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Minimizing Examinee Collusion with a Latin-Square Treatment Structure

M. H. Omar

***Abstract*—**Cheating on standardized tests has been a major concern as it potentially minimizes measurement precision. One major way to reduce cheating by collusion is to administer multiple forms of a test. Even with this approach, potential collusion is still quite large. A Latin-square treatment structure for distributing multiple forms is proposed to further reduce the colluding potential. An index to measure the extent of colluding potential is also proposed. Finally, with a simple algorithm, the various Latin-squares were explored to find the best structure to keep the colluding potential to a minimum.

***Keywords*—**Colluding pairs, Scale for Colluding Potential, Latin-Square Structure, Minimization of Cheating.

I. INTRODUCTION

CHEATING on tests is a serious academic problem and can potentially compromise precision in measuring examinee's true academic performance [1]. Although cheating on paper-and-pencil tests have been discussed in the academic literature [1], the best method to minimize this academic dishonesty has not been discussed extensively. Currently, many testing programs use multiple forms of a test to minimize cheating. These forms are often packaged in spiral order prior to administration [2].

One advantage of multiple test forms is that indeed cheating by collusion could be cut down. However, as will be presented in this article, the potential for collusion may still be quite extensive. As a result, most researchers concern themselves with finding ways to detect collusion *after* a test has been administered [3], [4].

This article offers an alternative approach to the current practice of spiraling to minimize academic collusion further *during* a test administration.

II. SPIRALING OF MULTIPLE TEST FORMS

A. Spiraling

Spiraling is an activity where multiple test forms are arranged in order such as A, B, C and D and handed out to examinees according to this order [2]. This ensures that either the same row or column will not have the same form. One of the main objectives of introducing multiple and parallel test forms of a test is to cut down the potential for cheating by academic collusion. However, in this section, it can be shown that the total potential for cheating during this spiraling activity can still be quite extensive and can be problematic without proper proctoring.

Consider Fig. 1 below. In a class of 25 students, if one test form is used, there can be up to 72 potentially colluding pairs of examinees. Examinees can collude row-wise (4*5 or 20 pairs), column-wise (4*5 or 20 pairs) or diagonal-wise (2*4*4 or 32 pairs). However, if 2 parallel forms such as in Fig. 1(b) are used, the total number of potential collusion can be reduced to 20 row-wise conspiring pairs. For 5 parallel forms such as in Fig. 1(c), the number of potentially colluding pairs remains at 20. This is because it is immaterial to examinees taking form B what other test form his/her neighbor takes as long as it is not form B. That is, even with large forms, the number of colluding pairs is still quite large. This makes the task of administrating the test quite difficult.

A A A A A	A B A B A	A B C D E
A A A A A	A B A B A	A B C D E
A A A A A	A B A B A	A B C D E
A A A A A	A B A B A	A B C D E
A A A A A	A B A B A	A B C D E
a. single form	b. 2 forms	c. 5 forms

Fig. 1 Illustration of the Maximum Number of Colluding Pairs by the Number of Spiraled Forms

In general, the maximum number of potential collusion is given by Theorem 1 below.

Theorem 1. Let R = the number of rows of examinee seats. Also let C = number of columns. If only one test form is used, then M, the maximum number of potentially colluding pairs, is

$$M = C(R - 1) + R(C - 1) + 2(R - 1)(C - 1). \quad (1)$$

Proof. The maximum number of potentially colluding pairs, M = row-wise colluding pairs + column-wise colluding pairs + diagonal-wise colluding pairs. This total number simplifies as in Theorem 1.

Corollary 1. If $C = R$, then the maximum number of colluding pairs, is given as

$$M = 2(R - 1)(2R - 1).$$

Proof. If $C = R$, then from theorem 1

$$M = R(R - 1) + R(R - 1) + 2(R - 1)(R - 1)$$

which when factorized, simplifies as stated in Corollary 1.

When spiraling is done, some improvement on the maximum number of potentially colluding pairs can be reduced as stated in the next theorem.

Theorem 2. Let R = the number of rows of examinee seats and C = number of columns of examinee seats. If two forms are used and these forms are spiraled across columns, then the number of potentially colluding pairs is given as follows.

$$M = C(R - 1). \qquad (2)$$

Proof. Since examinees seated in adjacent chairs receive different forms, collusion is not beneficial for these examinees. Thus, the maximum number of potentially colluding pairs is given by the first term in Theorem 1 only.

