(\mathbf{X}^*) = 155.5183$.

The result reported in [1] is the one obtained for maximization of single objective system reliability subject to four constraints shown in the problem description.

Problem P3

The three problems solved here deal with mixed-integer non-linear programming problems where the $\mathbf{x}= (x_1, x_2,\ldots,x_n, x_{n+1}, \ldots, x_{2n})$ is a vector of decision variables, first n variables are integers that are the redundancies of the n stages and next n elements are reliabilities of component of each subsystem (stage). Other authors have solved such problems [4] using goal programming, goal attainment method and random search technique based on *min-max* approach described in [1], [2] to which the results are compared here.

Problem P3,1 [2]

The system configuration considered here is the same as considered above for Problem P1 & P2 and multi-objective optimization problem is solved to locate the optimal values of component reliabilities and allocation for each of the three subsystems.

Statement of the Problem

Maximize $R_s(\mathbf{x}) = \prod_{i=1}^{3} R_i(x_i)$ where $R_1(x_i)$ = 0.88, 0.92, 0.98 and 0.99 for x_i = 1,2,3,4 i.e. reliability of the first subsystem depends upon of the number of redundant units comprising the subsystem.

$$R_2(x_2) = 1 - (1 - r_2)^{x_2}$$

$$R_3(x_3) = \sum_{i=2}^{x_3} \binom{x_3}{i} (r_3)^i (1-r_3)^{x_3-i}$$

and minimize: $C_s(\mathbf{x}) = 4 \exp\{0.02/(1-R_1(x_i)\} + c_2 x_2 + c_3 x_3$

Subject to:

$$g_1(\mathbf{x}) = 45.0 - [4 \exp\{0.02/(1-R_1(x_i)\} + c_2 x_2 + c_3 x_3] \ge 0$$

$$g_2(\mathbf{x}) = 65.0 - \{w_1 e^{x_1/8} + w_2(x_2 + e^{x_2/4}) + w_3(x_3 + e^{(x_3-1)/4})\} \ge 0$$

$$g_3(\mathbf{x}) = 230 - \{v_2 x_2 e^{x_2/4} + v_3(x_3 - 1)e^{(x_3-1)/4}\} \ge 0$$

$$g_4(X) = \prod_{i=1}^{3} R_i(x_i) - 0.90 \ge 0$$

also $0.45 \le r_i \le 0.99$ for I = 1, 2, 3 where g_1, g_2, g_3 and g_4 are cost, weight, volume and reliability constraints respectively and each of the coefficient c_i, w_i, v_i is of the form $\alpha_i \exp\{\beta_i/(1-r_i)\}$. The values of α_i and β_i for each of c_i, w_i, v_i are listed in Table I.

TABLE I
VALUES OF α_i AND β_i FOR DIFFERENT CONSTRAINTS

Stage / Constraints	1		2		3	
	α_1	β_1	α_2	β_2	α_3	β_3
1	4.0	0.02	4.5	0.002	1.2	0.15
2	0.16	0.22	2.5	0.03	2.5	0.21
3	0.0	0.0	4.0	0.14	2.1	0.32

Results

Following result is the minimum of multi-objective function found assigning weight λ_1=0.50 and λ_2=0.50 to the reliability and cost function.

- Optimal allocation $(\mathbf{x},\mathbf{r})$ = (3, 2, 5, 0.98, 0.90239, 0.69870); Optimal subsystem reliability = (0.98, 0.990422, 0.968728); Optimal system reliability = 0.94030; Optimal system cost = 20.508897.

- Optimal solution where the emphasis is on achieving maximizing reliability, the result is provided at the end of Phase2. Optimal allocation (**x,r**) = (3, 2, 6, 0.98, 0.90239, 0.700705); Optimal subsystem reliability = (0.98, 0.990472, 0.989184); Optimal system reliability = 0.960164; Optimal system cost = 22.602661.

