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MPSO based Model Order Formulation Scheme for Discrete PID Controller Design

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***Abstract*—**This paper proposes the novel model order formulation scheme to design a discrete PID controller for higher order linear time invariant discrete systems. Modified PSO (MPSO) based model order formulation technique has used to obtain the successful formulated second order system. PID controller is tuned to meet the desired performance specification by using pole-zero cancellation and proposed design procedures. Proposed PID controller is attached with both higher order system and formulated second order system. System specifications are tabulated and closed loop response is observed for stabilization process. The proposed method is illustrated through numerical examples from literature.

***Keywords*—**Discrete PID controller, Model Order Formulation, Modified Particle Swarm Optimization, Pole-Zero Cancellation

I. INTRODUCTION

DURING the past decades, the process control techniques in the industry have made great advances. Numerous control methods such as Neuro-fuzzy control [1], Fuzzy logic control [2] and also Genetic algorithm based control [3] has been studied. Among them, the best known is the proportional-integral-derivative (PID) controller, which has been widely used in the industry because of its simple structure and robust performance in a wide range of operating conditions. It has been a crucial problem to tune properly the gains of the PID controller because many industrial plants are often burdened with the characteristics such as higher order, time delay and nonlinearities [4].While modeling the complex systems like space vehicle mechanism, fuel injector and spark timing of auto mobiles it can be noted that the system order is increased. The analysis and synthesis of higher order systems are difficult and generally not desirable on economic and computational considerations. Thus, it is necessary to obtain a lower order model so that the obtained lower order maintains the characteristics of the original system. This helps in minimizing the variations during design and realization of suitable control system components to be attached to the original system. The computational and implementation difficulties involved in design of optimal and adaptive controller for higher order linear time invariant continuous system can also be minimized with the help of suitable reduced order models.Model order formulation is the process of deriving the lower order model from the higher order model. Model order formulation approximates the complex system by simple one. The main aim of the formulation is to find the best possible approximation of the output of the original system.

During the past four decades, numerous impressive varieties of new techniques [5] - [8] have been developed for obtaining lower order models from higher order linear system. Each of these methods have both advantages and disadvantages when tried on a particular system. Several methods have been developed for designing a PID controller. The first method used the classical tuning rules proposed by Ziegler and Nichols. In general, it is often hard to determine optimal or near optimal PID parameters with the Ziegler-Nichols formula in many industrial plants. Tschanner [9] has proposed the Jury stability conditions derived from Routh and Fuller tables. Yeung [10] has investigated the graphical procedure for selecting the parameters of the PID controllers for a given linear system. Zhuang et al., [11] have proposed the analytical procedures for obtaining optimum PID controller settings for minimization of time weighted integral performance criteria. Various methods are developed by employing frequency response matching techniques for designing the controllers.

Rattan et al., [12] proposed a method based on complex curve fitting and involves the matching of frequency response of closed loop system with the reference model. The digital controller design method proposed by Inooka et al., [13] is based on series expansion of pulse transfer function. Aguirre [14] introduced a method for the design of continuous time controllers by matching a combination of time moments and Markov parameters of the closed loop system. The main purpose of the approach is to reduce the excessive overshoot of the system to be compensated. To enhance the capabilities of traditional PID parameter tuning techniques, several intelligent approaches have been suggested to improve the PID tuning, such as those using Genetic Algorithms (GA) [15] and the Particle Swarm Optimization (PSO) [16]. With the advance of computational methods in the recent times, optimization algorithms are often proposed to tune the control parameters in order to find an optimal performance [17].

In this paper a simple algebraic scheme is proposed to design a PID controller for Linear Time Invariant discrete System (LTIDS). Adjunct Polynomial scheme is used for deriving the basic second order system from the original higher order system, and to obtain a fine tuned second order system depicting the original characteristics of the system, Modified Particle Swarm Optimization (MPSO) algorithm is proposed. Pole-zero cancellation method is employed for initialize the PID gain values. Matlab simulation procedures are used to obtain the optimal PID gain values. The robustness of the proposed scheme is compared with general PSO based formulated second order model.

