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Model Reference Adaptive Control and LQR Control for Quadrotor with Parametric Uncertainties

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Abstract—A model reference adaptive control and a fixed gain LQR control were implemented in the height controller of a quadrotor that has parametric uncertainties due to the act of picking up an object of unknown dimension and mass. It is shown that an adaptive controller, unlike the fixed gain controller, is capable of ensuring a stable tracking performance under such condition, although adaptive control suffers from several limitations. The combination of both adaptive and fixed gain control in the controller architecture can result in an enhanced tracking performance in the presence parametric uncertainties.

Keywords—UAV, quadrotor, model reference adaptive control, LQR control.

I. Introduction

THE technology of small unmanned aerial vehicles (UAVs) has grown in the past decade and this field of research has become increasingly popular due to the promising capabilities of these UAVs to carry out tasks in various fields such as mapping and surveillance, heavy lifting, and the role of first responder to access areas that are inaccessible by human beings. A quadrotor UAV is a convenient platform for research because it is mechanically simple, low cost, agile, and capable of hovering in space. However, the dynamics of a quadrotor is inherently unstable thus a feedback controller is essential for this type of vehicle to fly.

In previous attempts to fully optimize the use of quadrotors, designers have added to the quadrotor tools such as a gripper [1], [2], a Velcro adhesive [3], and a robotic arm [4]–[6], allowing the vehicle to physically interact with its surrounding. This enables tasks like picking up and dropping objects [2] and perching on a pole [1]. The capability of the robot to interact with its physical surrounding increases its potential and allows users to leverage the vehicle for more demanding jobs. However, this poses a serious challenge to researchers due to the complexity in the vehicle design and control. Not only is the augmented quadrotor configuration difficult to model mathematically due to highly coupled dynamics, but the dynamics of the vehicle changes significantly when physical contact is established. With a high level of parametric uncertainties in the dynamic model, designing an effective fixed gain controller for this system is difficult because the controller requires an accurate model in order to minimize performance error.

The work in this paper aims to facilitate research on control design for a quadrotor that is capable of interacting with the environment, and the main goal is to design a controller that can work well under significant parametric uncertainties in the system dynamics. Therefore the focus is more on the control part, whilst the vehicle design aspect is left for future work. The use of a robotic arm attached to the quadrotor for manipulative purposes being just one possibility. This paper is described as follows. Section I contains the introduction to this research with Part A describes the previous related work in this area, Part B describes the chosen quadrotor model, and Part C describes the quadrotor mathematical model including the motor dynamics. The chosen controller architecture is given in Section II and simulation results and discussion is presented in Section III. Finally, the conclusion is drawn in Section IV.

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A. Background

Previous works related to a multi-link arm attached to a quadrotor are limited. In [4], the author used the variable parameter integral backstepping control for the quadrotor with an attached multi-link arm. The changes in the vehicle dynamic behaviour from contact forces are not taken into account in the model and left to the integral term in the feedback controller to correct for. In similar work, a quadrotor with a 2 degree of freedom robotic arm is controlled using adaptive sliding mode control [5].

An adaptive controller, unlike a fixed gain controller, is capable of achieving good performance in the presence of significant parametric uncertainties, and even without the full knowledge of the plant [7]. The Model Reference Adaptive Control (MRAC) was originally proposed by Whitaker et. al [8] in 1958, and this control method is still actively studied today. In this control scheme, the controller parameters are allowed to be adjusted through an adjusting mechanism designed in the feedback control law to give a plant output performance similar to that of the reference model. A fast adaptation can be achieved because the performance index measures are obtained simply by a direct comparison of the output from the reference model and the plant thus this method does not require the identification of the plant dynamic performance, although a certain a priori knowledge of the plant structure is necessary to implement the adaptive control system [9]–[11]. This method also allows the designer to specify the response characteristics as desired through the design of the reference model. Unfortunately, the adaptive systems comes with a caveat that controller parameter drift can occur when the input signal is not rich or persistently exciting, which can cause sudden instability [12].

