

# Multi-criteria Optimization of Square Beam using Linear Weighted Average Model

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**Abstract**—Increasing energy absorption is a significant parameter in vehicle design. Absorbing more energy results in decreasing occupant damage. Limitation of the deflection in a side impact results in decreased energy absorption (SEA) and increased peak load (PL). Hence a high crash force jeopardizes passenger safety and vehicle integrity. The aims of this paper are to determine suitable dimensions and material of a square beam subjected to side impact, in order to maximize SEA and minimize PL. To achieve this novel goal, the geometric parameters of a square beam are optimized using the response surface method (RSM).multi-objective optimization is performed, and the optimum design for different response features is obtained.

**Keywords**—Crashworthiness, side impact, energy absorption, multi-objective optimization, Square beam, SEA

## I. INTRODUCTION

GLOBAL accident statistics demonstrate that nearly 30% of accidents and 35% of fatalities are caused by side impact [1, 2]. Side impact is more significant than frontal impact due to the reduced crash zone.

For this reason thin-walled structures is increasingly used and a lot of research work has been carried out in past decades on the energy absorption of thin-walled structures under loading [3-10]. Kecman conducted experimental and theoretical analysis of the bending performance of rectangular beams. Niknejad [11] studied the fold creation in square columns under axial loading.. The effect of web corrugation under bending was investigated by C. L. Chan et al [12]. However, they have not considered the side impact on a square beam. Most of the research has analyzed the axial crash of a square beam but neglected the lateral crash of a square beam, which is analyzed in this research. Langseth et al. [13, 14] studied local buckling and the crush behavior of square beams.

Finding the optimum point, considering maximum SEA and minimum PL with respect to their simultaneous limitation of deflection, is a major challenge. This optimum design point is critically important for vehicle components subjected to side impact. Meanwhile, a conflict between the criteria for these objectives is inevitable.

This paper aims to present optimization method to find the optimum point. The modeling, meshing and crash analysis were done using the LS-DYNA suite of programs, and at a crash speed of 5 m/s.

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The thickness of the square beam is 1 mm. Figure 1 shows the dimensions of the structure and the condition of the impactor.

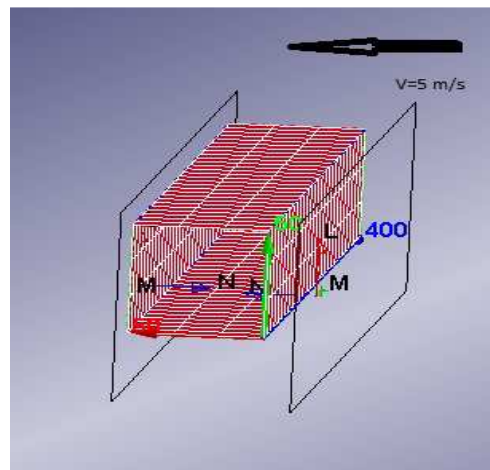


Fig. 1 Mapping nonlinear data to a higher dimensional feature space

Two steps in this research are considered. In the first step, the effects of steel and aluminum alloys are investigated to find the maximum SEA with reasonable deflection.. In the second step, to choose the optimum structure design, the optimization method is investigated. This optimum design should result in the maximum SEA and minimum impact force simultaneously, considering the limitations of deflection.

## II. SPECIFIC ENERGY ABSORPTION

The energy  $E$  which is absorbed by the objects during the collision can be obtained from the following Equations:

$$E = \int_v A(\epsilon) dv \quad (1)$$

where  $A(\epsilon)$  implies the total strain energy density of the corresponding structure. The specific energy absorption (SEA), which is the energy absorbed per unit mass of the structure part, can be defined by:

$$SEA = \frac{E_{total}}{M} \quad (2)$$

where  $E_{total}$  is the total energy and  $M$  is the mass of the corresponding structure under impact

## III. FINITE ELEMENT MODELING

The CAD data of the square beam is modelled, meshed and simulated using LS-DYNA 3.1 Beta software from LSTC Co. In the analysis, the square beam is constrained with a rigid wall on one side, while the other side is impacted by a rigid wall of 10 kg mass moving with a constant velocity of 5 m/s.

The four- node quadrilateral element (Belytschko-Tsay) is chosen because of its appropriate application in shell elements with the formulation of 3 integration points to mesh the model [15].

#### IV. MATERIALS PROPERTIES

The properties of aluminium, steel and magnesium are assigned to the square beam. The mechanical properties of the materials are given in Table I.

