Modeling and Control of Two Manipulators Handling a Flexible Beam

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Abstract—This paper seeks to develop simple yet practical and efficient control scheme that enables cooperating arms to handle a flexible beam. Specifically the problem studied herein is that of two arms rigidly grasping a flexible beam and such capable of generating forces/moments in such away as to move a flexible beam along a predefined trajectory. The paper develops a sliding mode control law that provides robustness against model imperfection and uncertainty. It also provides an implicit stability proof. Simulation results for two three joint arms moving a flexible beam, are presented to validate the theoretical results.

Keywords—Sliding mode control, cooperative manipulators**.**

I. INTRODUCTION

ROBOTIC applications, such as the lifting of flexible objects or lifting of objects with unusual geometry require the use of two cooperating manipulators. Utilizing multiple manipulators invites some issues that have to be dealt with, Inter-arm load balancing and internal force control become important [1][2][3][4][5]. What complicates matter even further is having a flexible object rather than a rigid object to manipulate. Manipulating flexible objects, however, stirs growing interest due to its potential applications in industry [6]. Some previous work has been done on the manipulation of flexible objects using dual arms; Zheng et al. [7] [8] studied the problem of trajectory planning and coordination of two manipulators to deform flexible beams. McCarragher et al. [9] addressed the same problem with a solution based on a hybrid position/force approach. James K. Mills et al. [10] addresses the problem of designing a control method for a multi-robot system designed for the assembly of flexible sheet metal parts, a practical algorithm is proposed based on the "rigid" body dynamics of the robot and payload. Meer, David William [11] [12] considered the manipulation of a particular flexible object. The control policy utilized in [11] [12] is based on a controller developed previously [13] for rigid objects, the object impedance controller. Al-Yahmadi[14] used a scheme that is capable of handling a flexible object both in free space and in contact tasks. The scheme is based on a controller previously developed for rigid objects by Bonitz and Hsia [15]. Dong Sun et al. [16] [17] used a hybrid impedance control algorithm to stabilize a flexible beam handled by two manipulators and simultaneously controlling its internal force.

The hybrid impedance control [16][17] cannot however, achieve trajectory tracking. In this paper position trajectory tracking is achieved using sliding mode control. Sliding mode control [18][19][20][21][22]is well suited to handle the highly nonlinear dynamic interaction present in the model describing the flexible beam.

II. NONLINEAR MODEL OF THE SYSTEM

Let the two planar manipulators with three revolute joints to be rigidly grasping the two ends of a beam of length *l*, mass per unit length ρA, and bending stiffness *EI*.

The *B*-spline based method will be used to approximate the dynamics of the flexible beam. In this method one seeks an approximate solution of the deflection of the beam in the form

$$\varepsilon(v,t) = \sum q_{fk} B_k^r(v)$$

where, $\varepsilon(v,t)$ is the deflection (transfers deflection) at time *t*, and at a spatial point *v*, and the deflection in this form is a sum of the product of two functions, one is a function of time q_{fk} and the other $B_k^r(v)$ is a function of the distance along the beam and $B_k^r(v)$ are piece-wise smooth polynomial functions of order *r* derived on the basis of a knot sequence $v_k, v_{k+1},...v_{k+r}$, and they are defined by the following recursion formula

$$B_k^r(v) = \frac{v - v_k}{v_{k+r} - v_k} B_k^{r-1}(v) + \frac{v_{k+r+1} - v}{v_{k+r+1} - v_{k+1}} B_k^{r-1}(v)$$

where,

$$B_k^0(v) = \begin{cases} 1 & \text{if } v_k \leq v \leq v_{k+1} \\ 0 & \text{otherwise} \end{cases}$$

Let $q_r = \begin{bmatrix} p_{ox}, p_{oy}, \theta \end{bmatrix}^T$ stand for the position and orientation of the center of mass of the un-deformed beam. Let a mobile coordinates frame $0\text{-}v\varepsilon$ be attached to that center of mass as seen in Figure 1.

