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Modeling and Simulating Human Arm Movement using a 2 Dimensional 3 Segments Coupled Pendulum System

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***Abstract*—**A two dimensional three segments coupled pendulum system that mathematically models human arm configuration was developed along with constructing and solving the equations of motions for this model using the energy (work) based approach of Lagrange.

The equations of motion of the model were solved iteratively both as an initial value problem and as a two point boundary value problem. In the initial value problem solutions, both the initial system configuration (segment angles) and initial system velocity (segment angular velocities) were used as inputs, whereas, in the two point boundary value problem solutions initial and final configurations and time were used as inputs to solve for the trajectory of motion. The results suggest that the model solutions are sensitive to small changes in the dynamic forces applied to the system as well as to the initial and boundary conditions used. To overcome the system sensitivity a new approach is suggested.

***Keywords*—**Body Configurations, Equations of Motion, Mathematical Modeling, Movement Trajectories.

I. INTRODUCTION

RESTORATION of upper extremity function for patients with upper extremity disabilities is a task of utmost importance since the loss of hand function affects the subject's ability to perform daily living activities significantly impacting self-care activities such as feeding, dressing, and grooming, and even limiting the use of assistive technology aimed at improving quality of life.

Human movement analysis and modeling using mathematics, coupled with theoretical concepts in physiology and mechanics is constantly expanding and becoming more and more important in human performance and rehabilitation studies.

Faced with the task of understanding a complex system, it is often useful to extract its most essential features and use them to create a simplified representation of the system, or a 'model' of the system. A model allows one to observe more closely the behavior of the system and to make predictions regarding its performance under altered input conditions and different system parameters. The validity of these models is established by comparing their predictions with data from real physical models.

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Mathematical description may help us understand and quantify the physical disability for subjects with upper extremities disability. Furthermore, mathematical models provide a framework to design and quantify the best training strategies and motor tasks to find optimal movement patterns that can prevent injury and improve performance.

The use of mathematical models to investigate the dynamics of human movement relies on two approaches: forward dynamics and inverse dynamics. The former uses forces to predict the motion and the later uses a system's motion to predict the forces required to produce the motion.

In the forward approach the differential equations of motion are usually solved as initial value problems, therefore both the initial system configuration (segment angles) and initial system velocity (segment angular velocities) are required inputs [1] [2] [3]. The forward approach can be used to predict new physically realizable motions.

The forward method requires muscle forces as inputs. Unfortunately, noninvasive measurements of such forces don't exist presently. Therefore, forward models must rely on estimates or even educated guesses of forces that must have acted on the system in order to produce an observed movement.

In inverse dynamics the motion is given and the force-angular momentum method of Newton-Euler is used to solves for the net joint reaction force and net joint torque (moment of muscle force) acting at the next joint end of a given segment using the joint torque acting at a similar joint end of the previous adjacent segment [4].

A primary disadvantage of inverse models is that they require kinematics as inputs. Therefore, the inverse method is used to analyze already performed movements, thus, they are incapable of generating system kinematics as output and, therefore, cannot be used to predict new movement patterns. That is, they can only compare existing motions.

To test the hypotheses that a 2 dimensional representation of 3 coupled pendulum system can mimic arm movement of throwing with lower sensitivity to small changes in the dynamic forces applied to the system as well as to the initial and boundary conditions, a mathematical model of human arm configuration was developed along with constructing and solving the equations of motions for this model using the energy (work) based approach of Lagrange.

II. MATERIALS AND METHODS

A. Idealization

The human arm was idealized as a two dimensional three segments coupled pendulum system where the branching pattern of those segments is shown in Figure (1) with three mass points one on each segment. The connected pattern corresponds to an acyclic graph and that one segment is anchored to the origin of the coordinate system Point (A).

The relation matrix (R) (Equation 1) shows the relationship of connection between segments of the system, where each row of R represents a point mass of the system, and has the information about the path from the origin of the system to the point mass. Each column represents a segment, and has the information about the usage of the segment for every path to point masses.

