

Modern Seismic Design Approach for Buildings with Hysteretic Dampers

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Abstract—The use of energy dissipation systems for seismic applications has increased worldwide, thus it is necessary to develop practical and modern criteria for their optimal design. Here, a direct displacement-based seismic design approach for frame buildings with hysteretic energy dissipation systems (HEDS) is applied. The building is constituted by two individual structural systems consisting of: 1) a main elastic structural frame designed for service loads; and 2) a secondary system, corresponding to the HEDS, that controls the effects of lateral loads. The procedure implies to control two design parameters: a) the stiffness ratio ($\alpha=K_{frame}/K_{total\ system}$), and b) the strength ratio ($\gamma=V_{damper}/V_{total\ system}$). The proposed damage-controlled approach contributes to the design of a more sustainable and resilient building because the structural damage is concentrated on the HEDS. The reduction of the design displacement spectrum is done by means of a damping factor (recently published) for elastic structural systems with HEDS, located in Mexico City. Two limit states are verified: serviceability and near collapse. Instead of the traditional trial-error approach, a procedure that allows the designer to establish the preliminary sizes of the structural elements of both systems is proposed. The design methodology is applied to an 8-story steel building with buckling restrained braces, located in soft soil of Mexico City. With the aim of choosing the optimal design parameters, a parametric study is developed considering different values of α and γ . The simplified methodology is for preliminary sizing, design, and evaluation of the effectiveness of HEDS, and it constitutes a modern and practical tool that enables the structural designer to select the best design parameters.

Keywords—Damage-controlled buildings, direct displacement-based seismic design, optimal hysteretic energy dissipation systems.

I. INTRODUCTION

FOR a long time, traditional seismic design was based on the strong-column/weak-beam principle; however, the lessons learned after the earthquakes of Northridge (1994) and Kobe (1995) resulted in significant changes in the design approach of steel moment resisting frame buildings [1]. The expected collapse mechanism was that the structural damage would be concentrated at the beams ends due to inelastic flexural deformation. The damages observed during those seismic events indicated that the energy dissipation capacity obtained in this manner was limited and even though the buildings did not collapse they caused high economic losses [1]. Therefore, new modern seismic design approaches; such

as the *damage-controlled structure* method, used here, have increased for the design of buildings located in high seismicity zones.

II. BACKGROUND

A. Direct Displacement-Based Design (DDBD)

Kim and Seo [2] proposed a seismic design method for low-rise steel buildings with buckling restrained braces (BRB) as hysteretic energy dissipation systems (HEDS). The method is based on a simplified Direct Displacement-Based Design (DDBD), which uses a linear displacement profile. The limitations of the procedure described in [2] is that it does not control directly the ductility demand and that it is limited to low-rise buildings (because of its displacement hypothesis). As an advantage, the procedure shows that DDBD is a rational approach for dimensioning structures with BRB.

Teran-Gilmore and Virto [3] introduced a displacement-based methodology for the preliminary design of concrete low-rise buildings with BRB. The study shows that the displacement-based design approach is an effective way of designing buildings with BRB. However, more aspects such as the efficiency of the BRB related to the stiffness contribution and ductility need to be examined.

B. Design Parameters

Fleming [4] proposes a design methodology for hysteretic dampers in buildings, using the philosophy of structural motion design proposed in [5]. A design algorithm that calibrates stiffness and yield force level of the damper is developed. The author [4] also includes a parametric study that shows the advantages and disadvantages of using different values for the design parameters. The optimal distribution of stiffness and strength between the systems is obtained, considering that these parameters are independent.

Vargas and Bruneau [6] show the results of a parametric study oriented to establish the key parameters in the behavior of nonlinear single-degree-of-freedom structures with HEDS which are designed considering the “structural fuse” concept. The authors proposed a general force-based seismic design procedure that is systematic and simple, and takes into account the key parameters that relate the frame and the damper.

