

Methodology of the Energy Supply Disturbances Affecting Energy System

J. Augutis, R. Krikstolaitis, and L. Martisauskas

Abstract—Recently global concerns for the energy security have steadily been on the increase and are expected to become a major issue over the next few decades. Energy security refers to a resilient energy system. This resilient system would be capable of withstanding threats through a combination of active, direct security measures and passive or more indirect measures such as redundancy, duplication of critical equipment, diversity in fuel, other sources of energy, and reliance on less vulnerable infrastructure. Threats and disruptions (disturbances) to one part of the energy system affect another. The paper presents methodology in theoretical background about energy system as an interconnected network and energy supply disturbances impact to the network. The proposed methodology uses a network flow approach to develop mathematical model of the energy system network as the system of nodes and arcs with energy flowing from node to node along paths in the network.

Keywords—Energy Security, Energy Supply Disturbances, Modeling of Energy System, Network Flow

I. INTRODUCTION

THE total energy system, from supply and generation to demand undergoes continuous change. Supply options (renewables, intermittent sources, clean fossil options with CO₂ removal and storage) emerge and are developed at the cost of other options. Energy use is growing (especially electricity and transport fuels) and is influenced by growth and decline of various economic sectors [1].

The nation's energy system is a complex, interconnected network in which a disruption in one part of the infrastructure can easily cause disruptions elsewhere in the system. Recently many policymakers and industry experts focused increased attention on the system's vulnerability to intentional attack, accident or natural disaster. Now, energy security has become an important consideration for state legislatures [2], [3]. Supply disruptions are the classic energy security concern [4].

Reference [2] shows the following some particular aspects of the system infrastructure that remain vulnerable: the electricity system (nuclear facilities, non-nuclear power plants, nuclear fuel storage and transportation, electric transmission

lines, electrical substations), the natural gas system (natural gas storage facilities, natural gas pipelines), and petroleum (crude oil storage and transport, fuel oil, refineries, petroleum product pipelines and terminals, cyber security, telecommunications systems). The vital importance of these elements of the energy system makes them critical to the nation's integrated energy system. Threats and disruptions to one part of the system affect another. For example, a disruption to a natural gas pipeline affects not only the companies and homeowners that use gas for heating, cooking or industrial process, but also the large number of new power plants that use gas to generate electricity [2]. According to International Energy Agency (IEA) [5] one way to look at energy security is to study the different energy sources (coal, oil, gas, and renewables), intermediate means (electricity, refineries), and transportation modes (grids, pipelines, ports, ships). All of these have risks of supply interruptions or failures, challenging the security of undisturbed energy supply. Disruptions – whether from terrorism or from natural disasters – to major power plants or to transmission lines could force the electric system to rely on other less efficient, greater emitting power plants [2]. The energy security disruptions in mentioned reviews have several types as supply, price, or economic disruptions. All the disruptions we define as energy supply disturbances in the proposed methodology. The following subsections in the paper presents developed methodology which is used to analyze energy system as a network and energy supply disturbance effects to the whole energy network. The paper focuses on the theoretical results rather than calculations.

II. ENERGY SYSTEM AS INTERCONNECTED NETWORK

The total energy system is described as interconnected graph – the system of nodes and arcs, with energy flowing from node to node along paths in the network. Such structure is a convenient form to use the network flow approach [6], [7].

A. Energy Network Nodes

Nodes in the energy network perform two functions: transmit energy flow to other network nodes and receives flow from other nodes. In spite of these functions two types of node sets are used in the mathematical model of energy system network. The set of all nodes we denote as N . The first one type includes three sets that define function of the node. According to this any node in the model can be linked to energy import, generation, storage, and consumption technologies. The following sets are used:

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1) *Source nodes* include energy or fuel extraction plants (coal mines, gas wells, etc.), storages of coal, gas, or other type of energy. The imported energy is defined in this set of nodes also. Generally the set of source nodes compose nodes supplying energy or primary energy resources for energy conversion or generation processes. The most important parameters of source nodes are quantities of fuel or primary energy supply, imported energy, and storage capacity over the time. The costs of fuel are significant parameters as well. The subset of source nodes is denoted as $N_S \subset N$.