The following values parameters values were used in the model; L_1, L_2, L_3, z_1, z_2, and z_3 are 0.36, 0.3, 0.2, 0.16, 0.13, and 0.1 meters respectively. Also, m_1, m_2, and m_3 are 2.07, 1.16, and 0.54 kilograms respectively [5].

$$R = \begin{bmatrix} z_1 & 0 & 0 \\ L_1 & z_2 & 0 \\ L_1 & L_2 & z_3 \end{bmatrix} \tag{1}$$

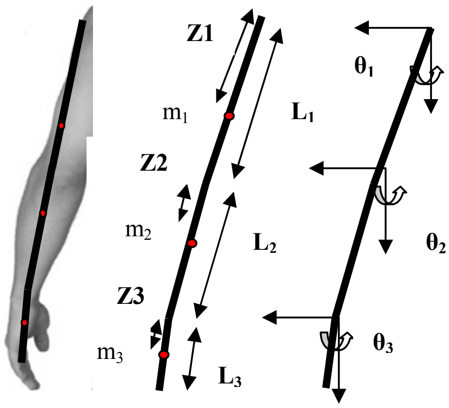

Fig. 1 Arm segments and mass distribution

B. Formulating Equations of Motion

The energy (work) - based approach of Lagrange was used to formulate the equations of motion of the suggested mechanical system. Since the choices of independent dynamic variables are position only then the equations of motion that are formulated are of second order.

The Lagrange method was used since it is usually more direct than either Newton's or D'Alembert's methods for arriving at the correct set of independent motion equations. The energy (Work) – based approach of Lagrange is based on the difference between the kinetic and potential energy of the system. More precisely the Lagrangian function is defined as:

$$L = K - P \tag{2}$$

where, K= the kinetic energy and L=the potential energy.

Using the above equation of Lagrangian function, the ith equation of motion corresponding to the ith degree of freedom takes the form found in Equation (3):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = 0 \tag{3}$$

In the suggested model, i=1, 2, 3 according to 3 segments used and θ_i represents the segment angle with the vertical axis viewing the segments from the lateral view.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0 \tag{4}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0 \tag{5}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_3}\right) - \frac{\partial L}{\partial \theta_3} = 0 \tag{6}$$

The kinetic and potential energies of the system are found according to Equations (8) and (9), respectively:

$$K = \frac{1}{2}\sum_{i=1}^{3} m_i (\dot{x}_i{}^2 + \dot{y}_i{}^2) \tag{7}$$

$$P = \sum_{i=1}^{3} g m_i y_i \tag{8}$$

C. Computer Simulation

Motion trajectories and segments angular velocities were evaluated iteratively on two test problems. The first is an initial value problem and the second is a boundary value problem. The implementation of the two methods has been done in MATLAB, utilizing built-in functions.

In the first test, we used the general purpose variable stepsize solver ODE45 in MATLAB to solve the system. For the second test, the build in function BVP4C was used.

III. Results

A. Equations of Motion

To formulate the system equations of motion, both the kinetic and potential energies of the system were found as shown in Equations (13) and (14):

$$x = \begin{bmatrix} z_1 \sin\theta_1 & L_1 \sin\theta_1 + z_2 \sin\theta_2 & L_1 \sin\theta_1 + L_2 \sin\theta_2 + z_3 \sin\theta_3 \end{bmatrix}^T \tag{9}$$

$$y = \begin{bmatrix} z_1 \cos\theta_1 & L_1 \cos\theta_1 + z_2 \cos\theta_2 & L_1 \cos\theta_1 + L_2 \cos\theta_2 + z_3 \cos\theta_3 \end{bmatrix}^T \tag{10}$$

