

# MHD Chemically Reacting Viscous Fluid Flow towards a Vertical Surface with Slip and Convective Boundary Conditions

Ibrahim Yakubu Seini, Oluwole Daniel Makinde

**Abstract**—MHD chemically reacting viscous fluid flow towards a vertical surface with slip and convective boundary conditions has been conducted. The temperature and the chemical species concentration of the surface and the velocity of the external flow are assumed to vary linearly with the distance from the vertical surface. The governing differential equations are modeled and transformed into systems of ordinary differential equations, which are then solved numerically by a shooting method. The effects of various parameters on the heat and mass transfer characteristics are discussed. Graphical results are presented for the velocity, temperature, and concentration profiles whilst the skin-friction coefficient and the rate of heat and mass transfers near the surface are presented in tables and discussed. The results revealed that increasing the strength of the magnetic field increases the skin-friction coefficient and the rate of heat and mass transfers toward the surface. The velocity profiles are increased towards the surface due to the presence of the Lorentz force, which attracts the fluid particles near the surface. The rate of chemical reaction is seen to decrease the concentration boundary layer near the surface due to the destructive chemical reaction occurring near the surface.

**Keywords**—Boundary layer, surface slip, MHD flow, chemical reaction, heat transfer, mass transfer.

## I. INTRODUCTION

THEORETICAL research on hydromagnetic boundary layer flow of chemically reacting viscous fluid has seen a tremendous increase in recent times due to their numerous engineering and industrial applications. In the cooling of electronic devices and emergency shutdown of nuclear reactors, heat is transferred. Other relevant applications are found in wire drawing, hot rolling, paper production, oil recovery, power generation, metal casting, etc. Ibrahim and Makinde [1], [2] examined a chemically reacting MHD boundary layer flow of heat and mass transfer past a low-heat-resistant sheet and with suction moving vertically downwards. Cao and Baker [3] investigated the slip effects on mixed convection flow and heat transfer from vertical plates whilst Zhu et al. [4] analytically studied the stagnation-point flow problem with heat transfer over a stretching sheet using the homotopy analysis method. Hassanien and Gorla [5] obtained results for the stagnation-point flow of micropolar fluids over non-isothermal surfaces whilst [6] considered the similarity

solutions of the Navier-Stokes equations for the stagnation-point flow towards a flat plate with slip. Kechil and Hashim [7] analyzed the boundary layer problem over a nonlinearly stretching sheet in a magnetic field with chemical reaction. A steady two-dimensional MHD stagnation point flow towards a nonlinearly stretching surface has been reported by [8] whilst a computational modelling of MHD unsteady flow and heat transfer over a flat surface with Navier slip and Newtonian heating investigated by [9]. Furthermore, [10] investigated the effects of chemical reaction on boundary layer flow past a vertically stretching surface in the presence of internal heat generation and observed that both the velocity and temperature profiles increased significantly, as the heat generation parameter increased. The influence of magnetic field on liquid metal free convection in an internally heated cubic enclosure was reported by [11]. Similarly [12] analyzed the mixed convection flow in non-Newtonian fluids along a vertical plate in porous media with surface mass transfer. Arthur et al. [13] recently analyzed the chemically reacting hydromagnetic flow over a flat surface in the presence of radiation with viscous dissipation and convective boundary conditions. Arthur and Seini [14] then analysed the MHD thermal stagnation point flow towards a stretching porous surface whilst [15] studied the heat and mass transfer problem over a vertical surface with convective boundary conditions in the presence of viscous dissipation and  $n^{\text{th}}$  order chemical reaction.

The present investigation examines the MHD chemically reacting viscous fluid flow towards a vertical surface with slip and convective boundary conditions. The paper is organized into the following sections: mathematical formulation of the problem is presented in Section II. The partial differential equations describing the problem are transformed to ordinary differential equations using suitable similarity variables. In Section III, the solution method is presented. Results and discussions are presented in Section IV whilst Section V concludes the paper.