2) *Generation nodes* include energy conversion or generation technologies like hydro or pumped storage, nuclear, combined heat power (CHP) plants, etc.

Parameters involve power plant characteristics that are necessary for generation nodes are the following: power plant type, minimal and maximal capacities, efficiency, availability, fuel type, ratio of generating heat and electricity (if it is a CHP unit) of power plant. Some characteristics will depend on the season of the year and may vary over the time. The subset of generation nodes is denoted as $N_G \subset N$.

3) *Demand nodes* include the demand centers in the regions and are characterized as the needs of consumers of all energy types analyzed. Generally this subset of nodes involves the end users. The most important parameter is the energy demand of the consumers in the regions over the time.

An unlimited amount of nodes, associated with the producers and consumers, can exist in each region. The end users can be electricity, heat, and gas consumers. Generation technology that uses natural gas from the network and heat pumps using electricity from the network are not considered as end users. The subset of demand nodes is denoted as $N_D \subset N$.

Second type of nodes includes sets that define the type of energy flowing or transported in the network, e.g. electricity, heat, natural gas, etc. and are noted as $N^e, N^h, N^{ng} \subset N$, etc. respectively. In general case we define a set E of all general energy types $\alpha, \beta, \dots, \omega \in E$ analyzed, where $\alpha, \beta, \dots, \omega$ corresponds to different energy type. The node subset of energy type α is denoted as $N^\alpha \subset N$. However, the same node may be included in the different sets of the first type of nodes. Node that is common to several networks has related to its energy conversion technology. The same node may be associated with multiple technologies simultaneously. It should be noted that both types of node sets may be overlapping. Summary of energy network nodes is presented in Table I.

TABLE I
TYPES OF ENERGY NETWORK NODES

	Energy type			
	α	β	...	ω
Source nodes	N_S^α	N_S^β	...	N_S^ω
Generation nodes	N_G^α	N_G^β	...	N_G^ω
Demand nodes	N_D^α	N_D^β	...	N_D^ω

B. Energy Generation

Energy generation is possible only in the generation nodes. One energy or fuel form is converted into other energy form, typically electricity and heat. Inflows into this node usually are from the source nodes, where produced or imported energy or fuel type is transmitted to the generation node to produce or generated other energy type.

Four types of generation can be classified according to the number of inputs and outputs to the node:

- single input and single output (e.g. plant converting natural gas to electricity);
- single input and multiple outputs (e.g. plant converting natural gas to electricity and heat);
- multiple inputs and single output (e.g. plant converting natural gas and fuel oil to electricity);
- multiple inputs and multiple outputs (e.g. plant converting natural gas and fuel oil to electricity and heat). This type of generation is typical for Lithuania Power Plant in Elektrėnai.

It leads to generation technologies can use one or more types of fuel and produce one or more types of energy. Let's define coefficient $\gamma_{\alpha\beta}$ that indicates the generation efficiency converting energy or fuel type α to type β . This coefficient indicates the share of one energy type converted into other. Cases of generation efficiency coefficient are summarized in Table II.

TABLE II
CASES OF GENERATION EFFICIENCY COEFFICIENT

Type of generation	Generation efficiency coefficient	Energy or fuel type
Lossless generation	$\gamma_{\alpha\beta} = 1$	$\alpha \neq \beta$
Lossy generation	$0 < \gamma_{\alpha\beta} \leq 1$	$\alpha \neq \beta$
No generation	$\gamma_{\alpha\beta} = 0$	Any α, β