$$\dot{x} = \begin{bmatrix} z_1 \dot{\theta}_1 \cos\theta_1 \\ L_1 \dot{\theta}_1 \cos\theta_1 + z_2 \dot{\theta}_2 \cos\theta_2 \\ L_1 \dot{\theta}_1 \cos\theta_1 + L_2 \dot{\theta}_2 \cos\theta_2 + z_3 \dot{\theta}_3 \cos\theta_3 \end{bmatrix} \tag{11}$$

$$\dot{y} = \begin{bmatrix} -z_1 \dot{\theta}_1 \sin\theta_1 \\ -L_1 \dot{\theta}_1 \sin\theta_1 - z_2 \dot{\theta}_2 \sin\theta_2 \\ -L \dot{\theta}_1 \sin\theta_1 - L_2 \dot{\theta}_2 \sin\theta_2 - z_3 \dot{\theta}_3 \sin\theta_3 \end{bmatrix} \tag{12}$$

$$K=\frac{1}{2}\begin{pmatrix} m_1 z_1^2\dot{\theta}_1^2+m_2L_1^2\dot{\theta}_1^2+m_2z_2^2\dot{\theta}_2^2+m_3L_1^2\dot{\theta}_1^2+m_3L_2^2\dot{\theta}_2^2+m_3z_3^2\dot{\theta}_3^2 \\ +2m_2L_1z_2\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2) \\ +2m_3L_1L_2\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2) \\ +2m_3L_2z_3\dot{\theta}_2\dot{\theta}_3(\cos\theta_2\cos\theta_3+\sin\theta_2\sin\theta_3) \\ +2m_3L_1z_3\dot{\theta}_1\dot{\theta}_3(\cos\theta_1\cos\theta_3+\sin\theta_1\sin\theta_3) \end{pmatrix} \tag{13}$$

$$P=g*\begin{pmatrix} m_1z_1\cos\theta_1+m_2L_1\cos\theta_1+m_2z_2\cos\theta_2+ \\ m_3L_1\cos\theta_1+m_3L_2\cos\theta_2+m_3z_3\cos\theta_3 \end{pmatrix} \tag{14}$$

Substituting Equations (13) and (14) in Equation (2) results in the Lagrangian function shown in Equation (15):

$$L=\begin{pmatrix} \frac{1}{2}m_1z_1^2\dot{\theta}_1^2+\frac{1}{2}m_2L_1^2\dot{\theta}_1^2+\frac{1}{2}m_2z_2^2\dot{\theta}_2^2 \\ +m_2L_1z_2\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2) \\ +\frac{1}{2}m_3L_1^2\dot{\theta}_1^2+\frac{1}{2}m_3L_2^2\dot{\theta}_2^2+\frac{1}{2}m_3z_3^2\dot{\theta}_3^2+ \\ m_3L_1L_2\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2) \\ +m_3z_3L_2\dot{\theta}_2\dot{\theta}_3(\cos\theta_2\cos\theta_3+\sin\theta_2\sin\theta_3)+ \\ m_3L_1z_3\dot{\theta}_1\dot{\theta}_3(\cos\theta_1\cos\theta_3+\sin\theta_1\sin\theta_3) \\ -gm_1z_1\cos\theta_1-gm_2L_1\cos\theta_1-gm_2z_2\cos\theta_2- \\ gm_3L_1\cos\theta_1-gm_3L_2\cos\theta_2-gm_3z_3\cos\theta_3 \end{pmatrix} \tag{15}$$

Thus, Equations (4), (5), and (6) are found to be Equations (16), (17), and (18) respectively, which represents the system of equations that is used to solve for the trajectory of motion and the angular velocities to produce them.