Result found in [2] is inferior to one reported above for multi-objective problem and also inferior to single objective of maximization of system reliability and minimization of system cost separately as reported below. The optimal allocations individually for each of objective function are:

- for Objective function 1 (i.e. maximizing system reliability) (3, 2, 6, 0.98, 0.902129, 0.7010), system reliability 0.960163 and system cost 22.611, weight 60.983 and volume 162.201
- for Objective function 2 (i.e. minimizing cost) (2, 2, 6, 0.92, 0.900375, 0.69871), system cost 22.6542 and system reliability 0.900707, weight 60.2326 and volume 159.7735

The optimal allocation reported in [2] is inferior for first objective function (i.e. reliability) and inadmissible for second objective function (i.e. cost) as the fourth constraint $g_4(x)$ is clearly violated.

Problem P3,2 [2]

The system consists of four subsystems in series with each subsystem consisting of identical units arranged in parallel. The problem is defined as:
Maximize:

$$R_s = \prod_{i=1}^{4} R_i = \prod_{i=1}^{4} (1-(1-r_i)^{x_i}),$$

or Minimize $Q_s = 1 - R_s$ and Minimize $C_s = \sum_{i=1}^{4} c_i x_i$

Subject to:

$$g_1(\mathbf{x}) = \sum_{i=1}^{4} c_i x_i \leq 400,$$

where $C_i = \alpha_i \exp(\beta_i/(1-r_i))$. The values of α_i and β_i for each of the subsystems are provided in Table II.

$$g_2(\mathbf{x}) = \sum_{i=1}^{4} w_i x_i \leq 75.0$$

$$g_3(\mathbf{x}) = \sum_{i=1}^{4} v_i x_i \leq 80.0$$

where W_i and V_i are of the form $\alpha_i r_i^{\beta_i}$ and also,

$$g_4(\mathbf{x}) = \prod_{i=1}^{4} R_i - 0.90 \geq 0$$

i.e., the system reliability should be at least equal to 0.90 and further, each of the subsystem reliabilities i.e., $R_i = (1-(1-r_i)^{x_i}) \geq 0.95$, i=1,…,4. Also, $0.40 \leq r_i \leq 0.99$ for i = 1,2,3,4 i.e., each of the component reliabilities in each subsystem is restricted to a value between 0.40 and 0.99.

TABLE II
VALUES OF α_i AND β_i FOR DIFFERENT CONSTRAINTS

Stage / Constraints	1 α_1	β_1	2 α_2	β_2	3 α_3	β_3	4 α_4	β_4
1	1.0	0.3	3.5	0.55	2.0	0.4	5.0	0.65
2	5.0	2.0	4.0	2.0	8.0	2.0	7.0	2.0
3	4.0	2.0	8.0	2.0	6.0	2.0	10.0	2.0

Results

- Optimal allocation = (11, 0.50399 10, 0.50996, 10, 0.51399 10, 0.49893); Optimal subsystem reliability = (0.999553, 0.999201, 0.999265, 0.999002); Optimal system reliability = 0.997025; Optimal system cost = 356.168 units; Resources consumed = (356.168, 62.933, 72.726).

The optimal allocations reported in [2] are as follows:

- Optimal allocation = (10, 0.573, 8, 0.587, 9, 0.547, 10, 0.532); Optimal subsystem reliability = 0.99979, 0.99916, 0.99920, 0.9995; Optimal system reliability = 0.99767; Optimal system cost = 370.94; Resource consumed = (370.95, 68.90, 79.79).

The optimal allocations with respect maximization of first objective function (system reliability) are:

- Optimal allocation = (12, 0.50644, 11, 0.51047, 11, 0.5164, 11, 0.49683); Optimal subsystem reliability = 0.99979, 0.99961, 0.99966, 0.99947; Optimal system reliability = 0.998543; Optimal system cost = 390.932 units; Resources consumed = (390.932, 69.332, 79.9985).

The optimal allocations with respect to minimization of second objective function (system cost) are:

- Optimal allocation = (6, 0.5260, 6, 0.43596, 6, 0.5130, 6, 0.4000); Optimal subsystem reliability = 0.98866, 0.96780, 0.98666, 0.95334; Optimal system reliability = 0.9000; Optimal system cost = 182.897 units; Resources consumed = (182.897, 32.2134, 34.837).