The total kinetic and potential energies for the beam can be expressed in terms of the *B*-splines and the nodal coordinates. Inserting these terms in Lagrange's equation, and taking the nodal coordinates, [q_{f1}, q_{f2}, ... , q_{fn}]^T, together with $q_r = \begin{bmatrix} p_{ox}, p_{oy}, \theta \end{bmatrix}^T$ as the set of generalized coordinates, leads to a set of coupled differential equations relating the nodal responses to the applied forces.

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{rf}^T & M_{ff} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{q}_f \end{bmatrix} + \begin{bmatrix} h_r \\ h_f \end{bmatrix} + \begin{bmatrix} c_r \\ c_{f+} \bar{K}_q q_f \end{bmatrix} = R^T \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad (1)$$

where, q_r is as defined earlier, and q_f are the nodal coordinates.

Now for the manipulators dynamics; the equation of motion of the arm j is given by

$$M_j(\phi_j)\ddot{\phi}_j + N_j(\phi_j,\dot{\phi}_j)\dot{\phi}_j + G_j(\phi_j)$$
$$= \tau_j + J_j(\phi_j)^T(-f_j) \tag{2}$$

Define $x = \left[\phi_1,\phi_2,q_r,q_f\right]^T$; this will give the following overall system dynamics, written in the matrix form

$$M(x)\ddot{x} + N(\dot{x},x) + G(x) = \tau + J^T f \tag{3}$$

, where, $M(x) = \begin{bmatrix} M_1(\phi_1) & 0 & 0 & 0 \\ 0 & M_2(\phi_2) & 0 & 0 \\ 0 & 0 & M_{rr} & M_{rf} \\ 0 & 0 & M_{rf}^T & M_{ff} \end{bmatrix}$,

and, $N(x,\dot{x}) = \begin{bmatrix} N_1(\phi_1,\dot{\phi}_1)\dot{\phi}_1 \\ N_2(\phi_2,\dot{\phi}_2)\dot{\phi}_2 \\ h_r \\ h_f \end{bmatrix}$, $G(x) = \begin{bmatrix} G_1(\phi_1) \\ G_2(\phi_2) \\ c_r \\ c_f + \bar{K}_q q_f \end{bmatrix}$

while, $\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ 0 \\ 0 \end{bmatrix}$, and, $J^T = \begin{bmatrix} -J_1^T & 0 \\ 0 & -J_2^T \\ R^T_{01} & R^T_{02} \\ R^T_{q1} & R^T_{q2} \end{bmatrix}$

It can be shown that for the Dynamics of the system derived using the Euler-Lagrange formulation has the following properties [23].

Property 1 The System inertia matrix M(x) is symmetric and positive definite, Further $M(x)$ and $M^{-1}(x)$ are bounded as follows: $\frac{1}{M^+} \leq M(x) \leq \frac{1}{M^-}$.

Property 2 The matrix $\dot{M}(x) - 2N(x,\dot{x})$ is skew symmetric, i.e. $\dot{x}^T\left(\dot{M}(x) - 2N(x,\dot{x})\right)\dot{x} = 0$ .and since $N(x,\dot{x})$ is quadratic in $\dot{x}$ it can be bounded from above by a quadratic function of $\dot{x}$. That is $N(x,\dot{x}) \leq C^+ \|\dot{x}\|^2$.

Property 3 The term $G(x)$ is bounded from above in general by a function of the generalized coordinates as follows: $G(x) \leq G^+$.

The first property is a mathematical statement of the following fact; the kinetic energy of a system is a quadratic form which is positive unless the system is at rest. The second property is referred to as the *passivity property*; this property implies that the total energy of the system is conserved in the absence of friction.