1) Single Input and Single Output

In this case generation nodes of the network an input energy or fuel type α is converting into β , where $\alpha, \beta \in E$. Inflows G_α^{in} and outflows G_β^{out} of energy are not independent and are considered to be related:

$$G_\beta^{out} = \gamma_{\alpha\beta} \cdot G_\alpha^{in} \quad (1)$$

The generation efficiency coefficient $\gamma_{\alpha\beta}$ defines the relation between inflows and outflows on generation nodes in the network. Due to conservation of energy flow, the outflow from the node must be lower than or equal to the inflow:

$$G_\beta^{out} \leq G_\alpha^{in} \quad (2)$$

Accordingly, the generation efficiency coefficient is limited to $0 < \gamma_{\alpha\beta} \leq 1, \forall \alpha, \beta$.

2) Multiple Inputs and Multiple Outputs

This generation can either be established by a single

generating unit or by a combination of multiple generators. However it is considered as one unit with dedicated inflows and outflows.

Stating all energy inflows $G_\alpha^{in}, G_\beta^{in}, \dots, G_\omega^{in}$ and outflows $G_\alpha^{out}, G_\beta^{out}, \dots, G_\omega^{out}$ in vectors G^{in} and G^{out} respectively, enables the formulation of multi-input and multi-output energy generation analogous to:

$$\begin{pmatrix} G_\alpha^{out} \\ G_\beta^{out} \\ \vdots \\ G_\omega^{out} \end{pmatrix} = \begin{pmatrix} \gamma_{\alpha\alpha} & \gamma_{\beta\alpha} & \dots & \gamma_{\omega\alpha} \\ \gamma_{\alpha\beta} & \gamma_{\beta\beta} & \dots & \gamma_{\omega\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{\alpha\omega} & \gamma_{\beta\omega} & \dots & \gamma_{\omega\omega} \end{pmatrix} \cdot \begin{pmatrix} G_\alpha^{in} \\ G_\beta^{in} \\ \vdots \\ G_\omega^{in} \end{pmatrix}, \quad (3)$$

or in matrix form:

$$G^{out} = \Gamma \times G^{in}. \quad (4)$$

Matrix Γ defines the generation efficiency coefficients between different energy or fuel types in the network.

Every energy production or generation technology in the generation nodes is characterized not only by technical parameters like fuel type, fuel price, capacities, and availability, but also by investment, operating, and maintenance costs.

C. Energy Network Arcs

Flows between network nodes are distributed by arcs connecting the network nodes. The set of all network arcs is denoted as A . The type of arc sets depends on the energy type flowing in the arc from one node to other. The arc subset of energy type α is denoted as $A^\alpha \subset A$. Each arc can be in different sets of energy type, e.g. natural gas from the source node i is transmitted to the generation node j to produce electricity. In this case we denote arc as (i^{ng}, j^e) . For simplicity the denotation (i, j) of arc will be used further.

Arcs represent the transmission and distribution lines in the electricity sector; pipelines and other transportation forms in the sectors of heat and natural gas.

Every arc includes operation and maintenance, investment, and transportation costs as well. The investment costs are estimated for a new generation technologies and arcs only. Let's define $\zeta^{(i,j)}$ as a total cost per unit of the energy flowing from node i to node j .

D. Network of Energy Flows

Let $\Omega = (A, N, F)$ be an energy flow network where $A = \{(i, j) | 1 \leq (i, j) \leq N_A\}$ is a set of arcs, $N = \{i | 1 \leq i \leq N_N\}$ is a set of nodes, and $F = \{F_{ij} | 1 \leq i, j \leq N_N, 1 \leq F_{ij} \leq N_F\}$ is a set of energy flows.