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II. Description of The Problem

A. PID Controller Transfer Function

The standard block diagram of PID controller is shown in Fig.1. PID controller can be mathematically represented as [18],

$$u(t) = K_P \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{d(t)} \right] \tag{1}$$

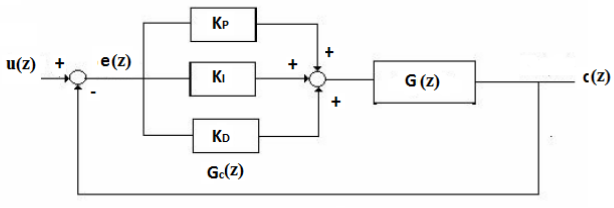

Fig. 1 General block diagram of discrete PID controller

Where $u(t)$ and $e(t)$ denotes the control and error signals of the system. K_P is the proportion gain, T_i and T_d represents the integral and derivative time constants respectively. The corresponding PID controller transfer function $G_c(z)$ is given as [19],

$$G_c(z) = \frac{\left(K_P + \frac{K_D}{T} + TK_I\right)z^2 - \left(K_P + \frac{2K_D}{T}\right)z + \frac{K_D}{T}}{z(z-1)} \tag{2}$$

K_P, K_I and K_D are represents the proportional, integral and derivative gain values of the controller.

B. Higher Order Transfer Function

Consider an n$^{\text{th}}$ order linear time invariant discrete system represented by,

$$G(z) = \frac{N(z)}{D(z)} = \frac{\sum_{i=0}^{n-1} A_i z^i}{\sum_{i=0}^{n} a_i z^i} \tag{3}$$

Where, *N(z)* is the numerator polynomial and *D(z)* is the denominator polynomial. Also A_i and a_i represents the constant coefficients of the z-terms of the numerator and denominator of *G (z)*. Equation (3) represented the higher order discrete system transfer function.

C. Lower Order Transfer Function

To find a m^{th} lower order model for the discrete system $R^m(z)$, where $m < n$ in the following form represented by (4) , such that the formulated lower order model retains the characteristics of the original system and approximates its response as closely as possible for the same type of inputs.

$$R^m(z) = \frac{N^m(z)}{D^m(z)} = \frac{\sum_{i=0}^{m-1} B_i z^i}{\sum_{i=0}^{m} b_i z^i} \tag{4}$$

Where, $N^m(z)$ and $D^m(z)$ are the numerator polynomial and denominator polynomial of the formulated lower order model respectively. Also B_i and b_i represent the constant coefficients of the z-terms of the numerator and denominator of $R^m(z)$. Equation (4) represented the lower order transfer function.

The main objective of the design is that to tune the gains (K_P , K_I and K_D) of the PID controller for a desired output. For reduce the computational complexities and difficulties of implementation, the higher order of the system is reduced into lower second order system. PID controller is tuned with respect to the design specification for a formulated second order model. Further the closed loop response of the new lower order model attached with PID controller is obtained, which depict the characteristics of the original higher order system response with PID controller.

III. Overview of Particle Swarm Optimization

The particle swarm optimization (PSO) technique appeared as a promising algorithm for handling the optimization problems. PSO is a population-based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling [20]. PSO is inspired by the ability of flocks of birds, schools of fish, and herds of animals to adapt to their environment, find rich sources of food, and avoid predators by implementing an information sharing approach. PSO technique was invented in the mid 1990s while attempting to simulate the choreographed, graceful motion of swarms of birds as part of a socio cognitive study investigating the notion of collective intelligence in biological populations.

The velocity of a particle is influenced by three components namely, inertial momentum, cognitive and social. The inertial component simulates the inertial behavior of the bird to fly in the previous direction. The cognitive component models the memory of the bird about its previous best position, and the social component models the memory of the bird about the best position among the particles. Mathematical model for PSO is as follows [20],

Velocity update equation is given by

$$V_{i+1} = \omega \times V_i + C_1 \times r_1 \times (P_{best_i} - S_i) + C_2 \times r_2 \times (g_{best_i} - S_i) \tag{5}$$

Position update equation is given by

$$S_{i+1} = S_i + V_{i+1} \tag{6}$$

Each particle tries to modify its velocity and position and based on (5) and (6) and reaches the target.