$$\begin{pmatrix} \left(m_1z_1^2+m_2l_1^2+m_3l_1^2\right)\ddot{\theta}_1+ \\ \left(m_2l_1z_2+m_3l_1l_2\right)\cos(\theta_1-\theta_2)\ddot{\theta}_2+ \\ \left(m_3l_1z_3\right)\cos(\theta_1-\theta_3)\ddot{\theta}_3 \end{pmatrix}=\begin{pmatrix} g\left(m_1z_1+m_2l_1+m_3l_1\right)\sin(\theta_1)- \\ \left(m_2l_1z_2+m_3l_1l_2\right)\sin(\theta_1-\theta_2)\dot{\theta}_2^2- \\ \left(m_3l_1z_3\right)\sin(\theta_1-\theta_3)\dot{\theta}_3^2 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \left(m_2l_1z_2+m_3l_1l_2\right)\cos(\theta_1-\theta_2)\ddot{\theta}_1 \\ +\left(m_3l_2^2\right)\ddot{\theta}_2+\left(m_3l_2z_3\right)\cos(\theta_2-\theta_3)\ddot{\theta}_3 \end{pmatrix}=\begin{pmatrix} g\left(m_2z_2+m_3l_2\right)\sin(\theta_2)- \\ \left(m_2l_1z_2+m_3l_1l_2\right)\sin(\theta_2-\theta_1)\dot{\theta}_1^2- \\ \left(m_3l_2z_3\right)\sin(\theta_2-\theta_3)\dot{\theta}_3^2 \end{pmatrix} \tag{17}$$

$$\begin{pmatrix} \left(m_3l_1z_3\right)\cos(\theta_1-\theta_3)\ddot{\theta}_1+ \\ \left(m_3l_2z_3\right)\cos(\theta_2-\theta_3)\ddot{\theta}_2+\left(m_3z_3^2\right)\ddot{\theta}_3 \end{pmatrix}=\begin{pmatrix} g\left(m_3z_3\right)\sin(\theta_3)- \\ \left(m_3l_1z_3\right)\sin(\theta_3-\theta_1)\dot{\theta}_1^2- \\ \left(m_3z_3l_2\right)\sin(\theta_3-\theta_2)\dot{\theta}_2 \end{pmatrix} \tag{18}$$

B. Computer Simulation

The system of Equations (16, 17, and 18) was presented in the form of Equation (19) as follow:

$$M(X)\dot{V}=F(X,V) \tag{19}$$

To be able to use MATLAB functions the suggested system of three second ordered-partial differential equations was transformed into six first ordered-partial differential equations to be solved iteratively. The solution of the system the vector Y shown below:

$$Y=\begin{bmatrix}\theta_1 & \theta_2 & \theta_3 & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3\end{bmatrix}^T \tag{20}$$

Different sets of initial conditions for the initial value problem solution were used to solve the dynamic system (Table I), as well as different sets of boundary conditions to solve the boundary value problem (Table II). All of which were chosen to represent movement trajectories of throwing an object looked at from the lateral view.

To assess the model validity in mimicking arm movement in throwing and to assess how sensitive the suggested model to the changes of the dynamic variables in the system as well as the initial and boundary conditions, a number of simulations were performed.

The equations of motion of the system were solved as an initial value problem with 100 iterations; the results showed that the solution was found for 0.1 second duration Figures (2A) and (3A), however the solution was truncated when extending the time to 1 second with the same initial conditions Figures (2B) and (3B).

The results from the boundary value problem solutions were found within the boundaries selected (Figures 4 and 5), however increasing the time and changing the boundaries selected resulted in singularities and large variations in the angular velocities to achieve the selected boundaries (Figures 6 and 7).

TABLE I
INITIAL VALUE PROBLEM SOLUTIONS

Line Space			Initial conditions						Solution
T_o	T_f	N	θ_{1o}	θ_{2o}	θ_{3o}	θ'_{1o}	θ'_{2o}	θ'_{3o}	
0	0.1	100	0	0	0	1	1	1	Fig. 2A
0	1*	100	0	0	0	1	1	1	Fig. 2B
0	0.1	100	0	0	0	0.1	0.1	0.1	Fig. 3A
0	1*	100	0	0	0	0.1	0.1	0.1	Fig. 3B

*Unable to meet integration tolerances without reducing the step size below the smallest value allowed