Because of the capability of learning whilst in flight and coping with uncertainties, adaptive control has been the popular choice for fault-tolerant or reconfigurable flight

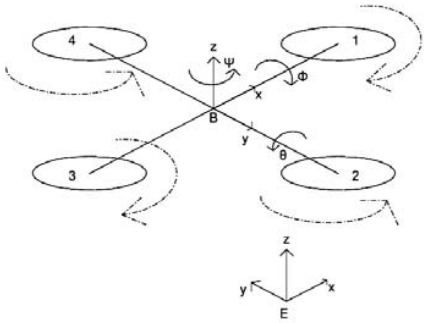

Fig. 1 OS4 Quadrotor Coordinate Sysem

control. In [13], the controller of a small quadrotor is augmented to include both a baseline fixed gain control and a model reference adaptive control. The whole system is equivalent to the baseline control in the nominal case, but in the case of a failure the adaptive control plays the role to maintain stability and regain the original performance. Although it is difficult to regain the original performance with a significant loss of thrust due to permanent damage in one of the four propellers, the author has demonstrated that adaptive control allows for safe hover and return.

In [14], an $\mathcal{L}_1$ adaptive control is implemented on a quadrotor used for aerobiological sampling because the controller can quickly compensate for large changes in vehicle dynamics. The drag force introduced by opening the sampling apparatus is treated as an uncertain disturbance thus rejected by the controller. The author used a high adaptive gain to ensure a fast convergence when disturbance exists. Although a high gain control does ensure a faster response in theory, it may cause instability due to motor saturation, or excitation of unmodeled dynamics. Furthermore, a high gain controller requires a high fidelity system dynamics, which can be difficult to obtain. Therefore, the adaptive gains are generally kept small to avoid instabilities.

The overall design configuration of the vehicle in this work has not yet been determined thus the dynamic model of an additional arm is excluded in the mathematical model of the quadrotor. However, it is presumed that the physical contact by means of picking up an object introduces unknown changes to the vehicle dynamic parameters. This means that the center of gravity and inertia changes and the overall mass increases. These changes are treated as uncertainties in the model, and a model reference adaptive controller is used to make a stable correction to compensate for unmodeled parametric changes. The performance of a model reference adaptive control is compared against that of a fixed gain LQR control and it can be shown that an LQR controller could not compensate for significant changes in the plant parameters, while the adaptive controller effectively retains the original performance when the plant parameters change. However, the response of the system controlled with adaptive controller during the transient learning phase is not satisfactory due to a large initial error. This problem is inherent to all adaptive controllers since the controller knows nothing of the plant, and the controller parameters has to be adjusted from zero. The response of a system controller with LQR controller, on the other hand, is very good when the plant is similar to how it was modelled initially. It can be shown that the combination of both LQR controller and adaptive controller is a viable solution. The controller benefits from optimal response of LQR controller in the nominal case, as well as self-correction capability from adaptive controller when there are changes in the plant parameters.

TABLE I
MASS PROPERTY OF OS4 QUADROTOR

Parameter	Value
Mass, m	0.53kg
Inertia on x-axis, I_{xx}	$6.28 \times 10^{-3}\ kgm^2$
Inertia on y-axis, I_{yy}	$6.28 \times 10^{-3}\ kgm^2$
Inertia on z-axis, I_{zz}	$1.121 \times 10^{-2}\ kgm^2$
Arm length, l	$0.232m$

B. Quadrotor Model

The model used to generate results in this paper was developed by Bouabdallah [15] for the OS4[1] project conducted in the Autonomous Systems Laboratory (EPFL) at the Swiss Federal Institute of Technology. The project focused on the design and control of a small quadrotor unmanned vehicle. A brief description of the model is given here, and the full details of model derivation can be found in [16]–[18].

C. Mathematical Model

The dynamic model is derived based on Newton-Euler formulation for rigid body. The reference system is shown in Fig. 1 where the earth fixed-frame, E, is defined as North, East, Up with respect to the Earth, and the body-fixed frame, B, is defined as X pointing towards the front of quadrotor, Y pointing towards the left of quadrotor, and Z-axis pointing up with respect to the rigid body. The right hand rule is applied for the sign convention for the respective angular rotations. The full equation of motion of the quadrotor is presented in [18] and the mass property of the OS4 is shown in Table I.