Where,

V_i = Velocity of particle
S_i = Current position of the particle
ω = Inertia weight
C_1 = Cognition acceleration coefficient
C_2 = Social acceleration coefficient
$Pbest_i$ = Own best position of particle
$gbest_i$ = Global best position among the group of particles
r_1, r_2 = Uniformly distributed random numbers in the range [0 to 1]

IV. MODIFIED PARTICLE SWARM OPTIMIZATION

In this new proposed modified PSO having better optimization result compare to general PSO by splitting the cognitive component of the general PSO into two different component. The first component can be called good experience component. This means the bird has a memory about its previously visited best position. This is similar to the general PSO method. The second component is given the name by bad experience component. The bad experience component helps the particle to remember its previously visited worst position. To calculate the new velocity, the bad experience of the particle also taken into consideration [21].

The new velocity update equation is given by

$$V_{i+1} = \omega \times V_i + C_{1g} \times r_1 \times (P_{best_i} - S_i) + C_{1b} \times r_2 \times (S_i - P_{worst_i}) + C_2 \times r_3 \times (g_{best_i} - S_i) \quad (7)$$

Where,

C_{1g} = Acceleration coefficient, which accelerate the particle towards its best position
C_{1b} = Acceleration coefficient, which accelerate the particle away from its worst position
$P_{worst\,i}$ = Worst position of the particle i
r_1, r_2, r_3 = Uniformly distributed random numbers in the range [0 to 1]

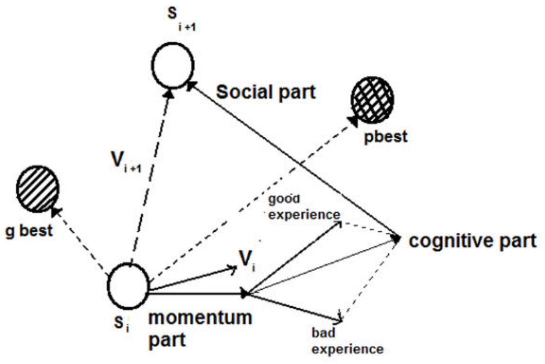

Fig. 2 Concept of Modified PSO searching point

Fig. 2 shows the searching behavior of Modified PSO approach. The inclusion of the worst experience component in the behavior of the particle gives the additional exploration capacity to the swarm. By using the bad experience component; the particle can bypass its previous worst position and try to occupy the better position.

The algorithmic steps for the modified PSO is as follows

Step 1 Select the number of particles, generations, tuning accelerating coefficients *C_{1g}*, *C_{1b}*, and *C_2* and random numbers *r_1*, *r_2*, *r_3* to start the optimal solution searching
Step 2 Initialize the particle position and velocity
Step 3 Select particles individual best value for each generation
Step 4 Select the particles global best value, i.e. particle near to the target among all the particles is obtained by comparing all the individual best values
Step 5 Select the particles individual worst value, i.e. Particle too away from the target
Step 6 Update particle individual best *(pbest),* global best *(gbest),* particle worst *(Pworst)* in the velocity equation (7) and obtain the new velocity
Step 7 Update new velocity value in the equation (6) and obtain the position of the particle
Step 8 Find the optimal solution with minimum ISE by the updated new velocity and position

V. STEPS FOR MPSO BASED MODEL ORDER FORMULATION TECHNIQUE

A. Adjunct Polynomial Scheme

The adjunct polynomial scheme is used to obtain the approximate second order model for the given higher order system. This scheme has the following steps

Step 1 Consider an n^th^ order linear time invariant discrete system represented by the transfer function G(z) in general form as,

$$G(z) = \frac{N(z)}{D(z)} = \frac{A_{n-1}z^{n-1} + A_{n-2}z^{n-2} + \ldots + A_1 z + A_0}{a_n z^n + a_{n-1}z^{n-1} + \ldots + a_2 z^2 + a_1 z + a_0} \quad (8)$$

