International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:6, No:8, 2012

Minimizing Makespan Subject to Budget Limitation in Parallel Flow Shop

Amin Sahraeian

***Abstract*—**One of the criteria in production scheduling is Make Span, minimizing this criteria causes more efficiently use of the resources specially machinery and manpower. By assigning some budget to some of the operations the operation time of these activities reduces and affects the total completion time of all the operations (Make Span). In this paper this issue is practiced in parallel flow shops. At first we convert parallel flow shop to a network model and by using a linear programming approach it is identified in order to minimize make span (the completion time of the network) which activities (operations) are better to absorb the predetermined and limited budget. Minimizing the total completion time of all the activities in the network is equivalent to minimizing make span in production scheduling.

***Keywords*—**parallel flow shop, make span, linear programming, budget

I. Introduction

THE part of production scheduling is flow shop scheduling, flow shop production system in turn is divided to three main categories: a) simple flow shop, b) hybrid flow shop and c) parallel flow shop. In real world most of the flow shops are in the form of hybrid, in which the execution of each job must go through multiple stages in one specific order and at each stage there are parallel machines available to process the jobs that have entered the stage [7]. Hojjati and Sahraeian also have solution for minimizing make span in hybrid flow shop systems [1]. It seems that the recent paper is suggested by Hojjati and et al which minimizes make span in hybrid flow shop by assigning some budget for crashing the activities with the goal of minimizing make span.

Most of the research works for scheduling problems are done in the simple flow shop [4], [8]. In these production systems there is only one machine in each stage for processing the different jobs.

In simple flow shop when there are two stages, Johnson suggests almost an optimal solution to minimize make span. When the number of stages increases to more than 3 stages Campbell and et al suggest their solution [6]. But parallel flow Shop is an np-hard problem and non-polynomial time algorithm is expected for these types of problems. The development of heuristic algorithms guarantees good solutions, especially for large size problems [2], [3], [5].

A. Sahraeian is with the Industrial Engineering Department, University of Payame noor, Asaluyeh, Iran (phone: +98- 772-7374710; fax: +98-772-7320858;e-mail: aminsahraeyan2008@gmail.com).

In this article a parallel flow shop is practiced and it is tried to crash the operations by assigning some budget, which results minimizing the total completion time of all the operations. Here at first the terminology of the approach is presented, then the general formula for n jobs with m machines in each flow shop and k flow shop are modeled, it is followed by a numerical example and finally the problem is solved using a linear programming algorithm. The general approach is to convert the parallel flow shop system to a network model. Finally by the use of linear programming, It is tried to minimize the completion time of the last node.

II. Nomenclature

The following terminology is used for modeling the problem:

M: number of machine.
N: number of Job.
K: number of flow shop.
r: job number.
m: machine number.
S: flow shop number.
Tj: starting time of node j.
Jrms: job r on machine m in flow shop s.
i,j : activity from node i to node j.
Di,j: normal duration time of activity from node i to node j.
$D_{f(i,j)}$: minimum crashing time of activity from node i to node j.
di,j: crashed duration time of activity from node i to node j.
Ci,j: slope of crashing cost of activity from node i to node j.
B: predetermined budget.

III. Converting Parallel Flow Shop into Network Model

We can illustrate a general form of parallel flow shop with n jobs m machines, and k flow shop as in Fig. 1.