Remark. If the number of forms is more than two but less than min(R,C), and spiraling is used then, the maximum number of potentially colluding pairs is still exactly equal to M in (2). This is because it is immaterial what test form examinees in adjacent columns receive as long as they are not the same forms.

To evaluate the effect of spiraling on reducing the maximum potential number of colluding pairs, a scale for these maximum numbers is needed.

B. Scale for Potential Collusion

An approach would be to divide the maximum number by the total number of examinees. This provides a natural interpretation where the maximum number of potentially colluding pairs is expressed in the terms of the number of examinees. The following theorem provides this index for the single form and spiraling situations.

Theorem 3. Let R = the number of row of examinee seats, C = the number of columns of examinee seats, and f = the number of spiraled forms. Then the maximum ratio of potentially colluding pairs K is given by

a) $K = 4 - 3/R - 3/C + 2/RC$ if $f = 1$,

b) $K = \dfrac{R-1}{R}$ if $2 \le f \le R$ and spiraling across columns,

c) $K = \dfrac{C-1}{C}$ if $2 \le f \le C$ and spiraling across rows.

Proof. If $f = 1$ form only, then dividing M in (1) with the total number of examinees, RC, gives

$$K = \frac{C(R-1) + R(C-1) + 2(R-1)(C-1)}{RC}$$

$$= \frac{(R-1)}{R} + \frac{(C-1)}{C} + 2\frac{(R-1)(C-1)}{RC}$$

$$= 1 - 1/R + 1 - 1/C + 2(1 - 1/R)(1 - 1/C)$$

which simplifies to Theorem 3(a).

When the number of spiraled forms is $2 \le f \le R$, where examinees seated in different columns get different forms,

$$K = \frac{C(R-1)}{RC}$$

which simplifies to Theorem 3(b).

When the number of spiraled forms is $2 \le f \le C$, the spiraled forms have to be distributed across rows so that examinees seated in different rows receive different forms. Since adjacent examinees sitting in each row will have different forms, collusion can really take place across columns only. Thus,

$$K = \frac{R(C-1)}{RC}$$

which simplifies to Theorem 3(c).

Remark. As the number of rows grow large ($R \rightarrow \infty$), spiraling (compare Theorem 3 part b to part a) reduces the maximum ratio of potentially colluding pairs by about $1/(4 - 3/C)$. Under spiraling, although the ratio K is much smaller than when a single form is used and is less than 1, it converges to 1 as the number of rows grows large.

Obviously, though spiraling reduces the maximum potential for collusion, the potential for this activity is still quite large and not ignorable. An alternative is thus needed.

III. A Latin Square Treatment Design: An Alternative to Straight Spiraling Order

A Latin-square treatment structure has been used quite extensively in many applications such as cryptology [5] and experimental designs [6]. However, it has not yet been explored in the context of standardized testing.

Consider the situation where the number of rows of examinee chairs is the same as the column of examinee seats (i.e. $R = C$). In this case, the examination venue is designed as a square. Let us consider what will happen if multiple forms were used and randomized using a Latin-Square treatment structure. For this Latin-Square treatment design, the same test form cannot occupy the same row or column twice. See Fig. 2 below for three different situations where a Latin-Square design is used. To easily see the potential colluding pairs, adjacent examinees with the same forms are highlighted and written in bold fonts.

A	**B**	C
C	**A**	**B**
B	**C**	**A**

a. 4 pairs, K= 4/9

C	**A**	**B**	D
D	**B**	**A**	C
A	**C**	**D**	B
B	**D**	**C**	A

b. 4 pairs, K= 4/16

A	B	C	D	E
C	D	**E**	**A**	B
B	**E**	**A**	**C**	D
D	**A**	**B**	E	**C**
E	C	D	**B**	A

c. 5 pairs K= 5/25

Fig. 2 The Maximum Number of Colluding Pairs by the Latin-Square Randomization of Spiraled Forms

Clearly, Latin-Square treatment designs can minimize the maximum number of colluding pairs. However, two questions remain. One, what is the minimum number for the maximum colluding pairs in each Latin-Square treatment design? Two, is the minimum the same for any Latin-Square?

IV. Can a Latin-Square Structure Further Minimize Colluding Pairs?

A Latin-square treatment structure has only the following restriction for placing of treatments in the squares.