There are no reported results in [2] for maximization of single objective system reliability and minimization of system cost to compare the above results.

Problem P3,3 [4]

This test problem deals with an example of a gas turbine where the speed is controlled by 4 control valves used to cutoff fuel supply. This control system is modeled as a 4-stage series system where all components have constant failure rate. The objective is to find optimal level of R_i and n_i at each stage such that:

i. f_1 (system reliability) is maximized
ii. f_2(cost) is minimized
iii. f_3(weight) is minimized

The multi-objective reliability apportionment problem is:

Find ***n*** and ***R*** that:
Minimize $[-f_1, f_2, f_3]$
Subject to: g_j (**R, n**) $\leq a_j, j = 1,\ldots,m$

The design constraints are:

1. $V = \sum_{i=1}^{N} v_i n_i^2 \leq V_{\lim}$,
2. $W = \sum_{i=1}^{N} w_i n_i \exp(n_i / 4) \leq W_{\lim}$
3. $R_{system} = \prod_{i=1}^{N} [1-(1-R_i)^{n_i}] \geq R$
4. $\sum_{i=1}^{N} C(R_i)[n_i + \exp(n_i / 4)] \leq C$

Additional Assumptions

The cost –reliability relation is:

$$C(R_i) = \alpha_i.[-t / \ln(R_i)]^{\beta_i}$$

t is the operating time during which the component must not fail. Additional constraints, for all i include: $1 \leq n_i \leq 10$ and $0.50 \leq R_i \leq 1 - 10^{-6}$

The design data used for the example problem is shown in [4]. The optimal results obtained by solving three individual objective optimization problems are shown in Table III.

TABLE III
OPTIMUM SOLUTIONS FOR SINGLE OBJECTIVE OPTIMIZATIONS

Objective	Optimal sol^n proposed method	Optimal sol^n reported in [4]	CPU (sec)
Max R_s	(7, 0.7492, 3, 0.9421, 4, 0.9257, 5, 0.8442): R_s = 0.99962, C = 399.991, w = 488.946, V = 165.0	(6, 0.81604, 6, 0.80309, 3, 0.98364, 5, 0.80373), R_s = 0.99961, C = 399.936, w = 495.652, V = 185.0	10.156
Min C	(4, 0.50, 4, 0.50, 5, 0.52276, 3, 0.50): R_s = 0.7500, C = 20.13985, w = 314.548 V = 141.0	(4, 0.50, 4, 0.50, 5, 0.5925,3, 0.50): R_s = 0.7604, C = 20.7252, w = 314.548, V = 141.0	4.187
Min W	(1, 0.97168, 1, 0.94673, 1, 0.96887, 1, 0.92916): R_s = 0.828154, C = 399.998, w = 34.6687, V = 8.0	(1, 0.96221, 1, 0.92315, 1, 0.98787, 1, 0.92065): R_s = 0.80786, C = 399.509, w = 34.668, V = 8.0	1.468

