Method for Solving Fully Fuzzy Assignment Problems Using Triangular Fuzzy Numbers

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Abstract—In this paper, a new method is proposed to find the fuzzy optimal solution of fuzzy assignment problems by representing all the parameters as triangular fuzzy numbers. The advantages of the proposed method are also discussed. To illustrate the proposed method a fuzzy assignment problem is solved by using the proposed method and the obtained results are discussed. The proposed method is easy to understand and to apply for finding the fuzzy optimal solution of fuzzy assignment problems occurring in real life situations.

Keywords—Fuzzy assignment problem, Ranking function, Triangular fuzzy numbers.

I. INTRODUCTION

THE assignment problem (AP) is a special type of linear programming problem in which our objective is to assign n number of jobs to n number of persons at a minimum cost (time). The mathematical formulation of the problem suggests that this is a 0-1 programming problem and is highly degenerate. All the algorithms developed to find optimal solution of transportation problem are applicable to assignment problem. However, due to its highly degeneracy nature, a specially designed algorithm, widely known as Hungarian method proposed by Kuhn [1], is used for its solution.

However, in real life situations, the parameters of AP are imprecise number instead of fixed real numbers because time/cost for doing a job by a facility (machine/person) might vary due to different reasons. Examples of these types of problems may be the case of assigning men to offices, crews (drivers and conductors) to buses, trucks to delivery routes etc. Over the past 50 years, many variations on the classical AP are proposed e.g. bottleneck assignment problem, generalized assignment problem, quadratic assignment problem etc.

Zadeh [2] introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Since then, tremendous efforts have been spent; significant advances have been made on the development of numerous methodologies and their applications to various decision problems. Fuzzy assignment problems have received great attention in recent years. For instance, Chen [3] proposed a fuzzy assignment model that did not consider the differences of individuals, and also proved some theorems. Wang [4] solved a similar model by graph theory. Dubois and Fortemps [5] proposed a flexible assignment problem, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. They also demonstrated and solved an example

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of fuzzy assignment problem. Sakawa et al. [6] dealt with actual problems on production and work force assignment of a housing material manufacturer and formulated two-level linear and linear fractional programming problems according to profit and profitability maximization respectively. By applying interactive fuzzy programming for two-level linear and linear fractional programming problems, they derived satisfactory solutions to the problems and compared the results.

Lin and Wen [7] proposed an efficient algorithm based on the labeling method for solving fuzzy assignment problems. The algorithm begins with primal feasibility and proceeds to obtain dual feasibility while maintaining complementary slackness until the primal optimal solution is found. Feng and Yang [8] investigated a two objective-cardinality assignment problem. A chance-constrained goal programming model is constructed for the problem and tabu search algorithm based on fuzzy simulation is used to solve the problem.

Majumdar and Bhunia [9] proposed an elitist genetic algorithm to solve generalized assignment problem with imprecise cost/time. Ye and Xu [10] proposed an effective method on priority-based genetic algorithm to solve fuzzy vehicle routing assignment when there is no genetic algorithm which can give clearly procedure of solving it. Liu and Gao [11] proposed an equilibrium optimization problem and extended the assignment problem to the equilibrium multi-job assignment problem, equilibrium multi-job quadratic assignment problem and used genetic algorithm to solve the proposed models.

In this paper, a new method is proposed to find the fuzzy optimal solution of fuzzy assignment problems by representing all the parameters as triangular fuzzy numbers. The advantages of the proposed method are also discussed. To illustrate the proposed method a fuzzy assignment problem is solved by using the proposed method and the obtained results are discussed. The proposed method is easy to understand and to apply for finding the fuzzy optimal solution of fuzzy assignment problems occurring in real life situations.

This paper is organized as follows: In section II, some basic definitions and arithmetic operations are reviewed. In section III, formulation of fuzzy assignment problem is presented. Also the application of ranking function for solving fuzzy assignment problem is presented. In section IV, a new method is proposed to find fuzzy optimal solution of fuzzy assignment problems and advantages of the proposed method are also discussed. In section V, to illustrate the proposed method, a numerical example is solved. The results are discussed in section VI and the conclusions are discussed in section VII.

II. PRELIMINARIES

In this section, some basic definitions and arithmetic operations are reviewed.

A. Basic Definitions

In this subsection, some basic definitions are reviewed.