Step 2 Calculate the transient gain (TG) and steady state gain (SSG) for the given higher order system in equation (8)

$$TG = \frac{A_{n-1}}{a_n} \quad (9)$$

$$SSG = G(z)\big|_{z=1} \quad (10)$$

Step 3 For simplicity, the approximate lower order model to be formulated using adjunct polynomial method is given by

$$R(z) = \frac{A_1 z + A_0}{a_2 z^2 + a_1 z + a_0} \quad (11)$$

Step 4 Scaling the equation (11),

$$R(z) = \frac{z + \left(\frac{A_0}{A_1}\right)}{z^2 + \left(\frac{a_1}{a_2}\right)z + \left(\frac{a_0}{a_2}\right)} \quad (12)$$

Step 5 To maintain the TG and SSG using the equations (9) and (10) in equation (12)

$$R(z) = \frac{(TG)z + \left[\left(\frac{A_0}{A_1}\right)\right]}{z^2 + \left(\frac{a_1}{a_2}\right)z + \left(\frac{a_0}{a_2}\right)} \quad (13)$$

Step 6 The coefficients of the approximated second order model R(z) by equation (13) as give as input to modified PSO. The MPSO used to search the better value of (A_0 / A_1), (a_1 / a_2) and (a_0 / a_2)

B. *Modified Particle Swarm Optimization*

Modified PSO algorithmic steps are applied after the approximate second order model R(z) obtained, shown in the equation (13), by using the modified particle swarm optimization algorithm the formulated lower order model is achieving the objective minimum ISE and follow the constraints.

VI. GENERAL ALGORITHM FOR DESIGNING THE PID CONTROLLER

Step 1 Read the open loop transfer function of the given higher order system
Step 2 Form the closed loop transfer function
Step 3 Obtain the step response of the closed loop system
Step 4 Check the response for the required specifications.
Step 5 If the specifications are not met, get the reduced order model by using proposed MPSO based formulation technique and design a controller for the reduced order model.
Step 6 Obtain the initial values of the parameters K_P, K_I and K_D by pole zero cancellation.
Step 7 Cascade the controller with the reduced order model and get the closed loop response with the initial values of the controller parameters.
Step 8 Find the optimum values for the controller parameters which satisfy the required specifications
Step 9 By applying the optimum values, cascade this controller with the original system.
Step 10 Obtain the closed loop response of the reduced order system with the controller.
Step 11 Obtain the closed loop response of the original system with the controller.

VII. NUMERICAL EXAMPLE

A. *Illustration 1*

Let us consider linear time invariant discrete system represented in the form of transfer function given in [22] as,

$$G(z)=\frac{0.1625\,z^{7}+0.125\,z^{6}-0.0025\,z^{5}+0.00525\,z^{4}-0.022625\,z^{3}-0.000875\,z^{2}+0.003\,z-0.0004125}{z^{8}-0.63075\,z^{7}-0.4185\,z^{6}+0.07875\,z^{5}-0.057\,z^{4}+0.1935\,z^{3}+0.09825\,z^{2}-0.0165\,z+0.00225} \tag{14}$$

Step-1

Calculate the transient gain (TG) and steady state gain (SSG) for the given higher order system in (14).

$$TG=\frac{0.1625}{1}=0.1625$$
$$SSG=G(z)\big|_{z=1}=1.0772 \tag{15}$$

Step- 2

Applying Adjunct polynomial scheme, [Appendix] to G(z) in (14) to get approximated second order model R(z),

$$R(z)=\frac{0.003z-0.0004125}{0.09825z^{2}-0.0165z+0.00225} \tag{16}$$

Step-3

On scaling (16),

$$R(z)=\frac{z-0.1375}{z^{2}-0.1679z+0.0229} \tag{17}$$

Step-4

To maintain TG, use the Equation (13) the result R(z) becomes

$$R(z)=\frac{0.1625z-0.1375}{z^{2}-0.1679z+0.0229} \tag{18}$$

Step-5

The MPSO algorithm is now invoked to search the values of *'z'* term (0.1679), constant terms (0.0229) and (0.1375) in R(z) represented by (18), so the characteristics of second order model matches the given higher order system given by (14). MPSO determines a better reduced second order model with the least integral square error. The transfer function of the reduced second order model obtained using MPSO scheme is,

$$R(z)=\frac{0.01625z-0.08386}{z^{2}-1.702z+0.7733} \tag{19}$$

Step-6

Performance specifications are considered with respect to the closed loop response of the compensated system to unit step input. The design specifications are chosen as