## II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a steady two-dimensional laminar boundary layer flow of a viscous, incompressible and electrically conducting fluid near the stagnation-point on a vertical surface, and assume that the velocity of the free stream is  $u_e(x)$ , the temperature and concentration of the plate are  $T_w(x)$  and  $C_w(x)$ , and the ambient fluid temperature and concentration

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are  $T_\infty$  and  $C_\infty$ , respectively, (Fig. 1).

The boundary layer equations describing the problem are the continuity, momentum, energy and the species concentration equations, respectively represented as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du}{dx} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (u - u_e) + g\beta_T (T - T_\infty) + g\beta_C (C - C_\infty) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c_p} (u - u_e)^2 \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \gamma (C - C_\infty), \tag{4}$$

where,  $u$  and  $v$  are velocities in the  $x$  and  $y$  directions, respectively,  $g$  is the acceleration due to gravity,  $T$  and  $C$  are the fluid temperature and concentration respectively while  $\beta_T$  and  $\beta_C$  are the thermal and volumetric expansion coefficients respectively. Furthermore  $\alpha$  is the thermal diffusivity,  $D$  is the mass diffusivity,  $B_0$  is the strength of the transverse magnetic field,  $\sigma$  is the electrical conductivity,  $\rho$  is the fluid density,  $\mu$  is the dynamic viscosity of the fluid,  $\gamma$  is the rate of chemical reaction,  $u_e$  is the free- stream velocity and  $c_p$  is the specific heat capacity at constant pressure. The boundary conditions similar to [16] are assumed:

$$u = L \frac{\partial u}{\partial y}, v = 0, T = T_w + k \frac{\partial T}{\partial y}, C = C_w, \text{ at } y = 0 \tag{5}$$

$$u \rightarrow u_e(x), T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } y \rightarrow \infty, \tag{6}$$

$$\text{Let } u_e = ax, T_w(x) = T_\infty + bx, C_w(x) = C_\infty + cx, \tag{7}$$

where  $a, b$  and  $c$  are constants.

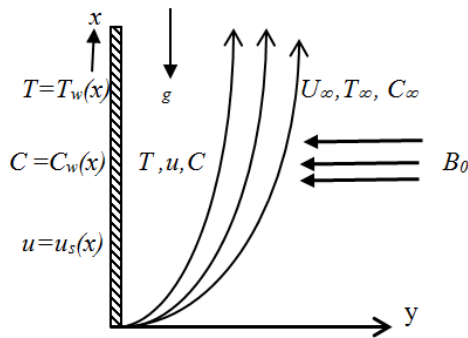


Fig. 1 Schematic diagram of the problem

We introduced the following similarity variables which transforms the partial differential equations to ordinary differential equations:

$$\eta = y \sqrt{\frac{a}{\nu}}, \psi = x \sqrt{a \nu} f, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}, \tag{8}$$

where,  $\eta$  is an independent dimensionless variable,  $y$  is the dimensionless temperature, and  $\psi$  is the stream function defined in the usual way as:

$$u = \frac{\partial \psi}{\partial y}, \text{ and } v = - \frac{\partial \psi}{\partial x}. \tag{9}$$

Equation (9) automatically satisfies the continuity equation (1). Substituting (8) and (9) into (2)-(4) transforms the modelled partial differential equations into higher order nonlinear ordinary differential equations:

$$f''' + ff'' - f'^2 - M(f' - 1) + 1 + Gt\theta + Gc\phi = 0, \tag{10}$$

$$\theta'' + \text{Pr} f\theta' - \text{Pr} f'\theta + \text{Pr} Ec f'^2 + \text{Pr} Ec M (f' - 1)^2 = 0, \tag{11}$$

$$\phi'' + Sc f\phi' - Sc f'\phi - Sc \beta \phi = 0, \tag{12}$$

with the boundary conditions transformed to

$$f'(0) = \delta f''(0), f(0) = 0, \theta(0) = 1 + Bi\theta'(0), \phi(0) = 1, \text{ at } y = 0, \\ f'(\infty) \rightarrow 1, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0, \text{ as } y \rightarrow \infty. \tag{13}$$