III. DIRECT DISPLACEMENT-BASED SEISMIC DESIGN

The design procedure known as Direct Displacement-Based Design (DDBD) is based on the idea that a structure should achieve a prescribed displacement or strain limit, for which a certain performance is expected for a given seismic intensity.

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The damage that occurs during seismic events can be related to the deformation of the structural elements and to the interstory drift. Both parameters can be easily linked to displacement; as a result, a direct relationship between damage and displacement can be obtained, unlike the relation between damage and force. In the DDBD approach the displacements and their control are critical since the beginning of the design process.

The DDBD method was originally proposed by Priestley in 1993. The fundamentals of DDBD are simple, and have been presented in several publications such as [7]. Here, only a brief discussion is included.

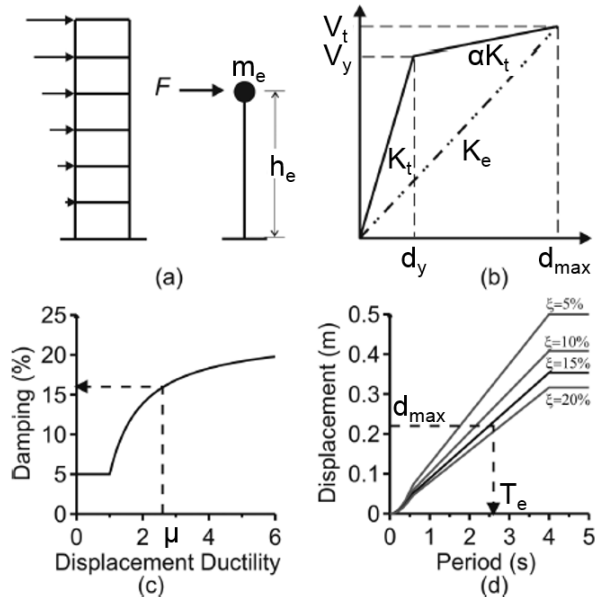


Fig. 1 Key steps of DDBD for MDOF systems: (a) ESDOF, (b) Effective stiffness and ductility, (c) Equivalent viscous damping vs ductility, (d) Design displacement spectra [7]

DDBD characterizes the multi-degree-of-freedom (MDOF) inelastic structure by an equivalent single-degree-of-freedom structure (ESDOF) (see Fig. 1 (a)). This is done by establishing a displacement profile, used to transform the stories peak displacements into the ESDOF design displacement d_{max} .

The stories peak displacement is set according to the deformation limits of the desired performance level, which can be related to structural or nonstructural compliances.

The design procedure, then characterizes the ESDOF structure using the secant stiffness K_e at maximum displacement d_{max} (see Fig. 1 (b)), and the inelastic response related to the ductility and energy dissipation capacity is represented by means of an Equivalent Viscous Damping (EVD). This consideration is known as the *substitute structure* [7].

Given that the ductility demand μ is required, it can be calculated knowing the yield displacement d_y of the ESDOF structure (see Fig. 1 (b)), as:

$$\mu = d_{max}/d_y \quad (1)$$

For a given level of ductility, the EVD can be estimated (see Fig. 1 (c)) and it can be used to calculate a damping modification factor to reduce the design displacement spectra. Once the design displacement has been determined, the effective period T_e is obtained (see Fig. 1 (d)).

The effective stiffness K_e of the ESDOF structure can then be found, as:

$$K_e = 4\pi m_e / T_e^2 \quad (2)$$

where m_e is the equivalent mass of the ESDOF structure, and is given by:

$$m_e = \sum \frac{m_i d_i^2}{d_{max}} \quad (3)$$

Then, the design lateral force is given by:

$$F = K_e d_{max} \quad (4)$$

As it is shown briefly, the design concept is simple, its complexity and particularity lies in the determination of the characteristics of the ESDOF, which depend on: 1) the displacements profile used to determine the design displacement, 2) the way to estimate the yield displacement according to the structural system, and 3) the development of the displacement design spectra and the response spectra damping reduction factor (which takes into account the presence of the HEDS).