The energy network $\Omega = (A, N, F)$ is described as interconnected graph – the system of nodes and arcs, with energy flowing from node to node along paths in the network. Such structure is a convenient form to use the network flow approach. Networks are usually modeled as graphs, including

the set of nodes N and the set of arcs A connecting the network nodes. For the generation node $i \in N_G$ in the network, the sum of all arc outflows F_i^{out} from this node must be equal to the power inflow F_i^{in} to node i if there are no losses in the arc:

$$\sum_{i \in N_G} F_i^{in} - \sum_{i \in N_G} F_i^{out} = 0. \quad (5)$$

The flow F from one node to other can be transmitted in both directions, i.e. from node i to node j and vice versa, from node j to node i . However, parameters of different direction flows may differ. The result of this leads to the situation of two cases: first, when parameters of the arc (i, j) are identical to the parameters of opposite direction arc (j, i) . Second, parameters are different for both flow directions.

In the first case, when parameters of the arc (i, j) are identical to the parameters of the arc (j, i) and when network links are assumed to transmit energy without losses, flows are equal on either side:

$$F_{ij} = -F_{ji}, \quad \forall i, j \in N. \quad (6)$$

Energy losses on arcs can be considered as the difference between inflow and outflow and are related to the transmission losses. If η_{ij} ($0 \leq \eta_{ij} < 1$) are losses in the arc (i, j) , then $F_j^{out} = F_j^{in}(1 - \eta_{ij})$, because losses in the arc are estimated in the location of inflow to node j . If there are no losses in the arc (i, j) then $\eta_{ij} = 0$. Due to losses η_{ij} in the arc (i, j) inequality $F_j^{in} \geq F_j^{out}$ is satisfied.

The efficiency $1 - \eta_{ij}$ indicates the share of energy is transmitted from node i to node j .

For the case of no physical connection between two nodes i and j efficiency $1 - \eta_{ij} = 0$:

$$\forall F_{ij} = 0 \Rightarrow \eta_{ij} = 1, \quad \forall i, j \in N. \quad (7)$$

Simplified chain of source, generation, and demand nodes and arcs is presented in Fig. 1.

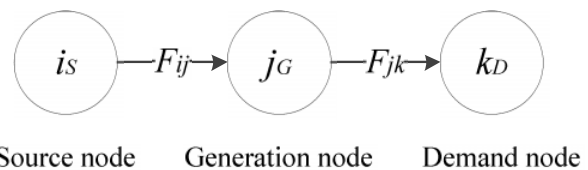


Fig. 1 Chain of network nodes

Network flows between the nodes may have certain constraints, i.e. the network arcs have capacities. This parameter is included in the model as two values: lower F_{ij}^{\min} and upper F_{ij}^{\max} bounds on the flow. Then the inequality should be satisfied for each flow:

$$0 \leq F_{ij}^{\min} \leq F_{ij} \leq F_{ij}^{\max}, \quad \forall i, j \in N. \quad (8)$$

Each node can be associated with more than one arc. The incoming flows to the node can be more than one as well. The sums of all inflows and outflows should be defined to meet the total power flow into each node of the network.

The sum of all inflows F_i^{in} of one energy type into the node i is equal to the total incoming power G_i^{in} to the node i of that energy form:

$$G_i^{in} = \sum_{\forall i \in N} F_i^{in}. \quad (9)$$

The sum of all outflows F_i^{out} of one energy type from the node i is equal to the total outgoing power G_i^{out} (which is satisfying demand or is transmitted to the other network nodes) from the node i of that energy form:

$$G_i^{out} = \sum_{\forall i \in N} F_i^{out}. \quad (10)$$

Simplified scheme of incoming to node i and outgoing from node i powers is presented in Fig. 2.

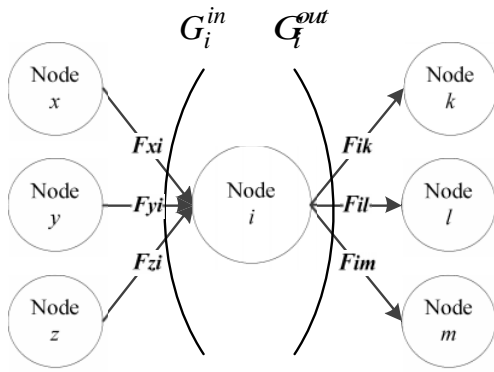


Fig. 2 Incoming and outgoing powers of the node

For simplicity a vector set of each energy type flows is defined:

$$F_{ij}^\alpha = (F_{1j}^\alpha \quad F_{2j}^\alpha \quad \dots \quad F_{N_N j}^\alpha)^T, \quad \forall i, j \in N, \forall \alpha \in E. \quad (11)$$

The number of these vectors is equal to the number of energy types in the network analyzed. Transition to vector and matrix forms is a convenient way to write equations and coefficients.