TABLE II
TWO POINT BOUNDARY VALUE PROBLEM SOLUTION

Line Space			Initial Guess						Boundary Conditions						Solution
T_o	T_f	N	θ_1	θ_2	θ_3	θ'_1	θ'_2	θ'_3	θ_{1o}	θ_{2o}	θ_{3o}	θ_{1f}	θ_{2f}	θ_{3f}	
0	0.1	100	0	0	0	1	1	1	0	0	0	Pi/4	Pi/4	Pi/4	Fig. 4
0	1	100	0	0	0	1	1	1	0	0	0	Pi/4	Pi/4	Pi/4	Fig. 5
0	2	100	0	0	0	1	1	1	0	0	0	Pi/4	Pi/4	Pi/4	Fig. 6
0	$3^{\#}$	100	0	0	0	1	1	1	0	0	0	Pi/4	Pi/4	Pi/4	No Solution
0	$1^{\#}$	100	0	0	0	1	1	1	0	0	0	Pi/4	3Pi/4	Pi	No Solution
0	1	100	0	0	0	1	1	1	0	0	0	Pi/4	3Pi/4	3Pi/4	Fig. 7

$^{\#}$A singular Jacobian encountered

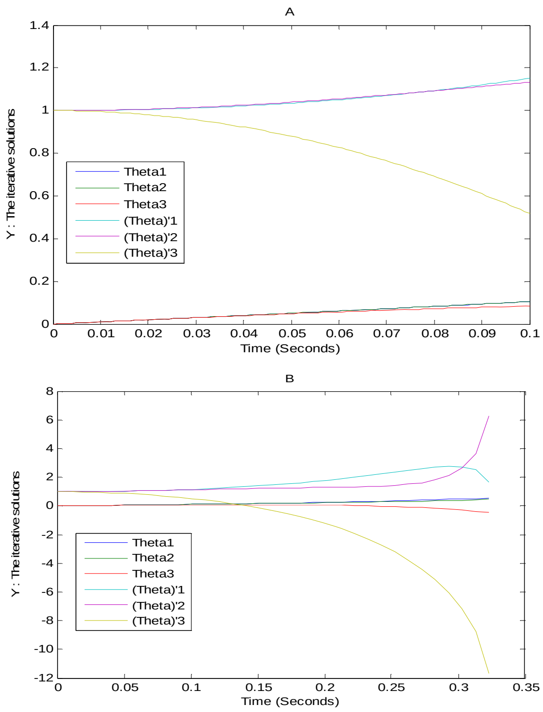

Fig. 2 Initial value problem solution sensitivity to the time interval selected to perform the solution. 100 iterations from t=0 to t=0.1sec (A), 100 iterations from t=0 to t=1 sec (B)

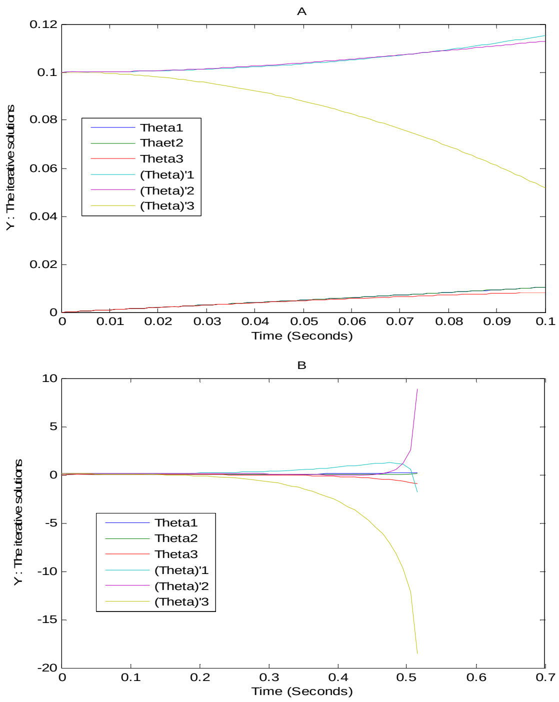

Fig. 3 Initial value problem solution sensitivity to the time interval selected to perform the solution. 100 iterations from t=0 to t=0.1sec (A), 100 iterations from t=0 to t=1 sec (B)