IV. DAMAGE CONTROLLED STRUCTURES

Damage-controlled structures are defined as the combination of structural systems and energy transformation devices that are integrated in such a way that damage due to a major loading is restricted to a specific set of elements that can be repaired easily [8].

In this study, the damage-controlled structural system is defined as the combination of (see Fig. 2):

- 1) A primary system which corresponds to the main frame with elastic behavior, which supports the vertical loads, but also provides part of the lateral stiffness.
- 2) A secondary system which corresponds to the passive energy dissipation system (in our case constituted by hysteretic dampers) that are designed to yield and absorb the seismic energy before the frame yields.

Each system shown in Fig. 2 is set independently to a desired performance level. In this study the limit states to be verified are: serviceability and near collapse.

The main system is required to have an elastic structural behavior of both limit states. The secondary system is not allowed to yield in the serviceability limit, and it is required to sustain significant inelastic deformation for the near collapse limit state.

In both limit states, the performance structural response indicator corresponds to the limits drifts related to structural and nonstructural damage.

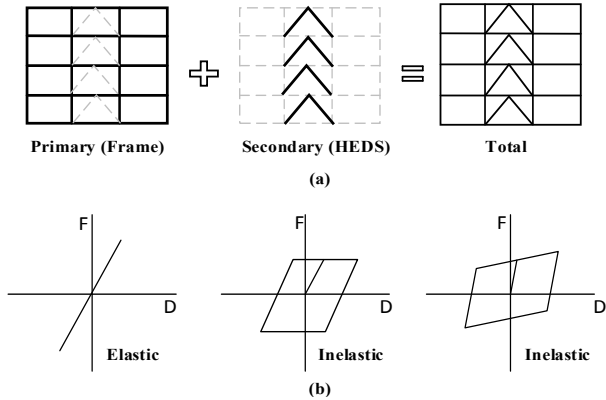


Fig. 2 Concept of a damage-controlled building: (a) Systems definition (b) Systems idealized behavior

V. DESIGN PARAMETERS

The structural model that represents the combined system and the key parameters between the interconnected variables of the systems is represented in Fig. 3 [6].

Fig. 3 shows the general shape of the base shear-displacement curve for a SDOF model of the combined system: frame and HEDS, which work in parallel.

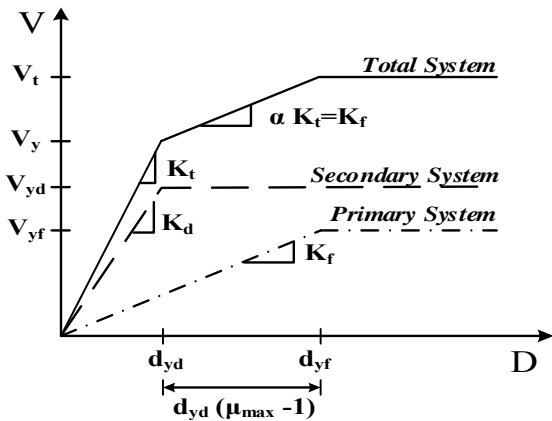


Fig. 3 General base shear- displacement curve for SDOF model [6]

In order to determine the optimal design parameters that relate the primary and the secondary systems, the following parameters are defined:

- 1) Stiffness Ratio (α): is the ratio between the frame stiffness K_f and the total initial stiffness K_t .

$$\alpha = \frac{K_f}{K_t} \quad (5)$$

where K_t is the sum of the frame initial stiffness K_f and the damper stiffness K_d .

- 2) Strength Ratio (γ): is the ratio between the damper yield force V_{yd} and the maximum force of the total system V_t .

$$\gamma = \frac{V_{yd}}{V_t} \quad (6)$$

- 3) Maximum Ductility Ratio (μ_{max}): is the ratio between the frame yield displacement d_{yf} and the damper yield displacement d_{yd} .

$$\mu_{max} = \frac{d_{yf}}{d_{yd}} \quad (7)$$

By establishing the value of the stiffness ratio in (5), the distribution of the total required stiffness K_t can be estimated in order to obtain the stiffness of both independent systems, as:

$$K_d = (1 - \alpha)K_t \quad (8)$$

$$K_f = \alpha K_t \quad (9)$$

and it allows to obtain the strength requirement for both systems:

$$V_{yd} = K_d * d_{yd} \quad (10)$$

$$V_{yf} = K_f * d_{max} \quad (11)$$

where V_{yf} represents the frame yield force.