Here, the prime symbol denotes differentiation with respect to  $\eta$ ,  $\text{Pr} = \nu / \alpha$  is the Prandtl number,  $\delta = L(a/\nu)^{1/2}$  is the velocity slip parameter, and  $G_t = \frac{g\beta_T(T_w - T_\infty)}{a^2 x}$ , and  $G_c = \frac{g\beta_C(C_w - C_\infty)}{a^2 x}$  are the thermal and solutal Grashof numbers.  $M = \frac{\sigma B_0^2}{\rho a}$  is the magnetic parameter.  $Ec = \frac{a^2 x^2}{c_p(T_w - T_\infty)}$  is the Eckert number.  $Sc = \frac{D}{\nu}$  is the Schmidt number and  $\beta = \frac{\gamma}{a}$  is the reaction rate parameter.  $Bi = k\sqrt{a/\nu}$  is the Biot number. The physical quantities of interest to engineers are the skin-friction coefficient  $C_f$ , the local Nusselt number,  $Nu_x$  and the local Sherwood number,  $Sh_x$  which are proportional to  $f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  respectively.

III. NUMERICAL PROCEDURE

The non-linear odes (10)-(12) with the boundary conditions (13) have been solved numerically using the fourth order Runge-Kutta integration scheme with a modified version of the Newton-Raphson shooting method. We let,

$$f = x_1, f' = x_2, f'' = x_3, \theta = x_4, \theta' = x_5, \phi = x_6, \phi' = x_7 \quad (14)$$

Equation (14) transforms (10)-(12) to first order systems of differential equations as:

$$\begin{aligned} f' &= x_1' = x_2, \\ f'' &= x_2' = x_3, \\ f''' &= x_3' = x_2^2 - x_1x_3 + M(x_2 - 1) - 1 - Gtx_4 - Gcx_6, \\ \theta' &= x_4' = x_5, \\ \theta'' &= x_5' = -Pr x_1x_5 + Pr x_2x_4 - Pr Ecx_3^2 - Pr EcM(x_2 - 1)^2, \\ \phi' &= x_6' = x_7, \\ \phi'' &= x_7' = -Scx_1x_7 + Scx_2x_6 + Sc\beta x_6, \end{aligned} \quad (15)$$

subject to the following initial conditions:

$$\begin{aligned} x_1(0) &= 0, x_2(0) = \delta_2, x_3(0) = s_1, x_4(0) = 1 + Bi\delta_2(0) = s_2, x_5(0) = 1, \\ x_2(\infty) &\rightarrow 1, x_3(\infty) \rightarrow 0, x_4(\infty) \rightarrow 0. \end{aligned} \quad (16)$$

In the shooting method, the unspecified initial conditions;  $s_1$  and  $s_2$  in (16) are assumed and (15) integrated numerically as an initial valued problem to a given terminal point. The accuracy of the assumed missing initial condition is checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If differences exist, improved values of the missing initial conditions are obtained and the process repeated. The computations are done by a written programme which uses a symbolic and computational computer language (MAPLE). A step size of  $\Delta\eta = 0.001$  was selected to be satisfactory for a convergence

criterion of  $10^{-7}$  in nearly all cases. The maximum value of  $\eta_\infty$  to each group of parameters is determined when the values of unknown boundary conditions at  $\eta = 0$  not change to successful loop with error less than  $10^{-7}$ . From the process of numerical computations, the local skin-friction coefficient, the local Nusselt numbers, and the local Sherwood numbers, which are, respectively, proportional to  $f''(0)$ ,  $-\theta(0)$  and  $-\phi(0)$  were worked out and their numerical values presented in tables.

IV. RESULTS AND DISCUSSION

A. Numerical Results

Numerical results are computed for the reduced skin-friction coefficient,  $f''(0)$ , the rate of heat transfer,  $-\theta(0)$  and the rate of mass transfer  $-\phi(0)$  at the surface. Table I shows the effect of Prandtl number (Pr), magnetic field parameter (M), Eckert number (Ec), thermal and solutal Grashof numbers (Gt) and (Gc), Schmidt number (Sc) and the reaction rate parameter ( $\beta$ ) on the skin-friction coefficient, the Nusselt number and the Sherwood number.