(i) Overshoot $\leq 1\%$
(ii) Settling time ≤ 1 seconds
(iii) Overshoot $\leq 1\%$

Step-7

The closed loop transfer function of the unity feedback system with G(s) can be represented as,

$$T(z)=\frac{G(z)}{1+G(z)} \tag{20}$$

the output response of T(z) is not stable within the specified design specification. So the PID controller is cascaded to the forwarded path to adjust the response.

Step-8

Applying pole- zero cancellation method to initialize the (K_P, K_I and K_D) values as, $K_P = 0.1554$, $K_I = 2.852$ and $K_D = 0.0193$

Step-9

Using the simulation procedure the initial parameters are tuned to get unit response of the compensated system to meet the required specification are, $K_P = -4.9498$, $K_I = 2.0201$ and $K_D = 3.1991$. The transfer function of the designed PID controller is as follows,

$$G_c(z) = \frac{3.1991z^2 - 4.9498z + 2.0201}{z^2 - z} \qquad (21)$$

Step-10

The closed loop transfer function of the PID controller represented by $G_c(z)$ in equation (21) is attached to the second order model represented by R(z) in equation (19) and $T_c(z)$ is obtained as,

$$T_c(z) = \frac{0.5199z^3 - 1.073z^2 + 0.7434z - 0.1694}{z^4 - 2.182z^3 + 1.403z^2 - 0.02994z - 0.1694} \qquad (22)$$

Step-11

The closed loop transfer function of the PID controller represented by $G_c(z)$ in equation (21) is attached to the original higher order system represented by G(z) in equation (14) and $T_c^{'}(z)$ is obtained as,

$$T_c^{'}(z) = \frac{\begin{array}{c}0.5199z^9 - 0.4045z^8 - 0.2985z^7 + 0.2817z^6 - 0.1034z^5 + \\ 0.1198z^4 - 0.03178z^3 - 0.01794z^2 + 0.00812z - 0.0008333\end{array}}{\begin{array}{c}z^{10} - 1.111z^9 - 0.1922z^8 + 0.1988z^7 + 0.1459z^6 + 0.1471z^5 + \\ 0.02455z^4 - 0.1465z^3 + 0.0008134z^2 + 0.005852z - 0.0008333\end{array}} \qquad (23)$$

The unit time responses of G(z), $T_c(z)$ and $T_c^{'}(z)$ are represented by equations (14), (22) and (23) are shown in Fig.3, Fig.4, and Fig.5 respectively. The comparison of the unit time response specifications are given in Table 1.

B. Illustration 2

Consider an eighth order system represented by its transfer function [23] given in Eq. (24) as,

$$G(z) = \frac{1.682z^7 + 1.116z^6 - 0.21z^5 + 0.152z^4 - 0.516z^3 - 0.262z^2 + 0.044z - 0.018}{8z^8 - 5.046z^7 - 3.348z^6 + 0.63z^5 - 0.456z^4 + 1.548z^3 + 0.786z^2 - 0.132z + 0.018} \qquad (24)$$

Step-1

Calculate the transient gain (TG) and steady state gain (SSG) for the given higher order system in eq. (24).