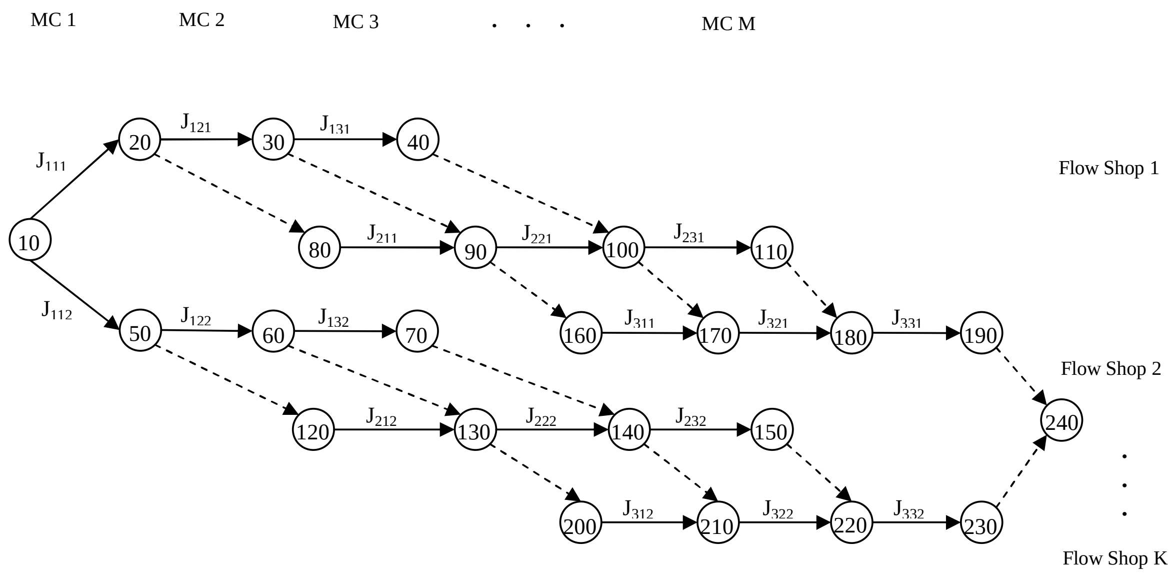

Fig. 1 General model of parallel flow shop

IV. Assumptions

The following assumptions are considered:

1. Shortest processing time (SPT) rule is used to assign the jobs to the machines.
2. Each machine starts at its earliest starting time possible.
3. The set up time is included in the processing time.
4. One unit of production for each job is considered.
5. Interruption of the machines is not allowed (no repairing during processing).
6. Each machine can process only one job at a time point.

Each operation has a predecessor which is shown in table 1. There are two sets of predecessors, one, the operational constraint, for which every job should be processed in its earlier flow shop, and second technological constraint for which each machine should operate the jobs in chronological .

TABLE I
PREDECESSORS FOR GENERAL MODEL

Flow shop	Activity	predecessor	Duration time
1	J_{rm1} r=1,2,…,N m=1,2,…,M	J_{rms} / r = r ,m = m-1 , s=1 J_{rms} / r = 1 or 2 or…or r-1, m = m, s = 1	D_{rm1}
2	J_{rm2} r=1,2,…,N m=1,2,…,M	J_{rms} r = r ,m = m-1 , s=2 J_{rms} r = 1 or 2 or…or r-1, m = m, s = 2	D_{rm2}
.	.	.	.
.	.	.	.
.	.	.	.

V. Problem Formulation

The problem can be formulated as follows:

$$MinZ = T_N - T_1$$

$$ST.$$

$$\sum\sum C_{i,j}(D_{i,j} - d_{i,j}) \le B$$

$$T_j - T_i \ge D_{i,j}$$

$$D_{f(i,j)} \le d_{i,j} \le D_{i,j}$$

$$T_i, T_j, d_{i,j} = \text{int}\,eger$$

VI. Numerical Example

The methodology is illustrated using a numerical example with 3 jobs, 2 flow shops and 3 machines in each Flow shop. According to SPT method, the sequence has been obtained as A-B-C with corresponding processing times as given in table 2.

TABLE II
THE PROCESSING TIME

Job	Flow shop 1			Flow shop 2		
	M# 1	M# 2	M# 3	M# 1	M# 2	M# 3
A	3	2	1	4	1	2
B	5	2	1	3	4	3
C	3	6	2	2	3	3

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:6, No:8, 2012

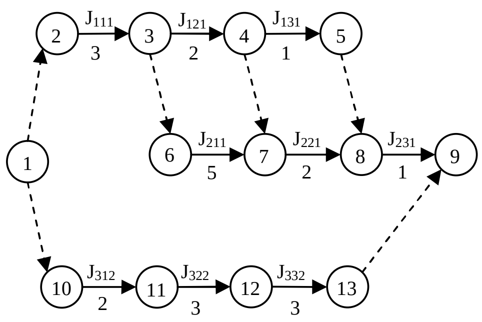

Fig. 2 The network of numerical example

TABLE III
PREDECESSOR OF THE NUMERICAL EXAMPLE

Node (i,j)	Activity (J_{rms})	Predecessor ---	Duration time ($D_{i,j}$)
2,3	J_{111}		3
3,4	J_{121}	J_{111}	2
4,5	J_{131}	J_{121}	1
6,7	J_{211}	J_{111}	5
7,8	J_{221}	J_{211}, J_{121}	2
8,9	J_{231}	J_{131}, J_{221}	1
10,11	J_{312}	---	2
11,12	J_{322}	J_{312}	3
12,13	J_{332}	J_{322}	3

Now we should assign some budget to some activities (operation) for which their time can be reduced. These are shown in table 4.