It is observed that both the skin-friction coefficient ( $f''(0)$ ) and the rate of mass transfer ( $-\phi(0)$ ) decrease whilst the rate of heat transfer ( $-\theta(0)$ ) increases with increasing values of the Prandtl number (Pr). The magnetic field parameter (M) leads to an increase in the skin friction coefficient mainly due to the presence of the induced force known as the Lorenz force caused by the transverse magnetic field. This hastens the flow towards the vertical plate thereby increasing rate of heat and mass transfers at the surface. The Eckert number (Ec) leads to an increase in both the skin friction coefficient ( $f''(0)$ ) and the mass transfer rate ( $-\phi(0)$ ) but decreases the rate of heat transfer ( $-\theta(0)$ ) at the surface.

TABLE I  
EFFECTS OF CONTROLLING PARAMETERS ON LOCAL SKIN FRICTION COEFFICIENT, LOCAL NUSSULT NUMBER AND LOCAL SHERWOOD NUMBER

Pr	M	Ec	Gt	Gc	Sc	$\beta$	Bi	$f''(0)$	$-\theta(0)$	$-\phi(0)$
0.72	0.1	0.1	0.1	0.1	0.24	1	0.1	1.37101	0.63750	0.65964
4.0	0.1	0.1	0.1	0.1	0.24	1	0.1	1.35702	1.03523	0.65852
7.2	0.1	0.1	0.1	0.1	0.24	1	0.1	1.35318	1.17990	0.65828
0.72	0.5	0.1	0.1	0.1	0.24	1	0.1	1.51115	0.64189	0.66496
0.72	1.0	0.1	0.1	0.1	0.24	1	0.1	1.66967	0.64601	0.67045
0.72	1.5	0.1	0.1	0.1	0.24	1	0.1	1.81423	0.64910	0.67503
0.72	0.1	0.5	0.1	0.1	0.24	1	0.1	1.37505	0.45364	0.65992
0.72	0.1	1.0	0.1	0.1	0.24	1	0.1	1.38009	0.22326	0.66028
0.72	0.1	1.5	0.1	0.1	0.24	1	0.1	1.38511	0.00773	0.66064
0.72	0.1	0.1	0.5	0.1	0.24	1	0.1	1.54838	0.65427	0.66816
0.72	0.1	0.1	1.0	0.1	0.24	1	0.1	1.76045	0.67299	0.67786
0.72	0.1	0.1	1.5	0.1	0.24	1	0.1	1.96373	0.68978	0.68671
0.72	0.1	0.1	0.1	0.5	0.24	1	0.1	1.57710	0.65843	0.67060
0.72	0.1	0.1	0.1	1.0	0.24	1	0.1	1.82560	0.68181	0.68318
0.72	0.1	0.1	0.1	1.5	0.24	1	0.1	2.06558	0.70275	0.69474
0.72	0.1	0.1	0.1	0.1	1.78	1	0.1	1.35188	0.63446	1.60868
0.72	0.1	0.1	0.1	0.1	2.64	1	0.1	1.34836	0.63400	1.91967
0.72	0.1	0.1	0.1	0.1	0.24	2	0.1	1.36755	0.63693	0.80943
0.72	0.1	0.1	0.1	0.1	0.24	3	0.1	1.36488	0.63649	0.93857
0.72	0.1	0.1	0.1	0.1	0.24	1	0.5	1.36214	0.50009	0.65920
0.72	0.1	0.1	0.1	0.1	0.24	1	1.0	1.35526	0.39411	0.65887

The thermal and solutal Grashof numbers (Gt) and (Gc) respectively increase all the important physical parameters considered in the problem. The Schmidt number (Sc) and the reaction rate parameter ( $\beta$ ) lead to decreasing skin friction coefficient ( $f''(0)$ ) and the rate of heat transfer ( $-\theta'(0)$ ) but increasing rate of mass transfer ( $-\phi(0)$ ). Conversely, the Biot

number is observed to decrease the skin friction coefficient, and the rate of heat and mass transfers near the surface.

The results of the study compared to previous works for varying values of Prandl number under similar conditions were obtained for  $f''(0)$  and  $-\theta'(0)$  as shown in Tables II and III respectively. The results are in good agreement with previously published works in the literature testifying to the robustness and validity of the numerical approach.