In order to determine the connections between each node and arc of the network, node connection matrix C of size $N_N \times N_N$ is defined:

$$C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1N_N} \\ c_{21} & c_{22} & \dots & c_{2N_N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N_N 1} & c_{N_N 2} & \dots & c_{N_N N_N} \end{pmatrix}, \quad (12)$$

where N_N is the number of nodes in the network and elements of this matrix are:

$$c_{ij} = \begin{cases} 1, & \text{if } (i, j) \in A, \\ 0, & \text{if } (i, j) \notin A. \end{cases} \quad (13)$$

If $c_{ij} = 1$, it means that the flow from the node i to the node j in the arc (i, j) exists. If $c_{ij} = 0$, then there is no physical connection between the nodes i and j in the network. Since the node to itself has no flow, then $c_{ij} = 0$, when $i = j$. Hence the diagonal elements of the node connection matrix (12) are equal to zero.

Each row of matrix (12) indicates the outflows of each node outgoing from that node (when matrix element is equal to 1) and each matrix column indicates the inflows of each node incoming to that node (when matrix element is equal to 1).

The node connection matrix needs to include efficiency coefficients $1 - \eta_{ij}$ when energy flows are with losses η_{ij} . Then the elements of the matrix (12) will be calculated as follows:

$$a_{ij} = \begin{cases} 1 \cdot (1 - \eta_{ij}) = 1 - \eta_{ij}, & \text{if } (i, j) \in A, \\ 0 \cdot (1 - \eta_{ij}) = 0, & \text{if } (i, j) \notin A. \end{cases} \quad (14)$$

The final expression of the node connection matrix is obtained:

$$C_{ij} = \begin{pmatrix} 0 & c_{12} & \dots & c_{1N_N} \\ c_{21} & 0 & \dots & c_{2N_N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N_N 1} & c_{N_N 2} & \dots & 0 \end{pmatrix}, \quad (15)$$

where:

$$c_{ij} = \begin{cases} 1 - \eta_{ij}, & \text{if } (i, j) \in A, \\ 0, & \text{if } (i, j) \notin A. \end{cases} \quad (16)$$

Knowing the energy flows between the network nodes (the vector F_{ij}^α (11) for each energy type) and the node connection matrix, a possibility is to calculate the distribution of each energy type flows between the network nodes at time moment t . Then:

$$F^\alpha(t) = C_{ij}(t) \times F_{ij}^\alpha(t), \quad \forall i, j \in N, \forall \alpha \in A, \forall t \in T. \quad (17)$$

Matrix $F^\alpha(t)$ shows the distribution of flows of energy type α in the network at time moment t .

III. ENERGY SUPPLY DISTURBANCES

Energy supply disturbances are external natural events or external events related to the human being activity, which may disturb energy system or some elements of the system work. A disturbance to the energy system consists of two parts: the external threats, e.g. a terrorist attack, and the weaknesses of the energy supply system.

Disturbances come in three types: technical, economical, and sociopolitical. Each of these disturbance types has one of four associated degrees of severity (0 – no disturbance, 1 – low disturbance, 2 – medium disturbance, 3 – high disturbance).

The importance and some other relevant information about

energy supply disturbances can be found in [8], [9].

The disturbances of energy supply are characterized in various parameters.