The OS4 quadrotor is propelled by four fixed-pitch propellers, each powered by a Brushless DC motor located at the end of each arm. The motor gives a maximum propeller rotation speed of 260 rad/sec. A first order transfer function describing the rotor dynamics is given below

$$G(s) = \frac{0.936}{0.178s + 1} \tag{1}$$

This work is focusing on the control of a quadrotor height channel only, and the full control of quadrotor is left for future work. By assuming small angle approximations and a near hover flight, the quadrotor height dynamics can be decoupled from the translation and rotational dynamics. A simplified model of the quadrotor height channel was built in the Simulink environment and the simulation was run at 100 Hz.

[1]Omnidirectional Stationary Flying Outstretched Robot

II. Controller Architecture

A baseline LQR controller is implemented on the quadrotor height channel and the response characteristic is obtained. Then, a model reference adaptive control is designed for the system, where the reference model is designed based on the quadrotor model plus LQR controller architecture response. This way, the reference model reaches the output performance of the nominal quadrotor height output performance when controlled with LQR.

A. LQR Control

Linear quadratic regulator (LQR) is an optimal method for choosing feedback gains for linear systems by minimizing a quadratic cost function in an infinite horizon. In a non-tracking problem, the goal of the LQR is to drive all the states to zero in the fastest amount of time, given a set of constraints described in the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$. To achieve tracking of a reference input, an LQR with feedback integral action [19] is used where the integrator is used to remove the steady state error. The control formulation is achieved by augmenting the linear system matrices with a new state that computes the integral of the error signal. Fig. 2 shows the architecture of LQR controller with integral action.

Given a linear time invariant state space model:

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}\mathbf{\Lambda}u \\ y &= \mathbf{C}x \end{aligned} \tag{2}$$

where $x \in \Re^n$, $u \in \Re^m$ and $y \in \Re^m$ are the states, input to the system, and system output respectively, $\mathbf{A} \in \Re^{n\times n}$, $\mathbf{B} \in \Re^{n\times m}$, and $\mathbf{\Lambda} \in \Re^{m\times m}$ is an unknown positive definite matrix representing uncertainties in the system. In order to design an LQR controller, the parameters in matrices $\mathbf{A}$ and $\mathbf{B\Lambda}$ must be known. If there are no uncertainties in the system, $\mathbf{\Lambda}$ is a diagonal matrix.

The open loop dynamics is augmented for tracking thus the error vector for the reference input, r, is represented as a new state z.

$$\dot{z} = r - y = r - \mathbf{C}x \tag{3}$$

The system representation is then augmented with the new state as follows [20].

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0_{n\times m} \\ -\mathbf{C} & 0_{m\times m} \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0_{m\times m} \end{bmatrix} u - \begin{bmatrix} 0_{n\times m} \\ \mathbf{I}_{m\times m} \end{bmatrix} r \tag{4}$$

where $\mathbf{I}$ is the identity matrix. The feedback control law that will bring $\dot{z}$ to zero is

$$u = -\begin{bmatrix} K_x & K_z \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} \tag{5}$$

The LQR control gain K is computed such that the quadratic cost function is minimized.

$$J = \int_0^\infty (x^T \mathbf{Q} x + u^T \mathbf{R} u)\, \mathrm{d}t \tag{6}$$

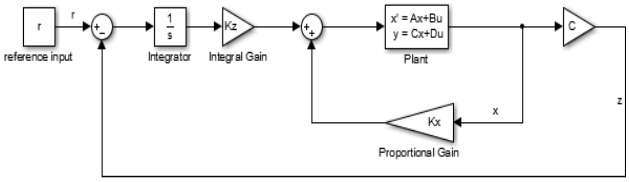

Fig. 2 LQR with integral action

1) Linearized Height Dynamics: The linearized state space representation of OS4 height dynamics is obtained in order to design an LQR controller. The linearization was performed at near hover trim condition.

$$\begin{aligned} x_1^* = x_2^* &= 0 \\ U_1^* &= mg \end{aligned} \tag{7}$$

In addition to this, all rotational angles are assumed to be zero, and motor dynamics are neglected. The matrices are augmented with a third state representing the tracking error in height. The linearized matrices are shown below.

$$\mathbf{A}_{aug} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad \mathbf{B}_{aug} = \begin{bmatrix} 0 \\ 1/m \\ 0 \end{bmatrix} \tag{8}$$