TABLE I  
UNITS FOR MAGNETIC PROPERTIES

Material types	E (Gpa)	Poisson's Ratio	Yield stress(Mpa)	Ultimate stress(Mpa)	Strain at failure	Density (kg/m <sup>3</sup> )
Aluminum 3105-H18	68.94	0.33	193	214	0.03	2720
Magnesium AZ31B	45	0.35	190	275	0.1	1740
Steel AISI1006	200	0.3	190	320	0.3	7860

#### V. THE EFFECT OF MATERIALS ON CRASHWORTHINESS

Fig. 2 shows the lateral deflection for the square beam made of different materials. The maximum deflection occurs for aluminum 3105 with deflection of 16 mm and 5 mm for aluminum 2011. The minimum deflection occurs to the steel due to its high rigidity compared with the aluminum alloys. Fig. 3 shows a comparison of the SEA for each material. It can be seen that the maximum SEA occurs with aluminum 3105, which is about 5.47276(N.mm/ton E+8). However, the amount of deflection for aluminum 3105 is high. Thus, aluminum 2011 is a good choice considering the less deflection compared with aluminum3105.in addition, the amount of SEA for aluminum 2011 is about 5.15484(N.mm/ton E+8) which is reasonable.Fig4 shows impact forces for these three material. it is observed that the maximum force belongs to the steel which is about 320434 N.

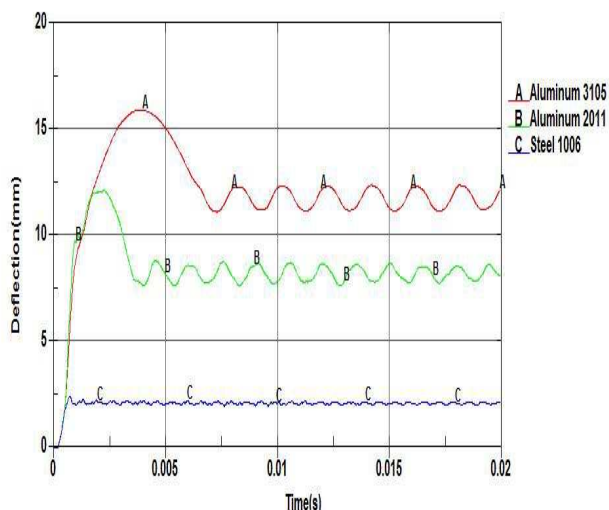


Fig. 2 deflection of aluminum alloys and steel beam

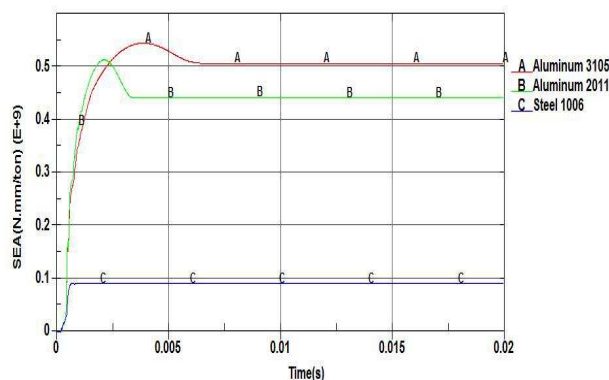


Fig. 3 SEA for aluminum alloys and steel beam square

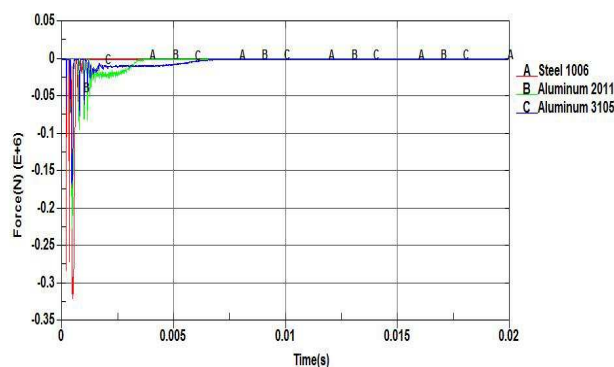


Fig. 4 Impact force for aluminum alloys and steel beam square

#### VI. PERFORMANCE OF SQUARE BEAM FOR DIFFERENT GEOMETRY

In this step the effect of changing geometry such as thickness and dimension of square beam on the two parameters of SEA and PL are investigated. The results are shown in Table II.