VI. STIFFNESS-BASED PROCEDURE FOR SIZING THE STRUCTURAL ELEMENTS

Once the required stiffness for each system has been estimated, a general stiffness-based procedure is proposed for sizing the structural elements. The approach allows the designer to establish directly the preliminary sizes of the structural elements of both systems, instead of the traditional trial-error approach.

The procedure is based on the calculation of the stiffness of each story, by using the required global stiffness (K_d or K_f) and the expected modal shape. This is based on the approach by Bertero [9], using the following expressions:

For the i -story:

$$K_i = (K/m_e) * m_i * \left(\frac{\delta_i}{\delta_i - \delta_{i-1}} \right) + K_{i+1} \left(\frac{\delta_{i+1} - \delta_i}{\delta_i - \delta_{i-1}} \right) \quad (12)$$

For the n -story:

$$K_n = (K/m_e) * m_i * \frac{\delta_n}{\delta_n - \delta_{n-1}} \quad (13)$$

where K = total stiffness of the corresponding system K_d or K_f ; δ = expected modal shape; m = mass of each story.

A. Primary System (Frame)

The lateral stiffness of the i -story of a frame structure can be estimated as follows, by considering the sections properties of columns and beams [9]:

$$K_i = \frac{N_c}{1 + \psi_i} * \left(12 \frac{E I_c}{L_c^3} \right) \quad (14)$$

$$\psi_i = \frac{N_c L_c}{N_b I_b} \frac{L H}{L_c^2} \quad (15)$$

where N_c = Number of columns per story; N_b = Number of beams per story; E = Modulus of elasticity of the material; I_c = Column section inertia; I_b = Beam section inertia; L_c = Free column length between stories; L = Beam span.

B. Secondary System (HEDS)

HEDS geometric properties can be related to the lateral stiffness that they provide to the combined system. Given the lateral stiffness K_{di} of the damper of the i -story, and following (16), it can be obtained the core cross-section area required for a BRB in inverted-V configuration (see Fig. 4).

$$Ad_i = \frac{K_{di} L}{4E \cos^2(\theta_i)} \tag{16}$$

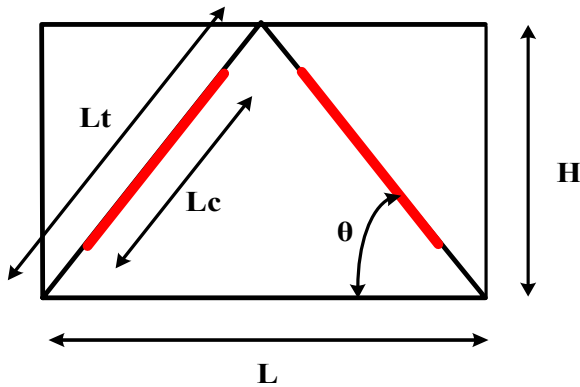


Fig. 4 BRB in inverted-V configuration

VII. APPLICATION EXAMPLE

A. Case Study

The design procedure is intended to be used in low-rise to medium-rise buildings whose dynamic response can be approximated by a shear beam structural model.

The case study example is an 8-story steel office building with buckling restrained braces (BRB) as HEDS. The structure is located in soft soil of Mexico City.

The building consists of 3x5 bays equally spaced at 8 m in each direction, as shown in Fig. 5 (a). Seismic gravity loads of 6.05 kPa for the typical stories, and of 4.95 kPa for the roof, were considered. The yield strength for BRB, beams and columns is 245 MPa. The design parameters chosen are $\alpha=0.25$ and $\gamma=0.3$.