2) LQR Control Parameters: The elements in the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ are chosen accordingly using the knowledge of the system. Some states are allowed to be penalized less than others. Any constraints such as motor saturation and maximum climb rate are taken into account when penalizing the respective states. The LQR controller is designed in a manner that allows the quadrotor to perform stable tracking whilst operating well within the maximum allowable flight envelope. The chosen weighting matrices are

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = 0.11 \tag{9}$$

The feedback gain is computed numerically using the *lqr* [21] command in MATLAB. The gains found were

$$K = \begin{bmatrix} 24.94 & 26.10 & -10.00 \end{bmatrix} \tag{10}$$

B. Model Reference Adaptive Control

An adaptive controller has an advantage over a fixed gain controller in cases where there are parametric uncertainties in the dynamic model. Adaptive control utilizes a reference model that describes the desired response characteristic for the system, and then utilizes adjustable parameters and mechanisms for adjusting the parameters [22] in the controller to drive the system's response to behave like that of the reference model. This method is called the model reference adaptive control. The architecture of an MRAC controller is shown in Fig. 3. This method gives a high speed adaptation because the measure of the difference between the performance of the reference model and that of the actual system is obtained directly by the comparison of the two outputs [9].

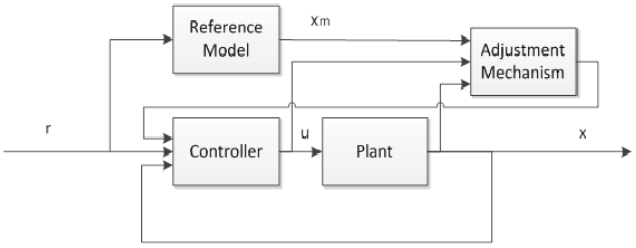

Fig. 3. Model Reference Adaptive Control

This controller can be applied to linear and nonlinear systems, and the exact knowledge of the dynamic model is not necessary. Consider a linear plant given in (2), and that matrix A is unknown and Λ contains information about uncertainties in the system. A reference model is designed for the unknown plant which describes the ideal response characteristics.

$$\dot{x}_m = \mathbf{A}_m x_m + \mathbf{B}_m r \tag{11}$$

where $x_m \in \Re^n$ is the reference model state, $r \in \Re^m$ is the reference input signal, $\mathbf{A}_m \in \Re^{n\times n}$ and $\mathbf{B}_m \in \Re^{n\times m}$.

The tracking error between the actual response and the reference model response is thus defined as

$$e = x - x_m \tag{12}$$

This error is to be minimized through an adjustment mechanism, which can be designed through several different methods such as the MIT rule [8] or Lyapunov synthesis. The general adaptive control law for the system is written as

$$u_{ad} = \hat{\theta}_x x + \hat{\theta}_r r + \hat{\theta}_d = \hat{\theta}\omega \tag{13}$$

where $\hat{\theta}_x$, $\hat{\theta}_r$ and $\hat{\theta}_d$ are the adjustable control parameters. The time varying adaptive parameter, $\hat{\theta}$, have the dimensions of $m \times p$ where $p = m + n + 1$, and $\omega^T = \begin{bmatrix} x^T & r^T & 1 \end{bmatrix}$ is the regressor vector of dimension p.

Using Lyapunov stability synthesis [13], the adjustment mechanism for the control parameters as written in (14) gives a stable system.

$$\dot{\hat{\theta}} = -\mathbf{\Gamma}\omega e^T \mathbf{P}\mathbf{B} \tag{14}$$

where $\mathbf{\Gamma} \in \Re^{p\times p}$ is a diagonal and positive definite matrix of adaptive gains, and $\mathbf{P} \in \Re^{n\times n}$ is a symmetric positive definite solution of the Lyapunov equation

$$\mathbf{A}_m^T P + \mathbf{P}\mathbf{A}_m = -\mathbf{Q} \tag{15}$$

where $\mathbf{Q} \in \Re^{n\times n}$ is an arbitrary positive definite matrix.

It can be further shown that all the signals are bounded, thus through Barbalat lemma [22], the system is asymptotically stable and the error is guaranteed to converge to zero. The advantage of using Lyapunov stability theory to find the update law is that it requires little computing power and it ensures the stability of the closed loop system. The MIT rule, for example, does not guarantee closed loop system stability. However, the Lyapunov technique does not necessarily ensure parameter convergence although this technique assures tracking error convergence [23].