TABLE II  
COMPARISON OF  $f''(0)$  FOR VALUES OF PR WHEN  $G_T = 1, Bi = 0, Ec = 0, M = 0$

Pr	Hassanien and Gorla [5]	Seini and Makinde [16]	Lok et al. [17]	Present study
0.7	1.70632	1.7063	1.7064	1.7063
1	-	1.6754	-	1.6754
7	-	1.5179	1.5180	1.5179
10	1.49284	1.4928	-	1.4928
20	-	1.4485	1.4486	1.4485
40	-	1.4101	1.4102	1.4101
50	1.40686	1.3989	-	1.3989
60	-	1.3903	1.3903	1.3903
80	-	1.3774	1.3774	1.3774
100	-	1.3680	1.3677	1.3680

TABLE III  
COMPARISON OF  $-\theta'(0)$  FOR VALUES OF PR WHEN  $G_T = 1, Bi = 0, Ec = 0, M = 0$

Pr	Hassanien and Gorla [5]	Seini and Makinde [16]	Lok et al. [17]	Present study
0.7	0.76406	0.7641	0.7641	0.7641
1	-	0.8708	-	0.8708
7	-	1.7224	1.7226	1.7224
10	1.94461	1.9446	-	1.9446
20	-	2.4576	2.4577	2.4576
40	-	3.1011	3.1023	3.1011
50	3.34882	3.3415	-	3.3415
60	-	3.5514	3.5560	3.5514
80	-	3.9095	3.9195	3.9095
100	4.23372	4.2116	4.2289	4.2116

*B. Velocity Profiles for Varying Parameters*

The effects of varying the magnetic field parameter (M) and the thermal and solutal Grashof numbers (Gt) and (Gc) respectively on the velocity profiles have been depicted in Figs. 2-4. The transverse magnetic field parameter (M) is observed to increase the velocity profiles as the fluid moves towards the vertical surface due to the Lorenz force attracting the fluid molecules. As depicted in Figs. 3 and 4, the velocity profile increases as the thermal and solutal Grashof numbers increase with pronounced overshoot close to the surface.

*C. Temperature Profiles for Varying Parameters*

The temperature profiles for varying controlling parameter values are depicted in Figs. 5-10. In Figs. 5-8, it is clear that the temperature profiles decrease when the Prandtl number (Pr), the thermal and solutal Grashof numbers (Gt) and (Gc) respectively, and the magnetic field parameter (M) are increased. A similar observation is made for Biot number (Bi) in Fig. 10. However, in Fig. 9, the Eckert (Ec) is observed to cause an increase in the temperature profiles towards the surface of the plate.