TABLE IV
THE ASSUMPTIONS OF THE EXAMPLE

Node (i,j)	Activity (J_{rms})	Predecessor	Duration time ($D_{i,j}$)	Minimum duration time ($D_{f(i,j)}$)	Cost Slope ($)
2,3	J_{111}	---	3	2	1550
3,4	J_{121}	J_{111}	2	2	---
4,5	J_{131}	J_{121}	1	1	---
6,7	J_{211}	J_{111}	5	3	1200
7,8	J_{221}	J_{211}, J_{121}	2	2	---
8,9	J_{231}	J_{131}, J_{221}	1	1	---
10,11	J_{312}	---	2	2	---
11,12	J_{322}	J_{312}	3	3	---
12,13	J_{332}	J_{322}	3	2	1700

VII. PROBLEM SOLUTION

Considering the information given for the problem in tables 3 and 4 and Fig. 2 the objective function and the constraints can be written as follows:

$$MinZ \;\; = T_9 - T_1$$

$$Subject \quad to:$$

$$1550 \times (3 - d_{23}) + 1200 \times (5 - d_{67}) + 1700 \times (3 - d_{1213}) \le 3000$$

$$\begin{array}{lll}
T_2 - T_1 \ge 0 & T_9 - T_8 \ge d_{89} & 3 \le d_{67} \le 5 \\
T_3 - T_2 \ge d_{23} & T_8 - T_5 \ge 0 & 2 \le d_{1213} \le 3 \\
T_4 - T_3 \ge d_{34} & T_{10} - T_1 \ge 0 & d_{34} = 2 \\
T_5 - T_4 \ge d_{45} & T_{11} - T_{10} \ge d_{1011} & d_{45} = 1 \\
T_6 - T_3 \ge 0 & T_{12} - T_{11} \ge d_{1112} & d_{78} = 2 \\
T_7 - T_6 \ge d_{67} & T_{13} - T_{12} \ge d_{1213} & d_{89} = 1 \\
T_7 - T_4 \ge 0 & T_9 - T_{12} \ge 0 & d_{1011} = 2 \\
T_8 - T_7 \ge d_{78} & 2 \le d_{23} \le 3 & d_{1112} = 3 \\
 & & T_i, T_j, d_{i,j} = \text{int}\,eger
\end{array}$$

The problem is solved by Lindo and the result is shown in table 5.

TABLE V
THE RESULT OF THE EXAMPLE

Activity ($d_{i,j}$)	Crashed time	Budget used
d_{23}	3	0
d_{34}	2	0
d_{45}	1	0
d_{67}	3	2400
d_{78}	2	0
d_{89}	1	0
d_{1011}	2	0
d_{1112}	3	0
d_{1213}	3	0
Objective	9	2400

By assigning different budgets, different results can be obtained, this is called the sensitivity analysis of the problem, the result can be shown as in tableVI.

TABLE VI
SENSITIVITY ANALYSIS OF THE NUMERICAL EXAMPLE

Budget ($)	Make span
0	11
2000	10
3000	9

VIII. CONCLUSION

It is shown that parallel flow shop problems can be converted to network model and using a linear programming formulation the critical activities are determined. Assigning some budget to activities that can be crashed by time, causes to reduce the completion time of all the project or make span, this by itself causes better use of the resources specially machinery and manpower, which by itself increase productivity.

For further research it is suggested to apply the methodology to some other systems like job shops.

ACKNOWLEDGMENT

The author is very grateful to Pars Oil and Gas Company for Financial Support.

REFERENCES

[1] S.M.H. Hojjati, and A. Sahraeyan," Minimizing Makespan Subject to Budget Limitation in Hybrid Flow-Shop," 39th International Conference on Computer and Industrial Engineering, Troyes, France 6-8 July 2009, pp 18-22.

[2] W. Bozejko, and J. Pempera,"Parallel tabu search algorithm for the permutation flow shop problem with criterion of minimizing sum of job completion times," Human System Interactions, Conference in Kralow 25-27 May 2008; 825-829.

[3] X. Zhang,and S.van de Velde," Approximation Algorithms for the Parallel Flow Shop Problem," European Journal of Operational Research (2011)B. Smith, "An approach to graphs of linear forms (Unpublished work style)," unpublished.

[4] M. Seda," Mathematical Models of Flow Shop and Job Shop Scheduling Problems," World Academy of Science, Engineering and Technology 31 2007.

[5] B. Al-Dulaomi, and A.Ali, . "A Novel Genetic Algorithm Approach for Solving Flow Shop Problem," International Journal of Computer Science and Network Security,2008, Vol. 8, No.9, pp. 229-235.

[6] H.G. Campbell, H.G. Dudeck, M.L. Smith," A Heuristic Algorithm for the n Job, m Machine Sequencing Problem," Management Science, No.16, pp. 630-637.

[7] J. Havill, and W. Mao," On-line Algorithms for hybrid Flow Shop Scheduling, "2002.

[8] A. Khodadadi," Solving Constrained Flow-Shop Scheduling Problem with Three Machines," International Journal of Academic Research, 2011, Vol. 3, No. 1, pp. 31 38-40.