1) MRAC Design Parameters: The reference model for the adaptive controller was designed to have a response characteristic similar to that of a quadrotor controlled with the LQR. Therefore, the closed loop dynamic model of the plant with the LQR controller is used as a reference model. The closed loop system is therefore equivalent to the baseline LQR control system in the nominal case, but in the case of response degradation due to parameter changes or failures, the adaptive controller will in effect retain the nominal performance. The reference model has the same structure as (4), with the control law u as specified in (5).

An integral part of designing an adaptive controller is to choose reasonable values of adaptation gains as specified in the diagonal matrix Γ. The gain affects the rate of adaptation and is typically kept small to avoid instabilities. The appropriate gains for this simulation have been chosen through several trial simulation runs, and it is observed that the gain values as given in (16) provide a satisfactory rate of adaptation without inducing significant oscillation.

$$\mathbf{\Gamma} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \tag{16}$$

III. Results

A. Quadrotor Simulation Test

The following test was conducted to investigate the capability of LQR and MRAC to maintain good performance under the presence of parametric uncertainties. The act of the quadrotor picking an unknown object is simulated as an additional mass added to the dynamic model. Fig. 4 shows the simulated quadrotor height response. At 25 seconds of simulation, the quadrotor mass is increased instantaneously by 0.3kg, which is more than half of the original weight. It is observed that both LQR and MRAC exhibit some degradation in performance at the time the mass is added. Although the LQR deteriorates significantly when compared with MRAC, both systems eventually regained performance close to the nominal case. This is because both controllers have an integral action on the altitude states, which effectively remove the tracking error. The major difference between the two is that MRAC is able to retain good performance with minimal oscillation during the instantaneous change in plant parameter. This is because the adjustable controller parameters changed accordingly to compensate for the additional mass immediately after the mass is added, as shown in Fig. 5.

The observation from this simulation shows that with a reasonable choice of adaptive gain, Γ, the output of the plant is successfully driven towards that of the reference model. This is possible even without a full knowledge of the plant parameters. However, the initial response is jerky and the jerk is consistent with the transient phase of the adjustable controller parameters. The tracking response then becomes ideal once the parameters reached to their respective constant values. Since the controller parameters all starts from zero value, the initial tracking error is high thus this causes rapid changes in the controller parameters. This is unfortunately, one