Definition 2.1 [12] The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 [12] A fuzzy set $\tilde{A}$, defined on universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is convex i.e., $\mu_{\tilde{A}}(\ x_1 + (1 -)x_2) \geq$ minimum$(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \ \forall \ x_1, x_2 \in R, \forall \quad \in [0,1]$

(ii) $\tilde{A}$ is normal i.e., $\exists \ x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$

(iii) $\mu_{\tilde{A}}$ is piecewise continous.

Definition 2.3 [12] A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0$, for all $x < 0$.

Definition 2.4 [12] A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & x = b \\ \frac{(x-c)}{(b-c)}, & b \leq x \leq c \end{cases}$$

where, $a, b, c \in R$

Definition 2.5 [12] A triangular fuzzy number $\tilde{A} = (a, b, c)$ is said to be non-negative triangular fuzzy number if and only if $a \geq 0$

Definition 2.6 [12] A triangular fuzzy number $\tilde{A} = (a, b, c)$ is said to be zero triangular fuzzy number if and only if $a = 0, b = 0, c = 0$

Definition 2.7 [12] Two triangular fuzzy numbers $\tilde{A} = (a, b, c)$ and $\tilde{A}_1 = (a_1, b_1, c_1)$ are said to be equal i.e. $\tilde{A} = \tilde{A}_1$ if and only if $a = a_1, b = b_1, c = c_1$

Definition 2.8 [13] A ranking function is a function $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers, defined on set of real numbers, which maps each fuzzy number into a real number.

Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number, then $\Re(\tilde{A}) = \frac{a + 2b + c}{4}$

B. Arithmetic Operations

In this subsection, addition and multiplication operations of triangular fuzzy numbers are reviewed [12].

Let $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$ be two triangular fuzzy numbers, then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 \simeq (a, b, c)$, where
$a = $ minimum $(a_1 a_2, a_1 c_2, a_2 c_1, c_1 c_2)$, $b = b_1 b_2$,
$c = $ maximum $(a_1 a_2, a_1 c_2, a_2 c_1, c_1 c_2)$

Remark 2.1 In this paper, at all places $\sum_{i=1}^{m} \tilde{\ }_i \otimes \tilde{A}_i$ and $\sum_{i=1}^{m} \ _i A_i$ represents the fuzzy and crisp additions respectively i.e. $\sum_{i=1}^{m} \tilde{\ }_i \otimes \tilde{A}_i = \tilde{\ }_1 \otimes \tilde{A}_1 \oplus \tilde{\ }_2 \otimes \tilde{A}_2 \oplus ... \oplus \tilde{\ }_m \oplus \tilde{A}_m$ and $\sum_{i=1}^{m} \ _i A_i = \ _1 A_1 + \ _2 A_2 + ... + \ _m A_m$, where $\tilde{A}_i$ and A_i are triangular fuzzy number and real number respectively.

III. FUZZY ASSIGNMENT PROBLEM

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do one job at a time and each job can be assigned to one person only. Let $\tilde{c}_{ij}$ be the approximate payment(cost), if i^{th} job is assigned to j^{th} person. The problem is to find an assignment $\tilde{x}_{ij}$ (which job should be assigned to which person) so that the approximate total cost for performing all jobs is minimum. The above problem may be formulated as follows:

Minimize $\sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

subject to $\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{1}, \ j = 1, 2, ..., n$

$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{1}, \ i = 1, 2, ..., n$

$\tilde{x}_{ij}$ is a non-negative triangular fuzzy number and $\tilde{1} = (1, 1, 1)$

A. Application of Ranking Function for Solving Fuzzy Assignment Problems

The fuzzy optimal solution of the fuzzy assignment problem:

Minimize $\sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

subject to $\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{1}, \ j = 1, 2, ..., n$

$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{1}, \ i = 1, 2, ..., n$

$\tilde{x}_{ij}$ is a non-negative triangular fuzzy number

is a triangular fuzzy number $\tilde{x}_{ij}$ which satisfies the following characteristics:

(i) $\tilde{x}_{ij}$ is a non-negative fuzzy number

(ii) $\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{1}, j = 1, 2, ..., n$ and $\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{1}, i = 1, 2, ..., n$

(iii) If there exist any non-negative triangular fuzzy number $\tilde{x}'_{ij}$ such that $\sum_{i=1}^{n} \tilde{x}'_{ij} = \tilde{1}, j = 1, 2, ..., n$ and $\sum_{j=1}^{n} \tilde{x}'_{ij} = \tilde{1}$, $i = 1, 2, ..., n$, then $\Re(\sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}) < \Re(\sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}'_{ij})$