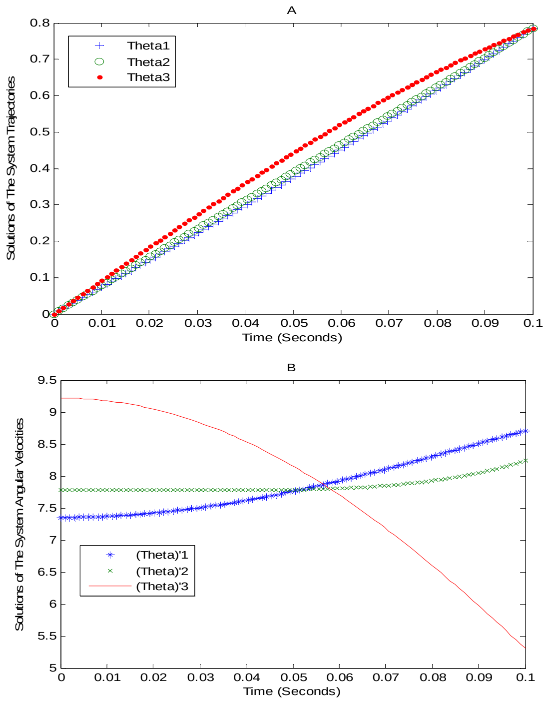

Fig. 4 Two point boundary value problem solution 100 iterations from t=0 to t=0.1 sec. Solution angular displacement (trajectories) (A) and Angular velocities (B)

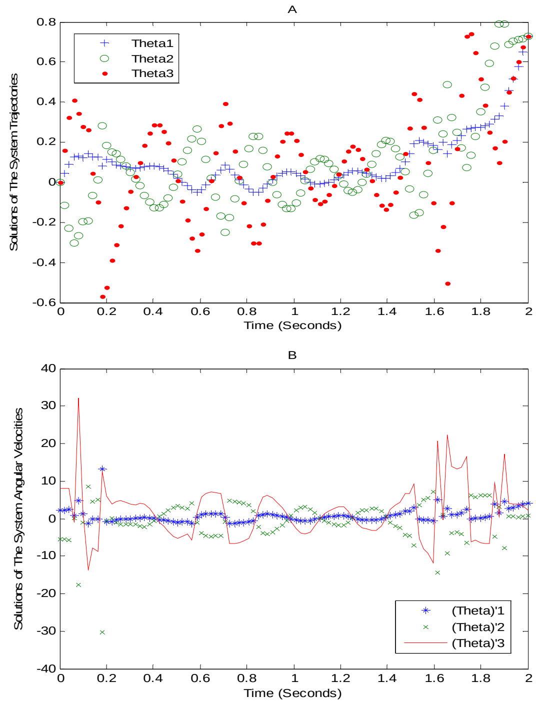

Fig. 6 Two point boundary value problem solution 100 iterations from t=0 to t=2 sec. Solution angular displacement (trajectories) (A) and Angular velocities (B).