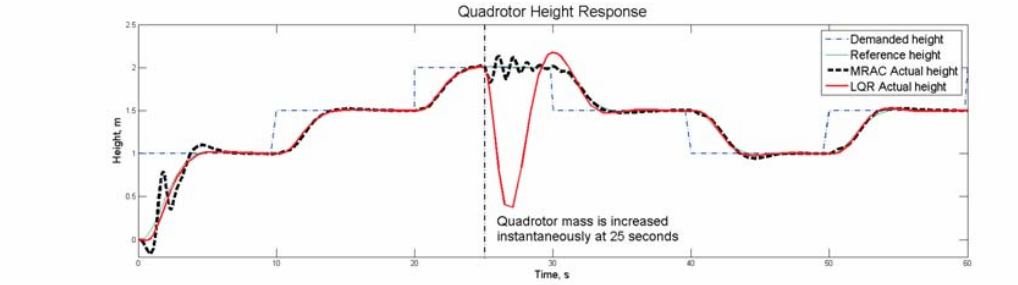

Fig. 4. Simulation of Quadrotor Height Response

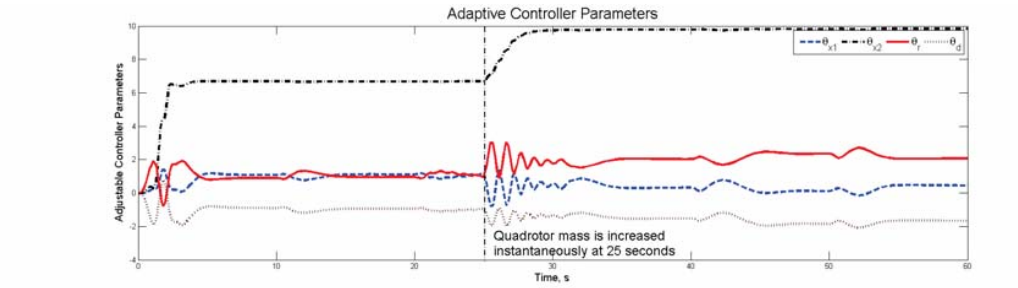

Fig. 5. MRAC Controller Parameters

of the disadvantageous inherent characteristic of MRAC. The controller does not know anything about the plant, and because of that tracking is initially poor. But once the tracking error has been identified, the adaptation process immediately takes in effect, and the error converges to zero through the appropriate change in controller parameters. This undesirable oscillation in the response can be minimized by choosing an appropriate initial conditions for the controller parameters close to their convergent values.

In this simulation, one of the reasons why the MRAC performance is good is because a relatively high adaptive gain is used. Although higher gains can give faster adaptation, it also cause more oscillation which might lead to instability due to excitation of unmodeled dynamics. Therefore, this design must be implemented with caution in practise.

It is worth mentioning that LQR controller can also be optimized further and the increased LQR gains can better compensate for the increased mass in the plant thus reducing the degradation in tracking response. However, since LQR is a fixed gain controller, it cannot provide the same level of robustness towards parametric uncertainties as the varying gain adaptive controller.

B. Quanser Experimental Test

Experimental tests were conducted to verify the underlying theories of LQR and MRAC. In order to minimize implementation difficulties that might arise from experimental tests on an actual quadrotor, the preliminary test runs were conducted on a Quanser 3-degree of freedom helicopter [24] as shown in Fig. 6. The helicopter consists of two propellers powered by electric motors located on each end of a rectangular frame that is mounted on one end of a long

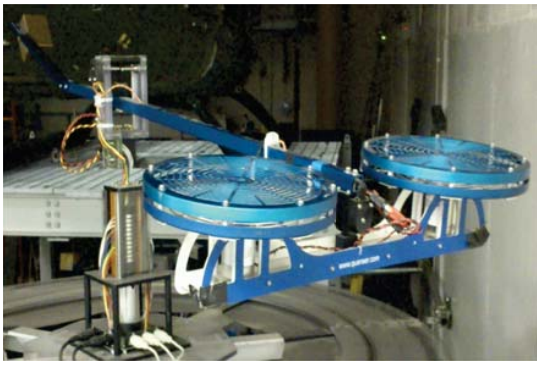

Fig. 6. Quanser 3-DOF Helicopter

arm and a counter weight attached to the other end of the arm. The helicopter frame is suspended from an instrumented joint, giving a total of 3 degrees of freedom motion in elevation, pitch, and travel. For this experiment, the LQR and MRAC is implemented on the elevation control, while the default PID baseline controller is used for pitch and travel control.

In the Quanser dynamic model, the weight of the helicopter body is specified as $70g$. An LQR controller was designed specifically for this system configuration, and the response was shown to be close to optimal in simulation. An MRAC controller is also designed by building a reference model based on the LQR response. In the experimental test, the helicopter weight is increased by shifting the counter weight further in on the arm. The new weight is $125g$, which is more than twice its initial weight. The experiment started with MRAC disconnected from the loop, giving LQR full