TABLE IV
COMPARISON OF OPTIMAL SOLUTIONS FOR MULTI-OBJECTIVE OPTIMIZATIONS

Starting Vector	Pareto-Optimum Sol^n Proposed	Optimal Sol^n Phase II Proposed Method	Optimal Sol^n Reference [4]	CPU Time (sec.)
(8, 0.80, 9, 0.85, 8, 0.85, 4, 0.95)	(3, 0.7965, 3, 0.8331, 3, 0.847 0.8477, 2, 0.907), R_s = 0.974967, C = 156.732, W = 150.1021, V = 62.0, F = -194.28 (2,0.8,2,0.7635,2,0.8364,2, 0.85) R_s= 0.86223 C=75.0 W=89.031 V=32.0, F=-241.365	(2, 0.80, 2, 0.76358, 2, 0.83648, 1, 0.85018), R_s = 0.7500, C = 59.898, W = 74.937, V = 26.0, F = -250.9635	(2, 0.9412, 2, 0.9091, 2, 0.9408, 2, 0.9128), R_s = 0.97739, C = 287.19, W = 89.031, V = 32.0, F = -171.384*	8.281
(10, 0.850, 8, 0.75, 7, 0.92, 5, 0.85)	(4, 0.8489, 3, 0.7489, 2, 0.919, 2, 0.8489), R_s = 0.95492, C = 110.1457, W = 152.8064, V = 54.0, F = -208.7546	(2, 0.8104, 2, 0.7389, 2, 0.8805, 1, 0.8469), R_s = 0.7500, C = 60.083, W = 74.937, V = 26.0, F = -250.9024	(2, 0.8650, 3, 0.8181, 2, 0.8425, 3, 0.8064), R_S = 0.94478, C = 104.472, W = 128.727, V = 52.0, F = -218.570*	0.484
(7, 0.86, 9, 0.90, 6, 0.95, 5, 0.855)	(3, 0.849, 3, 0.8497, 3, 0.9274, 3 0.8405), R_s = 0.98878, C = 156.0065, W = 171.477, V = 72.0, F = -187.47, (2, 0.8492, 2, 0.8496, 2, 0.9273, 2, 0.8404), R_s = 0.92591, C = 111.105, W = 89.031, V = 32.0, F = -229.474	(2, 0.8275, 1, 0.8649,1, 0.936, 2, 0.7867) R_s = 0.7500, C = 79.48, W = 60.84, V = 17.0, F = -249.152	(3, 0.94327, 2, 0.89276, 2, 0.95354, 3, 0.89132)*, R_s = 0.98492, C = 312.831, W = 128.727,V = 47.0, F = -149.824*	3.578

*Function values computed using Pareto-Optimum solution vector obtained for each optimization method shown in [4].

The proposed method agrees with the results for single objective optimization shown in [4] and provides slightly better optimum with lower consumption of resources in few cases.

The same algorithm is utilized to generate optimal solutions for multiple objective optimization problem and the results are compared with that generated by goal programming, goal attainment and fuzzy optimization [4], and shown in Table IV.

The starting vectors were chosen where for each stage the number of components and component reliability [4] are chosen between specified maximum and minimum allowable for each optimization run and the scalar multipliers 0.34, 0.33, 0.33 are used for the system reliability, cost and weight functions. A decision vector is said to be Pareto optimal, if it is not dominated by any other decision vectors. The set of all Pareto optimal decision vectors is called the admissible set of the problem. The local optimum obtained from 3^n enumeration in second phase is used as a seed for a new iterative cycle. The optimums found in each cycle are used to find the non-dominated vectors to form set of Pareto vectors. Other starting vectors can be used to generate additional Optimal Pareto vectors. A Pareto optimal vector (2, 0.80, 2, 0.7635, 2, 0.8364, 2, 0.85) providing a system reliability of 0.86223 and minimum cost of 75.0, weight 89.031 and volume 32.0 is the most desirable choice for this problem that has not been reported in literature. Other Pareto optimum vector shown in second column of Table IV is comparable to that reported in [4].

The goal programming formulation of multi-objective optimization using goal for each objective function obtained by minimizing individual objective functions shown in Table III as suggested in [4] were used but the results for optimal allocation remain unchanged from what is shown in Table IV.

IV. Conclusion

In this work an established method developed by the authors for solving integer and mixed-integer problems is utilized to solve multi-objective reliability allocation problems. The results for several test problems for integer and mixed integer problems have been compared and found to agree and in few instances superior to those reported in literature. The optimum result for multi-objective integer problem P1 reported is in fact for single objective reliability function. Once again for mixed allocation problem, reported result in [2] for optimum allocation, is for single objective system reliability. Our results for both problem P1 and P2 presented are superior to those reported in [1], [2]. In case of problem P3 the global optimum shown is the minimum cost and weight function found and lower limit of system reliability is reached irrespective of starting vector used for the search of global optimum. This global optimal solution can be also referred as Pareto optimal solution. In general, however, there exist a number of solutions that can be generated as suggested here using 3^n enumeration of the second phase of algorithm [13], [15] that user can use to select a compromise solution according to their preference. Further work will entail developing set of Pareto optimal solutions from which the system designer can select a superior compromise between mutually conflicting objective functions.