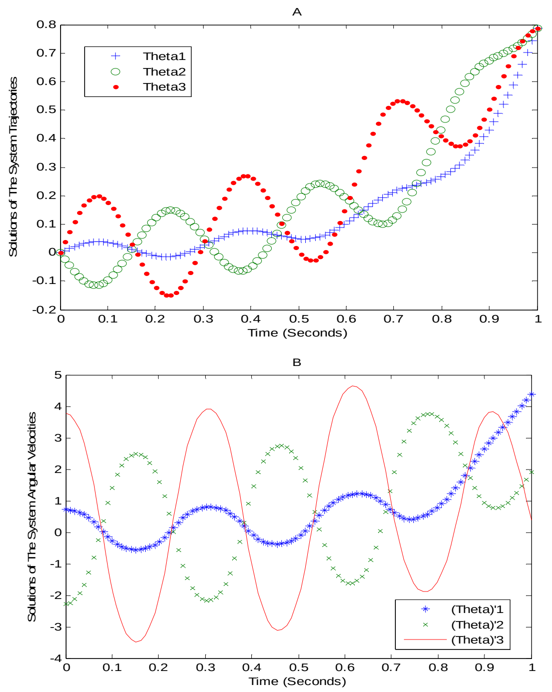

Fig. 5 Two point boundary value problem solution 100 iterations from t=0 to t=1 sec. Solution angular displacement (trajectories) (A) and Angular velocities (B)

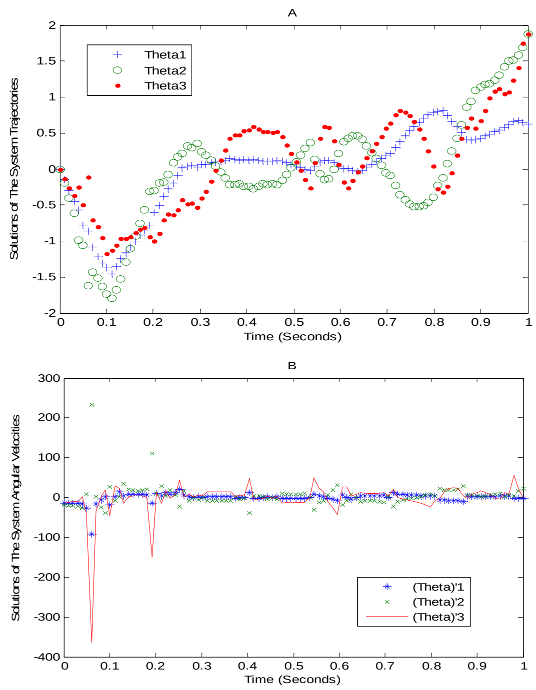

Fig. 7 Two point boundary value problem solution 100 iterations from t=0 to t=1 sec. Solution angular displacement (trajectories) (A) and Angular velocities (B)

IV. Discussion

Faced with the challenge of finding optimal arm movement patterns that will improve performance and reduce risks, it was hypothesized that constructing and solving the equations of motion using the energy based approach of a 2 dimensional 3 coupled pendulum system might mimic arm movement of throwing accurately.

We were able to show that such system can be constructed and can be stable and solvable with the condition of selecting the appropriate set of dynamical variables and conditions.

The data suggests that the system is sensitive to small changes in the dynamic forces applied to the system. This inherent instability of the motion requires a very accurate guess of the net moments that must have acted at each joint of the system in order to produce the movement. This is in consistent with the fact that many of the motor tasks for which an optimal solution is desired are highly skilled movements.

In order to get a smother solution (trajectory), it is suggested to break the phase into small phases, and solving each phase independently, i.e., breaking one two point boundary value problem in to two independent two point boundary value problem to solve them each one alone

To the best of our knowledge, this is the first study to investigate the use energy based approach in constructing and solving the equations of motion of an arm model, and to investigate the sensitivity of the solutions to the initial and boundary conditions selected.

Further investigations need to be conducted to elucidate the effects of these dynamic conditions on similar and more complex models.

V. Conclusion

The equations of motion for a 2 dimensional 3 coupled pendulum system were derived and solved iteratively, both as an initial value problem and a 2 point boundary value problem mimicking the arm movement in throwing. The solutions showed higher sensitivity to the time intervals in the initial value problem solutions.

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He has published several journal articles. His publications reflect his research interests in osseous repair, modeling, and rehabilitation engineering. He is also the founder and president of the Jordanian Biomedical Engineering Society.

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