$$TG = \frac{1.682}{8} = 0.21025$$

$$SSG = G(z)\big|_{z=1} = 0.994 \qquad (25)$$

Step- 2

Applying Adjunct polynomial scheme, [Appendix] to G(z) in (24) to get approximated second order model R(z),

$$R(z) = \frac{0.0442z - 0.018}{0.786z^2 - 0.132z + 0.018} \qquad (26)$$

Step-3

On scaling (26),

$$R(z) = \frac{z - 0.40909}{z^2 - 0.1679z + 0.0229} \qquad (27)$$

Step-4

To maintain TG, use the Equation (13) the result R(z) becomes

$$R(z) = \frac{0.21025z - 0.40909}{z^2 - 0.1679z + 0.0229} \qquad (28)$$

Step-5

The MPSO algorithm is now invoked to search the values of *'z'* term (0.1679), constant terms (0.0229) and (0.40909) in R(z) represented by (28), so the characteristics of second order model matches the given higher order system given by (24). MPSO determines a better reduced second order model with the least integral square error. The transfer function of the reduced second order model obtained using MPSO scheme is,

$$R(z) = \frac{0.2102z - 0.1354}{z^2 - 1.735z + 0.8118} \qquad (29)$$

Step-6

After choose the design specifications, pole-zero cancellation method is applied to initialize the (K_P, K_I and K_D) values as, K_P= 0.1114, K_I= 3.072 and K_D=0.0202

Step-7

Using the simulation procedure the initial parameters are tuned to get unit response of the compensated system to meet the required specification are, K_P = -4.8759, K_I = 2.1355 and K_D =3.0195. The transfer function of the designed PID controller is as follows,

$$G_c(z) = \frac{3.0195z^2 - 4.8759z + 2.1355}{z^2 - z} \qquad (30)$$

Step-8

The closed loop transfer function of the PID controller represented by $G_c(z)$ in equation (30) is attached to the second order model represented by R(z) in equation (29)and $T_c(z)$ is obtained as,

$$T_c(z) = \frac{0.6347z^3 - 1.434z^2 + 1.109z - 0.2891}{z^4 - 2.1z^3 + 1.113z^2 + 0.2973z - 0.2891} \qquad (31)$$

Step-9

The closed loop transfer function of the PID controller represented by $G_c(z)$ in equation (30) is attached to the original higher order system represented by G(z) in equation (24) and $T_c^{'}(z)$ is obtained as,

$$T_c^{'}(z) = \frac{\begin{array}{c}3.527z^9 - 2.326z^8 - 3.581z^7 + 3.866z^6 - 2.748z^5 + \\ 2.049z^4 + 0.3084z^3 - 0.8284z^2 + 0.1817z - 0.03844\end{array}}{\begin{array}{c}8z^{10} - 9.519z^9 - 0.6283z^8 + 0.3971z^7 + 2.78z^6 - 0.7437z^5 + \\ 1.287z^4 - 0.6096z^3 - 0.6784z^2 + 0.1637z - 0.03844\end{array}} \qquad (32)$$

The unit time responses of G(z), $T_c(z)$ and $T_c^{'}(z)$ are represented by equations (24), (31) and (32) are shown in Fig.6, Fig.7, and Fig.8 respectively. The comparison of the unit time response specifications are given in Table II.

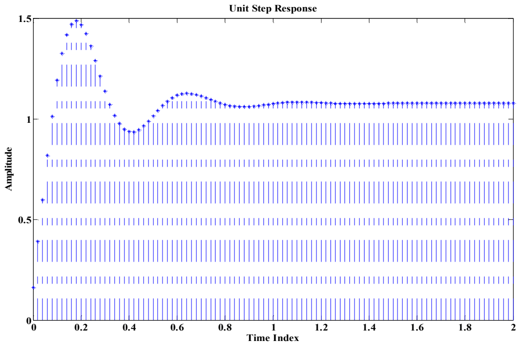

Fig. 3 Unit step response of higher order system for illustration 1

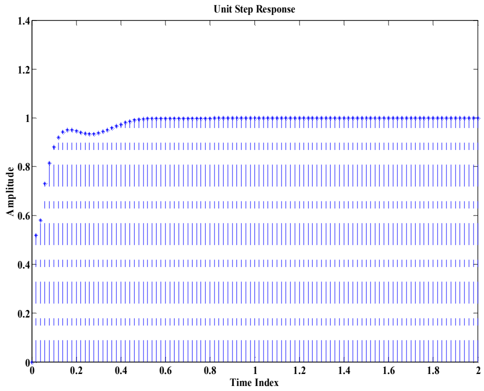

Fig. 4 Unit step response of formulated second order system with proposed PID controller for illustration 1