The concept of basic scenario is needed to describe disturbance parameters. The basic scenario shows a current situation of energy sector when primary energy supply disturbances do not exist till 2025, fuel and primary energy sources are supplied as it was predicted by demand for electric energy and heat production; also costs of primary energy sources or fuel change by average high forecasts.

The disturbances of energy supply that may result from both external and domestic events create a threat for whole energy system. This paper focuses on both types of energy supply disturbances: external and internal.

A. External Disturbance Parameters

1) *Fuel type or primary energy resource of restricted or disrupted supply* where $\varphi = \{\varphi_i | i=1,2,\dots,N_\varphi\}$ and N_φ is a number of fuel or energy types. The set φ includes elements of the set E .

Primary energy is understandable as energy found in nature that has not been subjected to any conversion or transformation process. It is energy contained in raw fuels as well as other forms of energy received as input to a system [10].

There are three distinct groups of primary energy resources: fossil fuels (coal, oil, natural gas, etc.), nuclear power (fission and fusion), and renewable energy (hydropower, biomass, wind power, solar power, geothermal, wave, and tidal power).

These three groups of primary energy resources compose any of energy system in general. Primary energy is transformed in energy conversion processes to more convenient forms of energy, such as electrical energy, refined fuels, or synthetic fuels such as hydrogen. In energy statistics these forms are called secondary energy.

2) *Part of primary energy supply deviation in percent from basic supply scenario* where $\delta = \{\delta_i | i=1,2,\dots,N_\delta\}$ and N_δ is a number of parameter values.

Potential supply reduction is from 0 % to 100 % for each type of primary energy compared with the supply amounts of the basic scenario in the appropriate period.

Probability distribution $P(\delta)$ should be defined to the parameter δ from the available statistical data and/or expert assessment.

3) *Duration of primary energy resources or fuel supply restriction or disruption* where $t = \{t_i | i=1,2,\dots,N_t\}$ and N_t is a number of parameter values. This parameter indicates the continuation of energy supply disturbance.

Probability distribution $P(t)$ should be defined to the parameter t from the available statistical data and/or expert assessment.

4) *Fuel or primary energy resources price deviation from projected price in the basic scenario* where $\sigma = \{\sigma_i | i=1,2,\dots,N_\sigma\}$ and N_σ is a number of parameter

values. Duration of price deviation is expected from the previous energy supply disturbance parameter.

Probability distribution $P(\sigma)$ should be defined to the parameter σ from the available statistical data and/or expert assessment.

5) *Starting moment of the external disturbance* where $\tau = \{\tau_i | i=1,2,\dots,N_\tau\}$ and N_τ is a number of parameter values. This parameter is important for installation of new technologies, such as construction of new power plants, networks, or grid connections, the connection of new consumers, etc.

Probability distribution $P(\tau)$ should be defined to the parameter τ from the expert assessment.

B. Internal Disturbance Parameters

This type of energy supply disturbances is related to reliability characteristics. Such disturbances are more of technical nature that occurs within the energy system and indicates the weaknesses of the energy supply system.

1) *Starting moment of the internal disturbance* where $\tau^{\text{int}} = \{\tau_i^{\text{int}} | i=1,2,\dots,N_{\tau^{\text{int}}}\}$ and $N_{\tau^{\text{int}}}$ is a number of parameter values.

2) *Duration of the internal disturbance* where $t^{\text{int}} = \{t_i^{\text{int}} | i=1,2,\dots,N_{t^{\text{int}}}\}$ and $N_{t^{\text{int}}}$ is a number of parameter values.

The meaning of parameters τ^{int} and t^{int} is the same as parameters τ and t respectively, but they can occur in different moment and be independent.

3) *Reliability factor* in this methodology expressed as *availability* where $\lambda = \{\lambda_i | i=1,2,\dots,N_\lambda\}$ and N_λ is a number of parameter values. This parameter is associated with the generators and supply networks. It indicates their reliability parameters. Availability parameter shows the maximum share of generator or network capacity available at the disturbance moment. Due to this parameter λ has limits: $0 \leq \lambda \leq 1$. If $\lambda=0$, then the generator or some network chain is unavailable at the disturbance moment and if $\lambda=1$, then aforesaid is operating in the normal conditions without any disruptions or failures. Probability distribution $P(\lambda)$ should be defined to the parameter λ from the available statistical data and/or expert assessment.