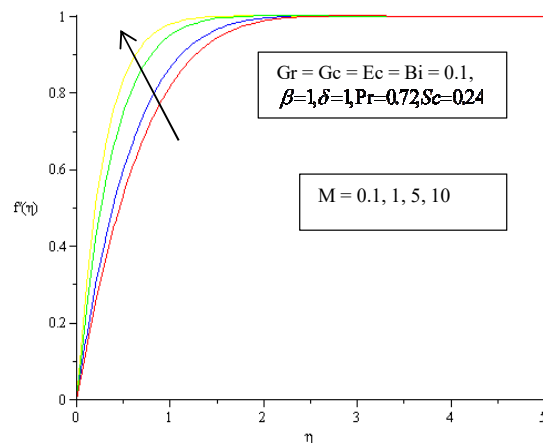


Fig. 2 Velocity Profiles for varying Magnetic Field Parameter

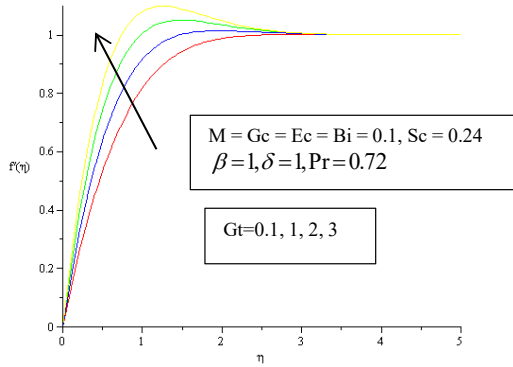


Fig. 3 Velocity Profiles for varying thermal Grashof number

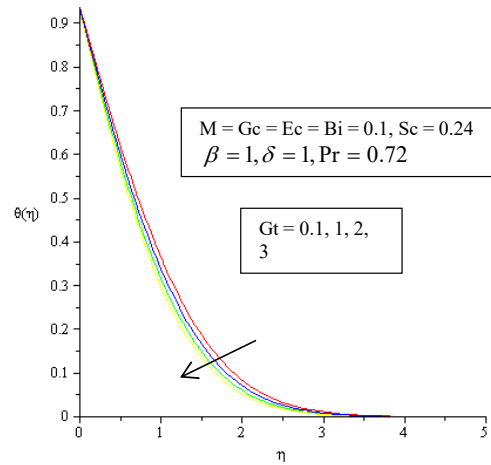


Fig. 6 Temperature Profiles with Thermal Grashof number

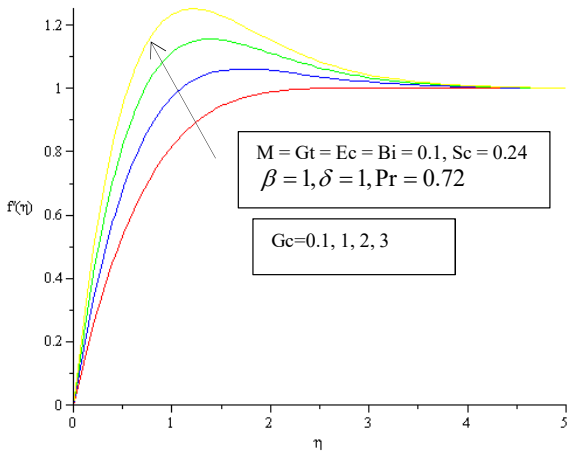


Fig. 4 Velocity Profiles for varying solutal Grashof number

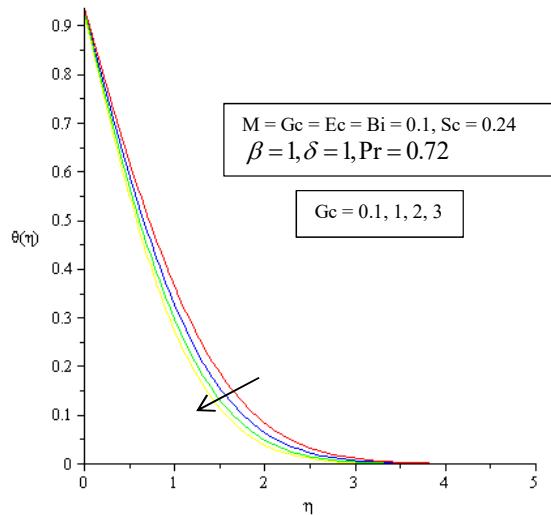


Fig. 7 Temperature Profiles with Solutal Grashof number

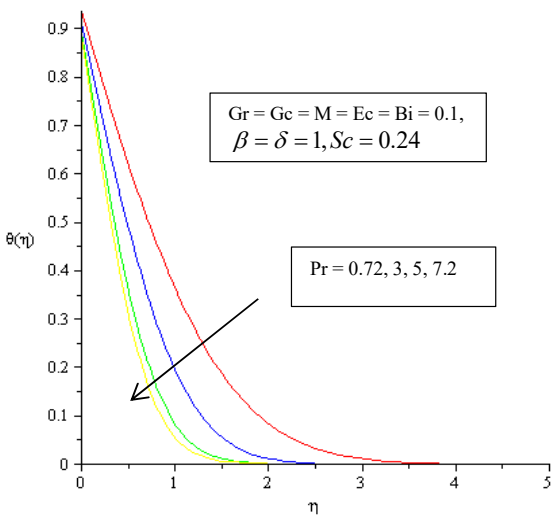


Fig. 5 Temperature Profiles for Varying Prandtl number

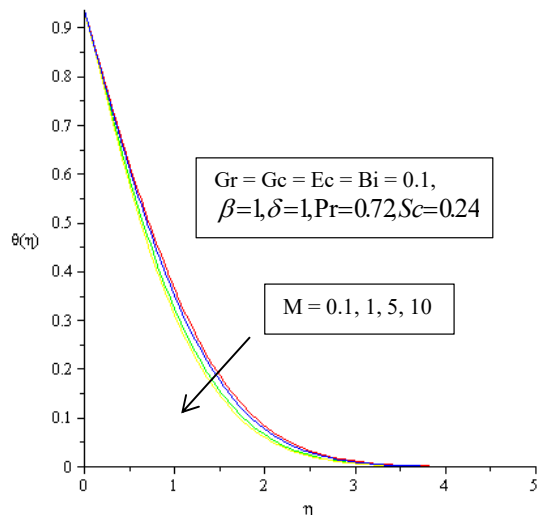


Fig. 8 Temperature Profiles with Magnetic Field Parameter

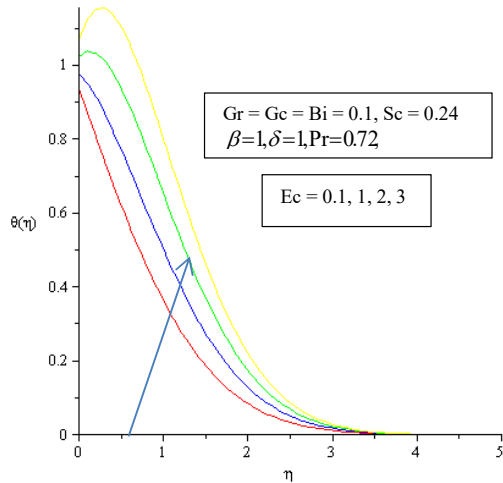


Fig. 9 Temperature Profiles with varying Eckert number

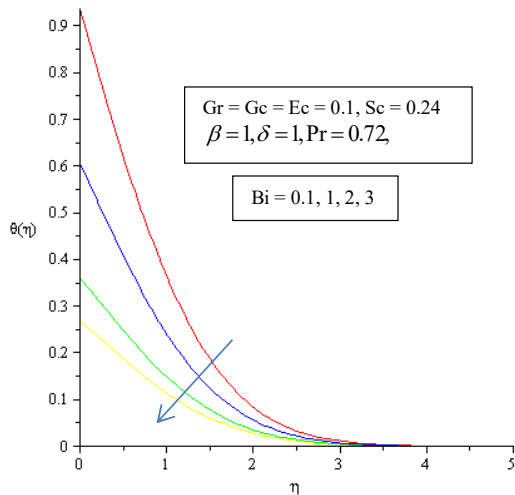


Fig. 10 Temperature Profiles for Varying Biot number

*D. Concentration Profiles for Varying Parameters*

Concentration profiles are depicted in Figs. 11-15. It is observed that increasing all the controlling parameters adversely affects the concentration profiles. However, the reaction rate coefficient and the Schmidt numbers are observed to have greater effects on the concentration profiles compared to the other controlling parameters.

V. CONCLUSION

The paper investigated the MHD chemically reacting viscous fluid flow towards a vertical surface with slip and convective boundary conditions. The boundary layer equations governing the flow were transformed to systems of ordinary differential equations using suitable similarity variables and solved numerically. The skin-friction coefficient and rates of heat and mass transfer from the surface were tabulated and discussed. The velocity, temperature, and concentration profiles are presented graphically and discussed. From the investigations, it is established that:

- i. The magnetic field strength increases the flow of fluid towards the surface.
- ii. The thermal and solutal Grashof numbers have a positive influence of the flow field.
- iii. The rate of heat transfer diminishes with increasing Grashof numbers as well as magnetic and prandtl numbers but increases with the Eckert number.
- iv. The concentration boundary layer thickness is observed to be largely affected by the Schmidt number and the reaction rate parameter.