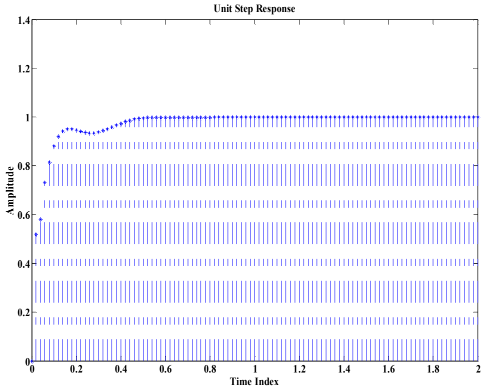

Fig. 5 Unit step response of higher order system with proposed PID controller for illustration 1

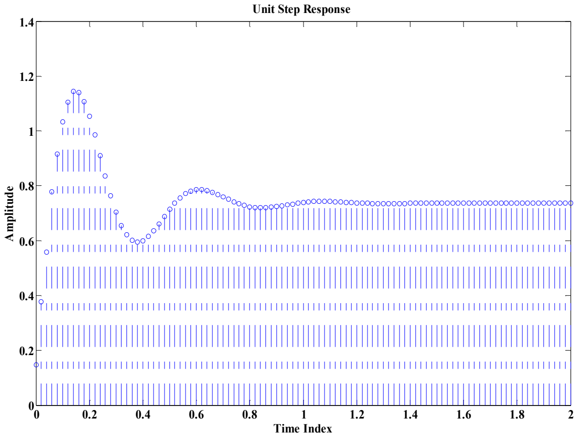

Fig. 6 Unit step response of higher order system for illustration 2

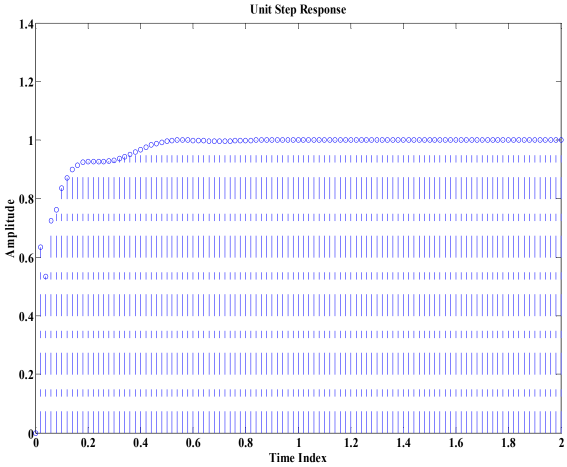

Fig. 7 Unit step response of formulated second order system with proposed PID controller for illustration 2

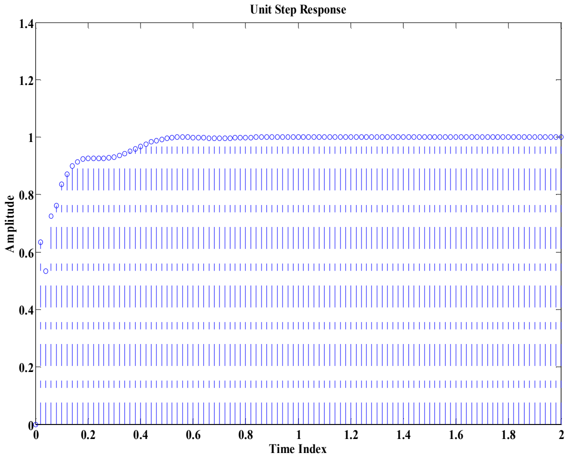

Fig. 8 Unit step response of higher order system with proposed PID controller for illustration 2