Internal and external disturbances of energy supply are independent, e.g. natural gas supply restriction due to the political decisions is not dependent on the random failure of generating unit. However, internal and external disturbances can occur at the same time and worsen the situation in the rare cases.

Summary of the energy supply disturbances is presented in Table III.

TABLE III
PARAMETERS OF THE ENERGY SUPPLY DISTURBANCES

Disturbance parameter	Values of the parameter	Probability distribution of the parameter value
<i>External disturbance</i>		
Fuel type or primary energy resource of restricted or disrupted supply	Fossil (φ_1)	Not required
	Nuclear (φ_2)	
	Renewable (φ_3)	
	...	
Part of primary energy supply deviation from basic supply scenario	δ_1 %	$P(\delta_1)$
	δ_2 %	$P(\delta_2)$

	$\delta_{N\delta}$ %	$P(\delta_{N\delta})$
Duration of primary energy resources or fuel supply restriction or disruption	t_1 months	$P(t_1)$
	t_2 months	$P(t_2)$

	t_{Nt} months	$P(t_{Nt})$
Fuel or primary energy resources price deviation from projected price in the basic scenario	σ_1 %	$P(\sigma_1)$
	σ_2 %	$P(\sigma_2)$

	$\sigma_{N\sigma}$ %	$P(\sigma_{N\sigma})$
Starting moment of the external disturbance	τ_1 year	$P(\tau_1)$
	τ_2 year	$P(\tau_2)$

	$\tau_{N\tau}$ year	$P(\tau_{N\tau})$
<i>Internal disturbance</i>		
Starting moment of the internal disturbance	τ_1^{int} year	$P(\tau_1^{int})$
	τ_2^{int} year	$P(\tau_2^{int})$

	$\tau_{N\tau}^{int}$ year	$P(\tau_{N\tau}^{int})$
Duration of the internal disturbance	t_1^{int} months	$P(t_1^{int})$
	t_2^{int} months	$P(t_2^{int})$

	t_{Nt}^{int} months	$P(t_{Nt}^{int})$
Availability	λ_1 %	$P(\lambda_1)$
	λ_2 %	$P(\lambda_2)$

	$\lambda_{N\lambda}$ %	$P(\lambda_{N\lambda})$

The sums of each parameter value probabilities should satisfy the following:

$$\begin{aligned}
 \sum_{i=1}^{N_\delta} P(\delta_i) &= \sum_{i=1}^{N_t} P(t_i) = \sum_{i=1}^{N_\sigma} P(\sigma_i) = \sum_{i=1}^{N_\tau} P(\tau_i) = \\
 &= \sum_{i=1}^{N_{\delta}^{int}} P(\tau_i^{int}) = \sum_{i=1}^{N_{t}^{int}} P(t_i^{int}) = \sum_{i=1}^{N_\lambda} P(\lambda_i) = 1.
 \end{aligned} \quad (18)$$

A set of various energy supply disturbances is estimated from different values of disturbance parameters according to the probability distributions. To evaluate probability of disturbance occurrence, probabilities of parameters values are needed. These probabilities should be assessed by experts and/or obtained from available statistical data.

The disturbances of energy supply are characterized as a stochastic (random) process $X = \{X(t), t \in T\}$ within or outside the energy system. Realization $x(t)$ is the individual disturbance of energy supply with particular set of parameters at the time moment t , where T is the set of time moments t .

IV. DISTURBANCE IMPACT ON ENERGY NETWORK

It is very important to determine how well the energy supply network reacts to disturbances it is facing. The influence of disturbances to energy system can be analyzed in respect of fuel accessibility, fuel and energy price changes, available generation capacity, meeting the demand of consumers, unserved demand.