TABLE II  
THE RESULT OF SEA AND PL

L(1) (mm)	L(2) (mm)	T (mm)	Mass(kg) E-4	SEA E+8(N.mm/ton)	PL (N)
50	50	1	2.26	5.15484	224945
50	50	0.8	1.81	6.54764	191935
50	50	0.6	1.35	8.68824	154678
50	50	0.4	.905	12.7332	108661
46	46	1	2.08	5.68518	207254
46	46	0.8	1.66	7.06207	173154
46	46	0.6	1.24	9.71116	156773
46	46	0.4	.833	13.8507	102317
42	42	1	1.9	6.12331	202121
42	42	0.8	1.52	7.62568	138150
42	42	0.6	1.14	10.0884	132121
42	42	0.4	.76	15.1942	130655
38	38	1	1.72	7.05006	190006
38	38	0.8	1.37	8.83633	178585
38	38	0.6	1.03	11.6397	146218
38	38	0.4	.688	16.4822	101925

#### VII. OPTIMIZATION PROBLEM DESCRIPTION

Structural optimization techniques have been used recently for optimizing the energy absorption and peak load of structures under impact.

There are a number of methods for optimization. The response surface method (RSM) is one of the methods most commonly used for crashworthiness optimization [16-20]. Yamazaki, Lee [21] and Allahbakhsh [22] have applied an RSM method for crashworthiness optimization. In this paper, for optimizing specific energy absorption and peak load, multi-objective optimization is applied. In the present paper, RSM as described by [23] is used and is described in this section.

Multi-objective optimization can be formulated in two different ways, one of which is the linear weighted average as given in Equation (3):

$$\left\{ \begin{array}{l} \text{Minimize } F_w = (1-w) \frac{f_1^*}{f_1} + w \frac{f_2^*}{f_2} \\ w \in [0,1] \text{ and } x^L \leq x \leq x^U \end{array} \right. \quad (3)$$

where  $f_1^*, f_2^*$  are the normalizing values of  $f_1 = SEA(x)$  and  $f_2 = PL(x)$  respectively [24-26].  $w$  is the weight factor for emphasizing the different importance of each of the objectives.

#### VIII. RESPONSE SURFACE MODEL

In this paper the second order polynomial function is used for  $SEA(x)$  and  $PL(x)$  and these can be expressed as Equations (4) and (5) respectively.

$$SEA = 1.079 \times 10^9 + 1.8 \times 10^8 x(1) - 7.953 \times 10^9 x(2) - 3.77 \times 10^6 x(1)^2 - 1.55 \times 10^8 x(1)x(2) + 9.60 \times 10^7 x(2)^2 \quad (4)$$

$$PL = 5.39 \times 10^5 - 2.06 \times 10^4 x(1) - 4.614 \times 10^4 x(2) + 2125 x(1)^2 - 474 x(1)x(2) - 5828 x(2)^2 \quad (5)$$

Where  $x(1)$  and  $x(2)$  are the dimension and thickness of structure respectively.

#### IX. DESIGN OPTIMIZATION RESULT

By varying weight  $w$  in Equation (3), the Pareto sets for the square beam are obtained as plotted in Fig. 4. The Pareto front provides a range of optimal solutions. The Pareto plot shows the relation between SEA and PL and any further improvement in SEA must sacrifice the PL and vice versa. In fact, any point in the Pareto frontier can be an optimal point, meaning that it is up to the designer to determine which factor is more important. For generating the Pareto frontier, the Genetic Algorithm (GA) multi-objective optimization solver of MATLAB is used.

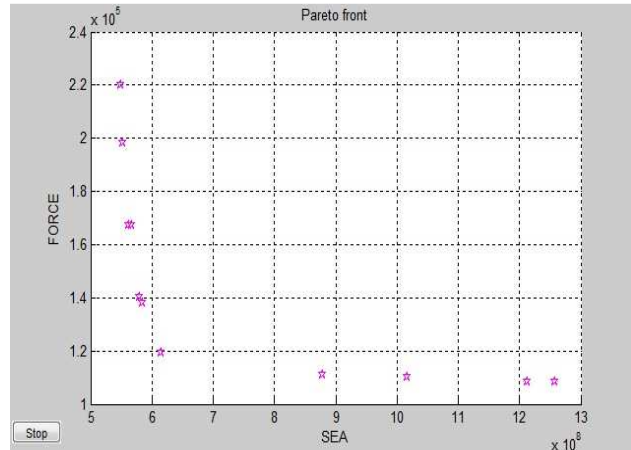


Fig. 4 pareto frontier graph

#### X. CONCLUSION

From the results obtained and the discussion presented, the following conclusions are made:

- Analyzing the effect of material on crashworthiness leads to choose aluminum 2011 due to its reasonable SEA and deflection compared to steel and aluminum 3105.
- The multi-objective Pareto graph enables the designer to make a better decision on the design point. Having various optimum points based on two contrary objectives (SEA, PL) enables the designer to have a group of solutions to find the optimum point, which is considered to be the maximum SEA and minimum PL with respect to deflection.