The analysis is simplified to a 2D structural model corresponding to one of the exterior frame (Fig. 5 (b)). The floor diaphragm is considered rigid, and given the regular plan and the symmetry of the building, the torsion effects are neglected.

The displacement demand corresponds to the design displacement spectra recommended by the Mexico City Building Code 2004 (MCBC) [10], for soft soil zone (defined as zone IIIb). The design drift limits were also established according to the MCBC considering structural and nonstructural damage for the serviceability and near collapse

limits. According to it, a serviceability limit drift of 0.4% and a near collapse drift limit of 1.5% were adopted.

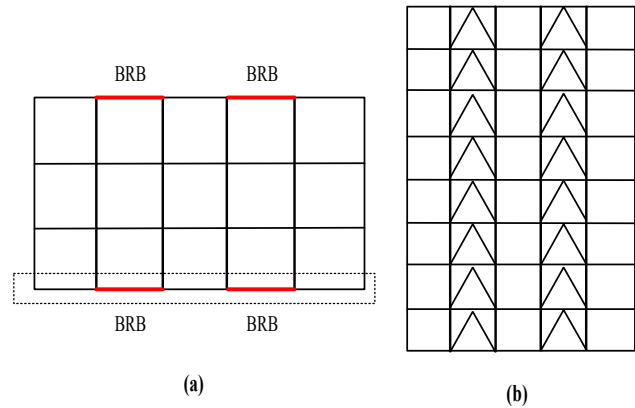


Fig. 5 Building Structural Model: (a) Floor plan (b) Typical Exterior Frame

B. Application of the DDBD Design Procedure

1) Equivalent Single-Degree-Of-Freedom (ESDOF)

The story displacement d_{ydi} that causes yielding of the BRB, depends on the BRB geometric configuration and on its yield strength f_{yd} , and it can be obtained as follows:

$$d_{ydi} = \frac{f_{yd} * \eta * L}{2E \cos^2(\theta)} \tag{17}$$

where η is the ratio between the yield length L_c and the total length L_t of the BRB.

A linear displacement profile was used to obtain the ESDOF yield displacement. The linear displacement profile was calculated using the linear mode shape [11].

The inelastic displacement profile is as that proposed by [7] for frame buildings whose behavior rely on the formation of a beam-sway mechanism. Using the displacement profile and the drift limit corresponding to the near collapse limit state, the ESDOF maximum displacement was obtained.

The transformation of the MDOF structure to the ESDOF using the specified displacement profile was done by means of:

$$ESDOF \text{ displacement} = \Sigma m_i d_i^2 / \Sigma m_i d_i \tag{18}$$

where d_i represents the displacement profile associated to yield or maximum displacement. Fig. 6 shows how the displacement profile predicts the story displacement for both limit states.

The ductility demand of the ESDOF structure can then be obtained using (1).

2) Displacement Design Spectra and Damping Modification Factor

Traditionally, DDBD uses an equivalent viscous damping (EVD) combined with an equivalent linear system to represent the energy dissipation of a nonlinear system during seismic loading. By knowing the EVD value it is possible to calculate

a damping modification factor to reduce the seismic design spectra.