In order to estimate the influence of energy supply disturbances to the energy system needs to simulate the operation of the energy system. Simulation can be achieved with optimization problems and tasks.

There are a number of reasonable objectives that can be used for optimizing energy systems. Energy cost is probably the most common criterion employed in the operational optimization of energy systems. The goal of the network flow problem is to satisfy energy demands with available fuel supplies at the minimal total cost without violating the bounds. The formulated problem falls into the category of generalized minimum cost flow problem [6], [11] and can be solved by applying some efficient algorithms [7], [12].

In general considering not only a single time period of the energy system but multiple time periods $t \in \{1, 2, \dots, N_t\}$ results in multi-period optimization. In the continuous case this optimization problem is generally stated as [12]:

$$\text{Minimize } \sum_{t=1}^{N_t} \Psi^t(x), \quad (19)$$

subject to:

$$g^t(x) = 0, \quad \forall t \quad (20)$$

$$h^t(x) \leq 0, \quad \forall t \quad (21)$$

where:

$x \in R^{(N_t \cdot u)}$ is the $(N_t \cdot u) \times 1$ vector of continuous optimization variables, i.e. the total number of variables is $N_t \cdot u$;

$t \in \{1, 2, \dots, N_t\}$ is the time period;

$\Psi^t(x): R^{(N_t \cdot u)} \rightarrow R$ is a scalar-valued objective function for time period t ;

$g^t(x): R^{(N_t \cdot u)} \rightarrow R^v$ is the $v \times 1$ vector of equality constrains at period t ;

$h^t(x): R^{(N_t \cdot u)} \rightarrow R^w$ is the $w \times 1$ vector of inequality constrains at period t .

From the security of energy supply point of view optimization of the energy system should not only confine to the costs, but also it should include vulnerability of the energy system expressed in the reliability and security terms. It directly leads to optimization with multiple objectives and can be performed by stating a composite objective function as a weighted linear combination of individual objectives. In this paper optimization with multiple objectives is refused. It is necessary to include in the model the level of unserved

demand $u(\alpha)$ and its costs $\zeta^{u(\alpha)}$, where $\alpha \in E$. The unserved demand is available due to the energy supply disturbances. The effect of supply disturbances to the energy system leads to unserved demand level and shows the percentage of consumers which are not satisfied with the energy needs. As a result the unserved demand criterion should be included in the optimization.

Each energy supply disturbance is as specific scenario in the energy system. Development and probability of each scenario describe parameters of the disturbance.

The mathematical expression for the frequency of a specific scenario $\Lambda_{j,k}$ is:

$$\Lambda_{j,k} \equiv \Lambda(ES_{j,k}) = \pi_j \Pr(ES_{j,k} | IE_j), \quad (22)$$

where:

π_j denotes the frequency of the j th energy supply disturbance or initiating event (IE) simulated in the model;

$\Pr(ES_{j,k} | IE_j)$ symbolizes the conditional probability for the end state of event sequence k in the model initiated by the disturbance IE_j , given that IE_j has occurred.

The main simulation results should include the total energy generation capacities, demand, unserved demand, percentage of the unsatisfied consumers, fuel consumption, investment, operational, and maintenance costs of the generation technologies and total costs of the energy system.

V.CONCLUSION

The paper reveals energy supply disturbance impact on energy system from energy security point of view. An integrated mathematical model was formulated and theoretical case of the proposed methodology was examined. Energy security subject and vulnerability of the energy system due to energy supply disturbances problem are very relevant. Theretofore we are developing the proposed model further and doing researches in the field of energy security. In the future we are planning to expand the model including barriers in the energy system and make it more stochastic and probabilistic. The pilot calculations would be necessary in the near future. The proposed methodology can be used to determine vulnerability of the energy system due to energy supply disturbances and this highlights threats to the energy security.

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