#### REFERENCES

- Dong G, Wang D, Zhang J, Huang S. Side structure sensitivity to passenger car crashworthiness during pole side impact. *Tsinghua Science & Technology*. 2007;12:290-5.
- Fildes B, Bostrom O, Haland Y, Sparke L. COUNTERMEASURES TO ADDRESS FAR-SIDE CRASHES: FIRST RESULTS. 2003.
- Abramowicz W, Wierzbicki T. Axial crushing of foam-filled columns. *International Journal of Mechanical Sciences*. 1988;30:263-71.
- Kecman D. Bending collapse of rectangular and square section tubes. *International Journal of Mechanical Sciences*. 1983;25:623-36.
- Kim T, Reid S. Bending collapse of thin-walled rectangular section columns. *Computers & Structures*. 2001;79:1897-911.
- Mamalis A, Manolacos D, Ioannidis M, Kostazos P. Bending of cylindrical steel tubes: numerical modelling. *International Journal of Crashworthiness*. 2006;11:37-47.
- Zhang Z, Liu S, Tang Z. Design optimization of cross-sectional configuration of rib-reinforced thin-walled beam. *Thin-Walled Structures*. 2009;47:868-78.
- Wierzbicki T, Abramowicz W. On the crushing mechanics of thin-walled structures. *Journal of Applied mechanics*. 1983;50:727.
- Langseth M, Hopperstad O. Static and dynamic axial crushing of square thin-walled aluminium extrusions. *International Journal of Impact Engineering*. 1996;18:949-68.
- Wierzbicki T, Recke L, Abramowicz W, Gholami T, Huang J. Stress profiles in thin-walled prismatic columns subjected to crush loading-II. Bending. *Computers & Structures*. 1994;51:625-41.
- Niknejad A, Liaghat G, Naeini HM, Behraves A. Experimental and theoretical investigation of the first fold creation in thin walled columns. *Acta Mechanica Sinica*. 2010;23:353-60.
- Chan C, Khalid Y, Sahari B, Hamouda A. Finite element analysis of corrugated web beams under bending. *Journal of Constructional Steel Research*. 2002;58:1391-406.
- Langseth M, Hopperstad O, Hanssen A. Crash behaviour of thin-walled aluminium members. *Thin-Walled Structures*. 1998;32:127-50.

- [14] Langseth M, Hopperstad O. Local buckling of square thin-walled aluminium extrusions. *Thin-Walled Structures*. 1997;27:117-26.
- [15] Halquist J. LS-DYNA keyword user's manual version 971. Livermore Software Technology Corporation, Livermore, CA. 2007.
- [16] Salehghaffari S, Rais-Rohani M, Najafi A. Analysis and optimization of externally stiffened crush tubes. *Thin-Walled Structures*. 2011.
- [17] Hou S, Li Q, Long S, Yang X, Li W. Design optimization of regular hexagonal thin-walled columns with crashworthiness criteria. *Finite elements in analysis and design*. 2007;43:555-65.
- [18] Acar E, Rais-Rohani M. Ensemble of metamodels with optimized weight factors. *Structural and Multidisciplinary Optimization*. 2009;37:279-94.
- [19] Xiang Y, Wang Q, Fan Z, Fang H. Optimal crashworthiness design of a spot-welded thin-walled hat section. *Finite elements in analysis and design*. 2006;42:846-55.
- [20] Yamazaki K, Han J. Maximization of the crushing energy absorption of cylindrical shells. *Advances in Engineering Software*. 2000;31:425-34.
- [21] Lee TH, Lee K. Multi-criteria shape optimization of a funnel in cathode ray tubes using a response surface model. *Structural and Multidisciplinary Optimization*. 2005;29:374-81.
- [22] Allahbakhsh H, Saemi J. Design optimization of square and circular aluminium extrusion damage columns with crashworthiness criteria. *Indian Journal of Engineering & Materials Sciences*. 2011;18:341-50.
- [23] Montgomery DC. *Design and analysis of experiments*: John Wiley & Sons Inc; 2008.
- [24] Fang H, Rais-Rohani M, Liu Z, Horstemeyer M. A comparative study of metamodeling methods for multiobjective crashworthiness optimization. *Computers & Structures*. 2005;83:2121-36.
- [25] Zarei H, Kröger M. Multiobjective crashworthiness optimization of circular aluminum tubes. *Thin-Walled Structures*. 2006;44:301-8.
- [26] Hou S, Li Q, Long S, Yang X, Li W. Multiobjective optimization of multi-cell sections for the crashworthiness design. *International Journal of Impact Engineering*. 2008;35:1355-67.