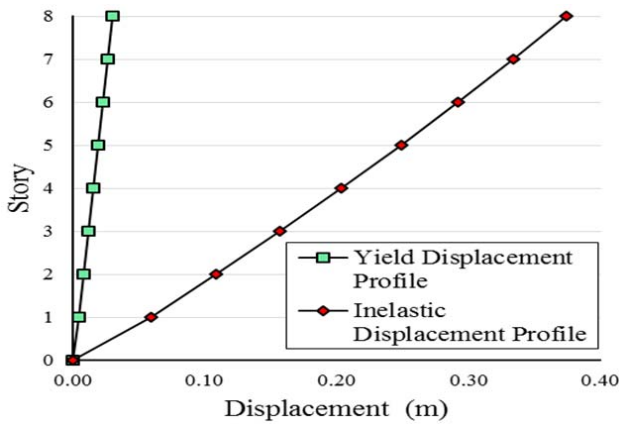


Fig. 6 Design Displacement Profiles

In this study a recently published damping modification factor (β_h) [12] for elastic structural systems with HEDS is used. The damping modification factor mathematical expression has been explicitly estimated for the soil conditions and ground motions typical of the valley of Mexico. The expression depends on the dominant ground period, the fundamental period of the structure, and the stiffness and strength ratios. The general expression was obtained from the ratio between uniform failure rate pseudo-acceleration spectra corresponding to systems with HEDS, $S_a(T, \alpha, \gamma)$, and uniform failure rate pseudo-acceleration spectra corresponding to systems with nominal damping ratio of 5%, $S_a(T, \xi = 5\%)$ [12].

$$\beta_h = \frac{S_a(T, \alpha, \gamma)}{S_a(T, \xi = 5\%)} \quad (19)$$

3) Effective Period and Initial Period

Using the damping modification factor β_h to reduce the design spectrum, the effective period T_e can be found using the ESDOF design displacement, (see Fig. 7).

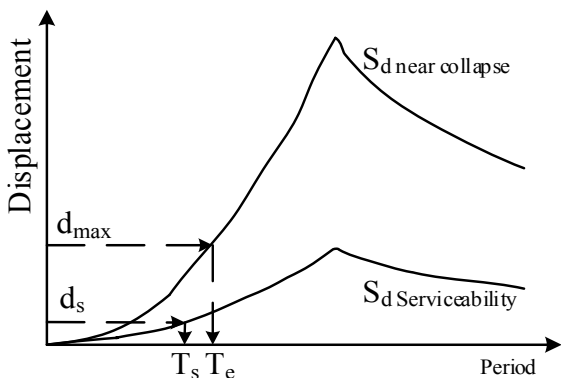


Fig. 7 Obtaining T_e and T_s from the design reduced displacement spectra

The effective stiffness can then be calculated using (2). The design procedure developed here proposes to transform the secant stiffness to the initial stiffness of the total system, and then, the elastic properties of the BRB and frame that can achieve the design displacement are obtained.

Hence, the effective stiffness K_e is transformed to the total initial stiffness K_t as:

$$K_t = K_e \frac{\mu_{max}}{\Omega} \quad (20)$$

$$\Omega = \frac{V_t}{V_y} = \alpha (\mu_{max} - 1) + 1 \quad (21)$$

where Ω represents an overstrength parameter which relates the system total shear to the system total yield force.

4) Serviceability Limit State

For the serviceability limit state, a linear displacement profile is used to calculate the design displacement using (17); then, the minimum period T_s for this condition is found. For this limit state a damping reduction factor $\beta_h = 1$ is used, because in the serviceability state the HEDS are not expected to yield.

The final design parameters are defined comparing the required initial period T_i (where T_i is the period associated to the initial stiffness (K_t) corresponding to the near collapse limit state and the required minimum period T_s associated with the serviceability limit state. The smallest of both periods is used to determine the required stiffness of the structure.

5) Case Study DDBD Results

Table I gives a brief summary of the key DDBD properties for each limit state.

TABLE I
DDBD AND DESIGN PARAMETERS DISTRIBUTION SUMMARY

	Limit State	
	Serviceability	Near Collapse
Stiffness Ratio, α	0.25	0.25
Strength Ratio, γ	0.75	0.30
Maximum Ductility, μ	1	8.04
Effective Period, T_e (s)	1.47	1.91
Initial Period, T_i (s)	0.96	0.86
Total Initial Stiffness, K_t (kN/mm)	26.95	33.70
Frame Stiffness, K_f (kN/mm)	6.74	8.42
Damper Stiffness, K_d (kN/mm)	20.21	25.27
Combined system total force, V_{yd} (kN)	878.80	3033.55
Frame yield force, V_{yd} (kN)	219.70	2209.47
Damper yield force, V_{yd} (kN)	659.10	824.08

C. Stiffness Based Procedure for Sizing the Structural Elements

1) Primary System: Main Frame

Using the procedure described in Section VI, the lateral stiffness of the primary system of each story was calculated. Table II shows a summary of the primary system section properties for columns and beams.