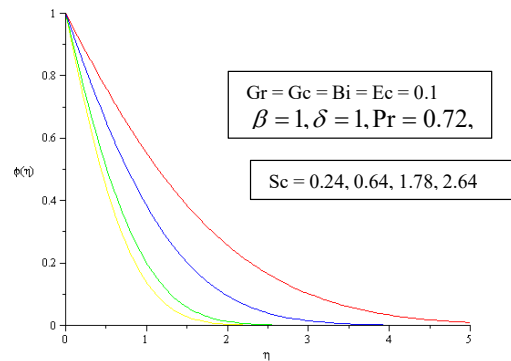


Fig. 11 Concentration Profiles with varying Schmidt number

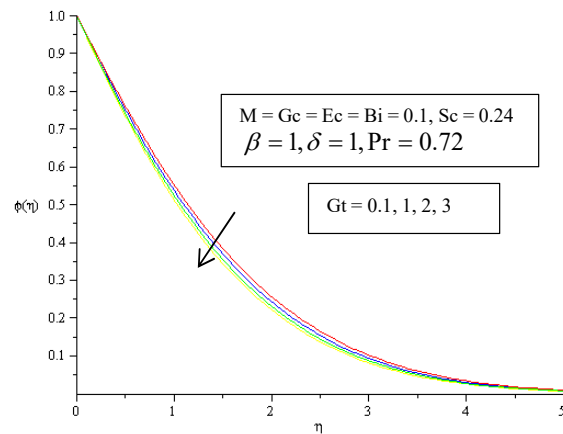


Fig. 12 Concentration Profiles with Thermal Grashof Number

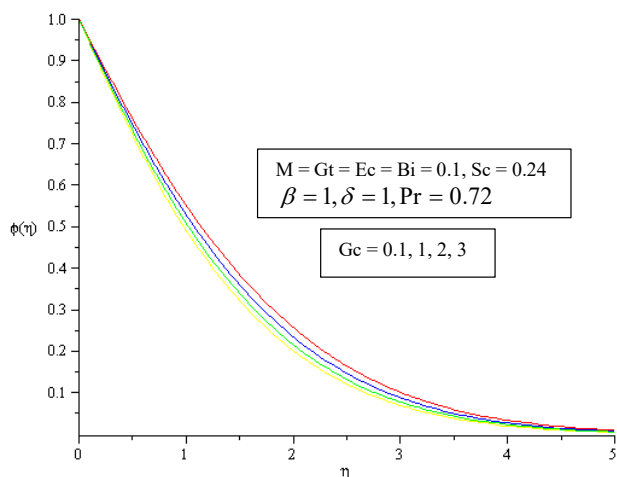


Fig. 13 Concentration Profiles with Solutal Grashof Number

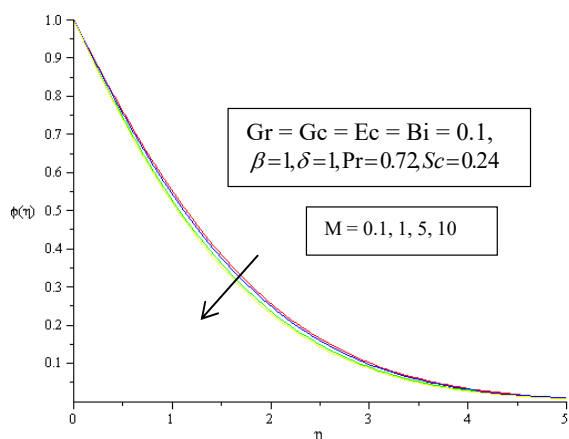


Fig. 14 Concentration Profiles with Magnetic Parameter

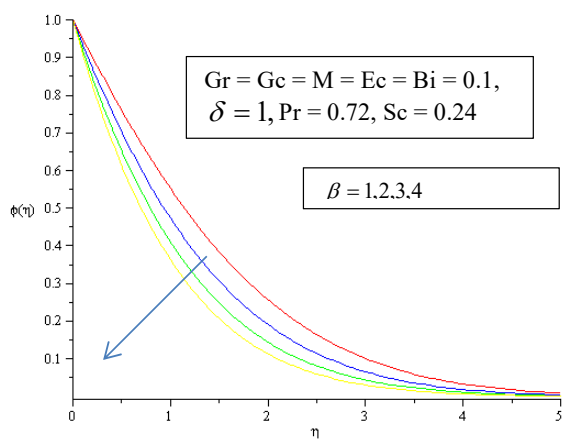


Fig. 15 Concentration Profiles with Reaction Rate Parameter

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