Table I
Comparison of Unit Time Response Specifications for Illustration 1

Specification details	Original system	Reduced order system using general PSO	Reduced order system using MPSO	Original system with proposed PID controller	MPSO based reduced order system with proposed PID controller
Rise time (sec)	0.0411	0.0014	0.0475	0.0451	**0.0529**
Settling time (sec)	0.3760	0.0559	0.2776	0.2044	**0.2084**
% Peak overshoot	38.0147	119.18	22.8671	0.0963	**0**
Steady state error	0.4813	1.0143	0.4755	0.5	**0.5**

Table II
Comparison of unit time response specifications for illustration 2

Specification details	Original system	Reduced order system using general PSO	Reduced order system using MPSO	Original system with proposed PID controller	MPSO based reduced order system with proposed PID controller
Rise time (sec)	0.0297	0.0035	0.0354	0.0667	**0.0683**
Settling time (sec)	0.4519	0.0719	0.3569	0.2495	**0.2160**
% Peak overshoot	55.0933	56.0359	38.8542	0	**0.0241**
Peak time (sec)	0.5756	1.3244	0.5065	0.5	**0.5**

VIII. Discussion

During MPSO algorithm process various parameters are used as, number of generations=100, number of particles=75. For successfully implement the MPSO, the values of the parameters inertia weight (w) = 0.5, accelerating factors C_{1g} ,C_{1b} and C_2 are choose by 0.5 .The MPSO algorithm was coded in Intel Pentium processor 4.0, 2.8 GHz, 256 MB RAM and it took 200 seconds by CPU for complete the simulation. Fig.3 and Fig.6 gives the unit step time response of the given eight order linear time invariant discrete systems, it exceed the desired design specifications. For achieving the desired output response without computational difficulties, MPSO technique is invoked to find the better second order system. Formulated second order system based on MPSO techniques effectively depicts the original characteristics of the higher order system. Closed loop response of the proposed PID controller with formulated second order system is given in Fig.4 for illustration 1 and Fig.7 for illustration 2. Fig 5 and Fig.8 represents the closed loop response of the higher order system with proposed controller for illustration 1 and 2 respectively. Table 1 and Table 2 show that the proposed modified particle swarm optimization gives the better system response for the higher order system. From the Figures its observed that the formulated lower order system depicts the characteristics of the original higher order system effectively and update the worst experience of the particle in the velocity equation gives better optimal solution compared with the general PSO model.

IV. Conclusion

The quality of a formulated lower order model is judged by designing the discrete PID controller. PID controller of the formulated lower order system effectively controls the original high order system. The main advantage of the proposed method is that it is easy of implementation and least elapsed time. This can also extended for other evolutionary techniques and hybrid methods and also its extended for further design of compensators as well as state variable controllers and observers for stabilization process.

Appendix

Consider an nth order linear time invariant discrete higher order system represented by its transfer function as

$$= \frac{A_{n-1}z^{n-1} + A_{n-2}z^{n-2} + \ldots + A_3z^3 + A_2z^2 + A_1z + A_0}{a_nz^n + a_{n-1}z^{n-1} + \ldots + a_3z^3 + a_2z^2 + a_1z + a_0} \tag{33}$$

The Adjunct Polynomial scheme for obtaining the approximated lower order models from the given higher order system is as follows:

$$\text{First order: } \frac{A_0}{a_1z + a_0} \tag{34}$$

$$\text{Second order: } \frac{A_1z + A_0}{a_2z^2 + a_1z + a_0} \tag{35}$$

Third order: $$\frac{A_2 z^2 + A_1 z + A_0}{a_3 z^3 + a_2 z^2 + a_1 z + a_0} \qquad (36)$$

(n-1) th order: $$\frac{A_{n-2} z^{n-2} + A_{n-3} z^{n-3} + \ldots + A_1 z + A_0}{a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \ldots + a_1 z + a_0} \qquad (37)$$

Equations (33) through (37), gives the lower order models formulated using adjunct polynomial scheme from the given higher order system *G(z)*. Based on the requirement, suitable lower order model can be selected and operates. It should be noted for a higher order system of order *'n', (n-1)* lower order models could be formulated. This method of selection of approximate lower order models helps to set the initial values of operating parameters to be used in the Modified Particle Swarm Optimization process.

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