TABLE II
PRIMARY SYSTEM PROPERTIES

Inter-story	Story Mass m_i (t)	Inelastic Mode Shape, δ_i	Story Frame Stiffness, K_{di} (kN/mm)	Column Section Inertia, I_c (cm ⁴)	Beam Section Inertia, I_b (cm ⁴)
8	6.85	1.00	7.26	1081.89	2163.78
7	10.28	0.88	16.81	2505.43	5010.86
6	10.28	0.75	25.02	3729.67	7459.35
5	10.28	0.63	31.90	4754.62	9509.24
4	10.28	0.51	37.44	5580.27	11160.55
3	10.28	0.39	41.64	6206.63	12413.26
2	10.28	0.26	44.50	6633.69	13267.39
1	10.28	0.14	40.28	4383.00	8766.00

TABLE III
SECONDARY SYSTEM PROPERTIES

Inter-story	Story Mass m_i (t)	Inelastic Mode Shape, δ_i	Story Damper Stiffness, K_{di} (kN/mm)	BRB Core Yield Area, A_{di} (mm ²)
8	6.85	1.00	21.77	397.17
7	10.28	0.88	50.43	919.77
6	10.28	0.75	75.07	1369.20
5	10.28	0.63	95.70	1745.47
4	10.28	0.51	112.31	2048.58
3	10.28	0.39	124.92	2278.52
2	10.28	0.26	133.51	2435.30
1	10.28	0.14	120.84	2657.16

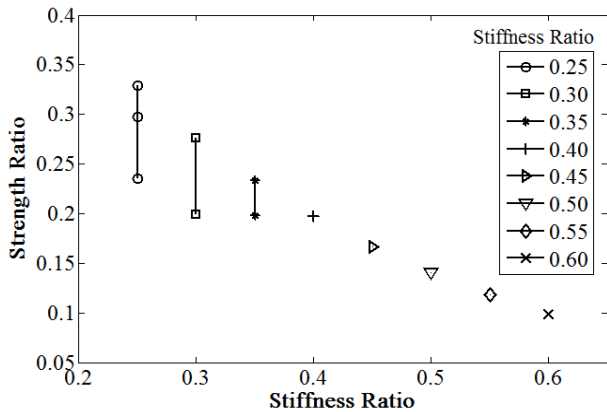


Fig. 8 Solution for different combinations of α and γ

2) Secondary System: BRB Core Area and Strength

The lateral stiffness of the secondary system of each story was calculated, then using (16) the required core area for each inter-story was found. Table III shows a summary of the core cross-section areas.

VIII. OPTIMAL DESIGN PARAMETERS

In the previous section, an application example was developed using the design parameters $\alpha = 0.25$ and $\gamma = 0.3$. In this section the best design parameters were determined in terms of HEDS efficiency and frame structural demand. For this purpose, several preliminary designs were carried out by analyzing the influence of the following design parameters ranges: α from 0.25 to 0.60, and γ from 0.2 to 0.40.

A. Solution for Different Combinations of α and γ Values

Fig. 8 shows the interval of solutions of the preliminary design alternatives for different combinations of α and γ values.

According to the results shown in Fig. 8, each stiffness ratio (α) has an interval of maximum strength ratio (γ) that can be achieved. This is because the strength contribution of the damper is limited to the yield displacement and to the yield strength capacity of the damper configuration in the frame. As the stiffness ratio grows, the strength contribution of the damper decreases, which means that the frame should have a greater stiffness and strength capacity, which diminishes the advantages of the HEDS.

B. Equivalent Viscous Damping (EVD)

The EVD of the combined system can be estimated using [13]:

$$EVD = \frac{2}{\pi} \left[1 - \frac{\gamma^2}{(1-\gamma)\left(\frac{1-\alpha}{\mu_f}\right)} - \frac{1-\gamma}{\mu_f} \right] \quad (21)$$

where μ_f represents the frame ductility taken as 1, because the frame is expected to work elastically under any level of seismic loading.