II. CONTROLLER DESIGN

Making use of the constraint equations, i.e. making use of the fact that the two end effectors' positions x_1 and x_2 are related to the coordinates $\left[q_r,q_f\right]^T$ describing the dynamic of the flexible beam; in the following manner

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = R \begin{bmatrix} \dot{q}_r \\ \dot{q}_f \end{bmatrix}$$

and noticing that x_1 and x_2 are in turn related to joint coordinates of the two manipulators; the model can be rearranged to have the form

$$\ddot{\phi} = A + B\tau$$

One can achieve that as follows: Assuming the end conditions of a clamped-clamped beam, one will have $R = \begin{bmatrix} R_{01_{3\times3}} & 0_{3\times(3+n)} \\ R_{02_{3\times3}} & 0_{3\times(3+n)} \end{bmatrix}$ and solving for $\ddot{q}_f$ from

$$M_{rf}^T\ddot{q}_r + M_{ff}\ddot{q}_f + h_f + c_f + K_q q_f = 0 \tag{4}$$

gives

$$\ddot{q}_f = -M_{ff}^{-1}\left\{M_{rf}^T\ddot{q}_r + h_f + c_f + K_q q_f\right\} \tag{5}$$

Substituting $\ddot{q}_f$ in

$$M_{rr}\ddot{q}_r + M_{rf}\ddot{q}_f + h_r + c_r = R_{01}^T f_1 + R_{02}^T f_2 \tag{6}$$

and rearranging gives

$$M_0\ddot{q}_r + h_0 = R_{01}^T f_1 + R_{02}^T f_2 \tag{7}$$

where

$$M_0 = M_{rr} - M_{rf}M_{ff}^{-1}M_{rf}^T$$

$$h_0 = h_r + c_r - M_{rf}M_{ff}^{-1}\left(h_f + c_f + K_q q_f\right)$$

from which one has

$$f_1 = R_{01}^{-T}\left(M_0\ddot{q}_r + h_0 - R_{02}^T f_2\right)$$

hence,

$$M_1\ddot{\phi}_1 + N_1 + G_1 = \tau_1 - J_1^T R_{01}^{-T}\left(M_0\ddot{q}_r + h_0 - R_{02}^T f_2\right)$$

But, the two end effectors positions x_1 and x_2 are related to the generalized coordinates of the flexible beam $\left[q_r,q_f\right]^T$ in the following manner

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = R \begin{bmatrix} \dot{q}_r \\ \dot{q}_f \end{bmatrix}$$

hence,

$$J_1\dot{\phi}_1 = R_{01}\dot{q}_r$$
$$J_1\ddot{\phi}_1 + \dot{J}_1\dot{\phi}_1 = R_{01}\ddot{q}_r + \dot{R}_{01}\dot{q}_r$$

i.e.

$$\ddot{q}_r = R_{01}^{-1}\left(J_1\ddot{\phi}_1 + \dot{J}_1\dot{\phi} - \dot{R}_{01}R_{01}^{-1}J_1\dot{\phi}_1\right) \tag{8}$$

which gives the following overall equation of the system

$$\hat{M}\ddot{\phi} + \hat{N} = \tau + \hat{R}^T f \tag{9}$$

where,

$$\hat{M} = \begin{bmatrix} M_1 + J_1^T R_{01}^{-T} M_0 R_{01}^{-1} J_1 & \\ & M_2 \end{bmatrix}$$

$$\hat{N} = \begin{bmatrix} N_1 + G_1 + J_1^T R_{01}^{-T} M_0 R_{01}^{-1}\left(\dot{J}_1 - \dot{R}_{01}^{-1}J_1\right)\dot{\phi}_1 & \\ & N_2 + G_2 \end{bmatrix}$$

and

$$\hat{R}^T = \begin{bmatrix} 0 & J_1^T R_{01}^{-T} R_{02}^T \\ 0 & -J_2^T \end{bmatrix}$$

$$\ddot{\phi} = \hat{M}^{-1}\left(-\hat{N} + \begin{bmatrix} J_1^T R_{01}^{-T} R_{02}^T \\ -J_2^T \end{bmatrix} f_2 + \tau\right)$$

which can be written as

$$\ddot{\phi} = A + B\tau$$

One can choose a suitable control law τ which makes ϕ tracks a given trajectory ϕ^d Using sliding mode control, by defining the state error and sliding surface as follows;

$$\begin{aligned} e &= \phi^d - \phi \\ r &= \Lambda e + \dot{e} \end{aligned} \tag{10}$$

and applying the control law

$$\tau = \hat{B}^{-1}\left(\tau_{eq} + \Gamma \, sign(r)\right), \text{ where } \tau_{eq} = \ddot{\phi}^d + \Lambda\dot{e} - \hat{A}$$

and $\hat{A}$, $\hat{B}$ are estimates of A and B respectively.