1) *each form occupies a column only once and*
2) *each form occupies a row only once.*

However, this does not guarantee that examinees cannot collude. Thus, another restriction is needed to further minimize the potential for collusion. This restriction is to avoid or at the very least, minimize instances where examinees with the same form are placed in adjacent seats.

In essence, an algorithm to produce the minimum colluding pairs can be outlined as below:

1) *each form occupies a column only once*
2) *each form occupies a row only once and*
3) *each form must be placed in the maximum number of nonadjacent seats.*

With this algorithm in mind, consider the following Latin-Square cases.

B. Case 1: a 3x3 Latin-Square Design

With the algorithm discussed earlier, it appears that there are only limited possibilities for the 3x3 Latin-Square structure as shown in Fig. 3 below. This is because the first part of

A B C	A **B C**	**B A** C
C A B	**B C A**	**C B A**
B **C A**	**C A** B	A **C B**

a. 4 pairs, *K*= 4/9 b. 4 pairs, *K*= 4/9 c. 4 pairs *K*= 4/9

Fig. 3 The Maximum Number of Colluding Pairs by the 3x3 Latin-Square Structure of Spiraled Forms

the algorithm will always force one of the three forms to be in the diagonal due to limited space. Thus, the maximum number of potentially colluding pairs is always 4 for this case.

C. Case 2: a 4x4 Latin-Square Design

With the algorithm discussed earlier, it appears from Fig. 4 below that there are some possibilities for the 4x4 Latin-Square designs. Due to adequate space, in this 4x4 case, the

A B C D	C **A B** D	A B C D
D A B C	D **B A** C	C **D A B**
C D A B	A **C D** B	**D A B** C
B C D A	B **D C** A	B C D A

a. 9 pairs, *K*=9/16 b. 5 pairs, *K*=5/16 c. 3 pairs, *K*=3/16

Fig 4. Maximum Number of Colluding Pairs by Different 4x4 Latin-Square Randomization of Spiraled Forms

first two parts of the algorithm no longer forces one of the four forms to be in the diagonal. Thus, the third part of the algorithm can do a better job at minimizing the colluding pair potential. For this 4x4 case, the maximum potentially colluding pairs is 9. However, this can be reduced further to the bare minimum of 3 potentially colluding pairs as shown in Fig. 4(c). The task of proctoring an exam is thus simplified by concentrating on rows 2 and 3 during the invigilation of the exam. Test administrators can also increase the seating space between the second and third rows to make collusion physically challenging for these examinees.

D. Case 3: a 5x5 Latin-Square Design

With the algorithm discussed earlier, it appears that some possibilities exist for the 5x5 Latin-Square structure as shown in Fig. 5 below. Fig. 5(a) contains the worst number of potentially colluding pairs as highlighted. Fig. 5(b) shows a much smaller colluding pair potential. Fortunately, Fig. 5(c)

A B C D E	A B C D E	A B C D E
E A B C D	C D **E A** B	D E A B C
D E A B C	B **E A C** D	B C D E A
C D E A B	D **A B** E **C**	E A B C D
B **C D E A**	E C D **B** A	C D E A B

a. 16 pairs, *K*= 16/25 b. 5 pairs, *K*= 5/25 c. 0 pairs, *K*= 0

Fig. 5 The Maximum Number of Colluding Pairs by the 5x5 Latin-Square Structure of Spiraled Forms

shows zero colluding pairs which was not achievable in the other spiraling designs.

V. Conclusion

This article presented the issue of cheating on examinations through collusion by adjacently seated examinees. One hazard of cheating is it compromises the integrity and precision of measurement.

Thus multiple forms are often used to counter this problem. These forms are administered in spiraled order such that examinees seated in adjacent rows or columns will have different forms. In this article, it is shown that the maximum potential number of colluding pairs was still quite large even with spiraling.

However, if spiraling is combined with randomization using the Latin-square treatment structure, the maximum potential number of colluding pairs can be reduced even further. In particular, the 5x5 Latin-square treatment design can reduce the potential number of colluding pairs to the bare minimum of zero.

For other Latin-square structures, although the absolute minimum potential for collusion is not zero, knowing where potential collusion is concentrated can help focus the proctoring efforts.

This knowledge is important in minimizing cheating activities during administration of standardized examinations.

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