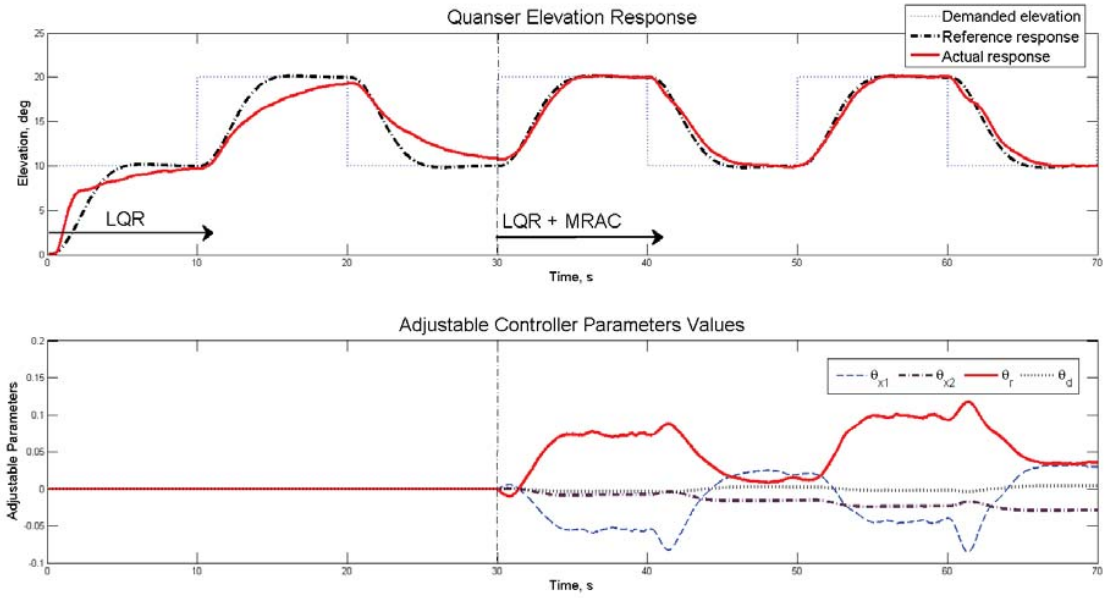

Fig. 7. Experimental Elevation Test on Quanser Helicopter: Quanser weight is increased by 78% of the modelled weight

control authority. And then at 30 seconds, MRAC is connected and working in parallel with the LQR. The Quanser elevation response is shown in Fig. 7. The plot shows clearly that the response degraded from the ideal response when the system is controlled by LQR. Immediately after MRAC is connected, the response immediately improved and tracked the ideal response properly with minimal error. The response improvement is due to the adjustable controller gains that change on-line when MRAC is switched on.

C. Discussion

The LQR controller was designed for a linearized system, in which motor dynamics and other uncertainties were neglected, but in real life the effect of motor dynamics is inherent to the system. Meanwhile, the MRAC controller was also designed without having full knowledge of the actual plant. Overall, the tracking response with both LQR and MRAC are satisfactory. It is observed that the LQR controller is effective for tracking if the gains are chosen in an optimal manner. However, the gains only work well exclusively for the linearized plant thus it will not work if the plant deviates far from the trim condition. Both simulation and experimental results show that the fixed gain controller is incapable of maintaining good performance when the model parameters are changed.

Adaptive controller, on the other hand, is capable of learning on-line, which maintains the desired performance in the presence of parametric uncertainties. Although MRAC is capable of driving the response characteristic to the desired response with time, it usually starts with an unsatisfactory response with high overshoot if the initial error is large. This is always the case at the start of the experiment when the adjustable parameters are far from the true value. The controller can make a lot of correction in a short amount of time, and the value of the adaptive gains play a crucial role in reducing or increasing this effect. Interestingly, in the experiment as shown in Fig. 7, the oscillatory response when MRAC is first switched on is effectively removed. When both LQR and MRAC are put together in a controller, the fixed gain controller will ensure that the response is within the desired criteria thus a huge shift in adjustable parameter values can be avoided.

There are several challenges identified in the design aspect of MRAC. The most prominent one is the difficulty in choosing the right adaptive gains, Γ. Currently in this experiment, the gains are tuned via trial and error. It is observed that too low gains give inadequate adaptation, while too high gains induce unwanted oscillation in the response. Another challenge is to deal with drifting of the adjustable controller parameters. This drift poses a threat of instability in the long run because the parameters might drift to a value that causes the poles of the system to cross over to the right half plane. The limiting factor for both controller implementations is motor saturation. This limits the degree of changes in the system parameters, since the motor must be able to generate the amount of thrust as commanded by the controller in order to achieve a stable tracking. This limitation can also cause gain windup in the adaptive controller.

IV. Conclusion

The results from both simulation and experimental experiment support the author's claim that adaptive control is a viable method for ensuring the stability of a vehicle with changing parameters. However, adaptive control alone is not ideal because fast adaptation can lead to undesirable

oscillations and instability. The combination of a fixed gain and an adaptive controllers result in enhanced tracking performance and robustness to failures and parametric uncertainties.

The future work will include the use of indirect method of adaptive control to estimate the changes in the plant parameters and strategies for dealing with the requirement of persistency of excitation.

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