It was shown earlier that this control law will ensure that the sufficient condition for sliding mode control will be achieved if $\|\Gamma\| > \dfrac{E^+}{C^-}$, where $C^- \le C \le C^+$ and $\left\|(I - C)\tau_{eq} - \left(A - \hat{A}\right)\right\| \le E^+$

For which
$$C = B\hat{B}^{-1}$$

III. SIMULATION RESULTS

Consider two identical robots and each robot is made up of three rigid links (shoulder, upper arm and forearm) of mass 3kg and length 1 m. the links are interconnected by three revolute joints.

The two robots are moving a flexible beam. The parameters of the flexible beam are as shown in Table I.

The simulations are as seen in Figures 2, and 3. The simulation results show that position tracking control can be achieved using sliding mode control. It shows further that the vibration is damped as seen from the fact that the flexible coordinates settles down to constant values.

TABLE I
PARAMETERS OF THE SIMULATED MODEL

Symbol, Quantity	Numeric value
Density, ρ	$26600\dfrac{kg}{m^3}$
Mass, m_0	$0.2034kg$
Length, l	$0.5\ m$
Cross-sectional area, A	$1.5\times10^{-4}m^2$
Young's Modulus, E	$7.1\times10^{10}Nm^{-2}$
Moment of Inertia, I	$3.125\times10^{-10}m^4$

IV. CONCLUSIONS

The paper addresses the problem of deriving a mathematical model that describes the system, and deriving a control law that is able to move the flexible beam along a given trajectory while suppressing the vibrations that are excited during the motion of the system The simulation results show that perfect trajectory tracking is achieved using the sliding mode controller.

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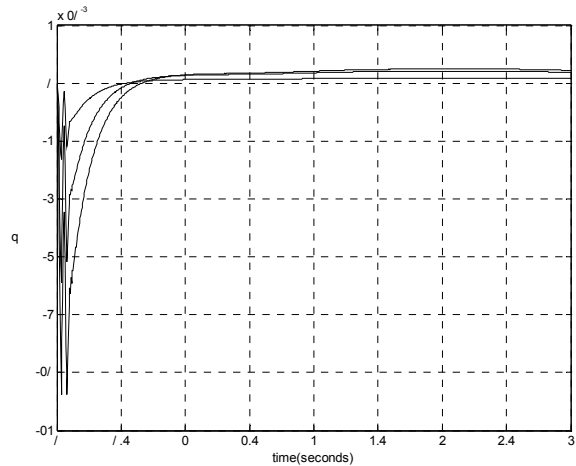
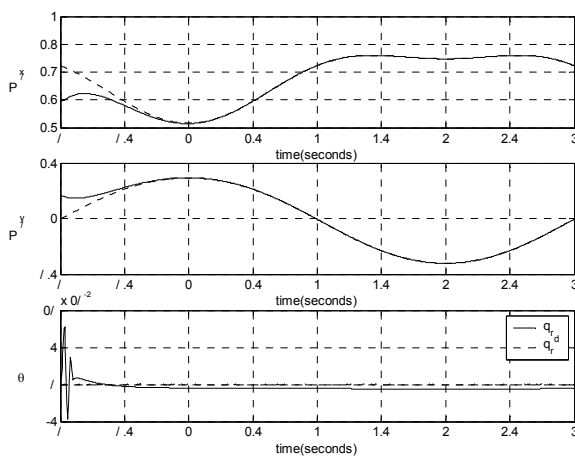Fig. 3 Time response of the rate of change of the first three flexible coordinates

Fig. 2 Position Tracking of P_{ox} , P_{oy} and θ