References

[1] Misra, Krishna B., Sharma, Usha, An Efficient Approach for Multiple Criteria Redundancy Optimization Problems, *Microelectronics Reliability*, Vol 31, No.2/3, pp. 303-321, 1991

[2] Misra, Krishna B., Sharma, Usha, Multicriteria Optimization for Combined Reliability and Redundancy Allocation in Systems Employing Mixed Redundancies, *Microelectronics Reliability*, Vol 31, No.2/3, pp. 323-335, 1991

[3] Ravi, V., Reddy, P., Zimmermann, H., Fuzzy Global Optimization of Complex System Reliability, *IEEE Transactions on Fuzzy Systems*, 8, 241-248, 2000.

[4] Dhingra, Anoop K., Optimal Apportionment of Reliability & Redundancy in Series Systems Under Multiple Objectives, IEEE Transactions on Reliability, Vol 41, NO 4, 1992 December.

[5] Rao, S.S., Dhingra, A.K., Reliability and redundancy apportionment using crisp and fuzzy multi objective optimization approaches, Reliability Engineering and System Safety 37 (1992) 253-261.

[6] Taboada, Heidi A., Baheranwala, Fatema, Coit, David W., Wattanapongsakorn, Naruemon, Practical solutions for multi-objective optimization: An application to system reliability design problems, Reliability Engineering and System Safety 92 (2007) 314-322.

[7] Taboada, Heidi A., Coit, David W., Development of a multiple objective genetic algorithm for solving reliability design allocation problems, Proceedings of the 2008 Industrial Engineering Research Conference, J. Fowler and S. Mason, eds.

[8] Kennedy, J. & Eberhart, R. C. (1995), Particle swarm optimization. In IEEE International Conference on Neural Networks (Vol. IV, pp. 1942–1948). Piscataway, NJ, Seoul, Korea.

[9] Garg, H., Sharma, S.P., Multi-objective reliability-redundancy allocation problem using particle swarm optimization, Computers & Industrial Engineering 64 (2013) 247-255.

[10] Zhang E., Wu, Y., Qingwei, C., A practical approach for solving multi-objective reliability redundancy allocation problems using extended bare-bones particle swarm optimization.

[11] Khalili-Damghani Kaveh, Abtahi Amir-Reza, Tavana Madjid, A new multi-objective particle swarm optimization method for solving reliability redundancy allocation problems, Reliability Engineering and System Safety, 111(2013),58-75

[12] Garg, H. Rani, Monica, Sharma, S.P., Vishwakarma, Intuitionistic fuzzy optimization technique for solving multi-objective reliability optimization problems in interval environ,ment. Expert systems with applications, 41 (2014), 3157-3167.

[13] Srivastava V.K., Fahim A, A Two-Phase optimization procedure for integer programming problems, Computers and Mathematics with Applications 42 (2001) 1585-1595

[14] Srivastava Vijaya K., Spinello, Davide, Efficiency of Robust Heuristic Gradient Based Enumerative and Tunneling Algorithms for Constrained Integer Programming Problems, International Conference of Mathematical and Computational Sciences, May 2017, Montreal, Vol 11, No 5, 201-206

[15] Srivastava Vijaya K, Fahim A, An optimization method for solving mixed discrete-continuous programming problems, Computers and Mathematics 53 (2007) 1481-1491.

[16] Bozorg-Haddad Omid, Solgi Mohammad, Loáiciga A. Hugo, Meta-Heuristic and Evolutionary Algorithms for Engineering Optimization, 2017 John Wiley & Sons, Inc.

[17] Klingman W.R., Himmelblau D.M., Nonlinear programming with the aid of a multiple-gradient summation technique, Journal of the Association of Computing Machinery 11 (40), 400-415, October 1964.