Fig. 9 shows the range values of EVD that can be expected for constant values of the stiffness ratio (α) and different values of the strength ratio (γ). From Fig. 9, it is observed that a lower stiffness ratio α corresponds to a higher EVD for the same strength ratio γ . In Fig. 9 the dashed lines correspond to the theoretical values calculated using (21) for the whole range of design parameters. Fig. 9 shows that there is a theoretical γ value that maximizes the EVD, and therefore the dampers efficiency.

However, for the example presented here the optimal value cannot be physically obtain, unless the yield strength and the displacement capacity of the damper are modified. In most practical cases, this cannot be easily done, because it depends on the available sections of the dampers in the market, as well as on the geometric configuration of the frame.

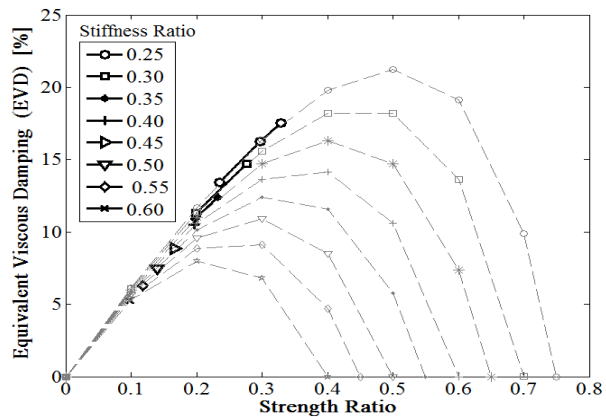


Fig. 9 EVD percentage for different combinations of α and γ

C. Ductility

The maximum ductility the damper can achieve is a way to measure its efficiency to dissipate seismic energy. Based on this idea, Fig. 10 shows the ductility versus the strength ratio for different values of the stiffness ratio (α).

It is observed in Fig. 10, that lower values of the stiffness ratio (α) can achieve larger values of ductility, also it can be seen that the ductility tends towards a constant value of 6 for lower values of the strength ratio γ , no matter the stiffness ratio. From the results shown in Fig. 10 it can be concluded that a low stiffness ratio combined with a high strength ratio value is the best option in terms of ductility.

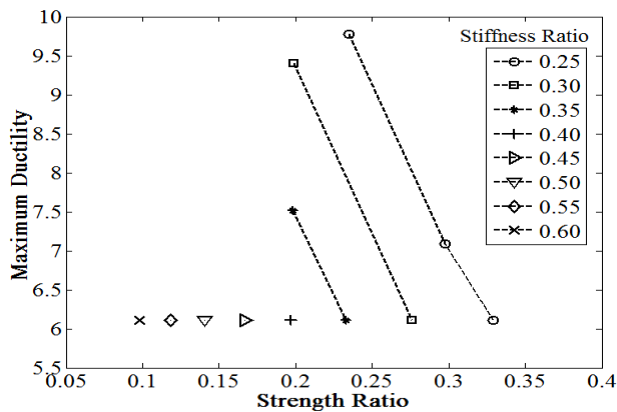


Fig. 10 Ductility relationship for different values of α and γ

For the example, from Fig. 10 the highest ductility is achieved for the design combination $\alpha = 0.25$ and $\gamma = 0.25$. However, using the design combination $\alpha = 0.30$ and $\gamma = 0.20$ almost the same ductility value could be expected. The advantage of using a smaller stiffness ratio is reflected on smaller brace core areas, and it could result in a better cost-efficient structure.

IX. CONCLUSION

A new design DDBD damage-controlled approach for buildings with HEDS has been presented. The procedure involves design parameters that interconnect the combined system.

A case study example was presented to show how the methodology works. It is concluded that, for the example analyzed here, the optimal design parameters correspond to the combination of low stiffness ratio and high strength ratio, because in this way the damper efficiency is maximized in terms of ductility, and the displacement demand is reduced because a higher EVD is developed.

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