Remark 3.1 Let $\tilde{x}_{ij}$ be a fuzzy optimal solution of fuzzy assignment problem and there exists one or more $\tilde{x}'_{ij}$ such

that

(i) $\tilde{x}'_{ij}$ is a non-negative triangular fuzzy number

(ii) $\sum\limits_{i=1}^{n} \tilde{x}'_{ij} = \tilde{1}, j = 1, 2, ..., n$ and $\sum\limits_{j=1}^{n} \tilde{x}'_{ij} = \tilde{1}, i = 1, 2, ..., n$

(iii) $\Re(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}) = \Re(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}'_{ij})$

then $\tilde{x}'_{ij}$ is said to be alternative fuzzy optimal solution of fuzzy assignment problem.

IV. Proposed Method

In this section, a new method is proposed to find the fuzzy optimal solution of fuzzy assignment problems, occurring in real life situations, by representing all the parameters as triangular fuzzy numbers. The steps of proposed algorithms are as follows:

Step 1 The fuzzy assignment problem is:

Minimize $\quad \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

subject to $\quad \sum\limits_{i=1}^{n} \tilde{x}_{ij} = \tilde{1}, \quad j = 1, 2, ..., n$

$\quad\quad\quad\quad \sum\limits_{j=1}^{n} \tilde{x}_{ij} = \tilde{1}, \quad i = 1, 2, ..., n$

$\tilde{x}_{ij}$ is a non-negative triangular fuzzy number
where,
$\tilde{c}_{ij} = (c_{ij1}, c_{ij2}, c_{ij3})$ is fuzzy payment to j^{th} person for doing i^{th} job
$\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$ is a non-negative triangular fuzzy number

Step 2 Now our objective is to find $\tilde{x}_{ij}$, which satisfies the following properties:

(i) $\tilde{x}_{ij}$ is a non-negative triangular fuzzy number

(ii) $\sum\limits_{i=1}^{n} \tilde{x}_{ij} = \tilde{1}, j = 1, 2, ..., n$ and $\sum\limits_{j=1}^{n} \tilde{x}_{ij} = \tilde{1}, i = 1, 2, ..., n$

(iii) If there exists any non-negative triangular fuzzy number $\tilde{x}'_{ij}$ such that $\sum\limits_{i=1}^{n} \tilde{x}'_{ij} = \tilde{1}, j = 1, 2, ..., n$ and $\sum\limits_{j=1}^{n} \tilde{x}'_{ij} = \tilde{1}, i = 1, 2, ..., n$ then $\Re(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}) < \Re(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}'_{ij})$
i.e.

Minimize $\Re(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij})$

subject to $\sum\limits_{i=1}^{n} \tilde{x}_{ij} = \tilde{1}, \quad j = 1, 2, ..., n$

$\quad\quad\quad\quad \sum\limits_{j=1}^{n} \tilde{x}_{ij} = \tilde{1}, \quad i = 1, 2, ..., n$

$\tilde{x}_{ij}$ is a non-negative triangular fuzzy number

Step 3 Let $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij} = (a_0, b_0, c_0)$, then fuzzy linear programming problem (FLPP), obtained in step 2 , may be written as:

Minimize $\Re(a_0, b_0, c_0)$
subject to
$(\sum\limits_{i=1}^{n} x_{ij1}, \sum\limits_{i=1}^{n} x_{ij2}, \sum\limits_{i=1}^{n} x_{ij3}) = (1, 1, 1), j = 1, 2, ..., n$
$(\sum\limits_{j=1}^{n} x_{ij1}, \sum\limits_{j=1}^{n} x_{ij2}, \sum\limits_{j=1}^{n} x_{ij3}) = (1, 1, 1), \quad i = 1, 2, ..., n$

$x_{ij2} - x_{ij1}, x_{ij3} - x_{ij2}, x_{ij1}, x_{ij2}, x_{ij3} \geq 0, \ \forall \ i, j$

Step 4 The FLPP, obtained in step 3, is converted into following crisp linear programming problem:

Minimize $\quad \frac{1}{4}(a_0 + 2b_0 + c_0)$

subject to $\quad \sum\limits_{i=1}^{n} x_{ij1} = 1, \quad j = 1, 2, ..., n$

$\quad\quad\quad\quad \sum\limits_{i=1}^{n} x_{ij2} = 1, \quad j = 1, 2, ..., n$

$\quad\quad\quad\quad \sum\limits_{i=1}^{n} x_{ij3} = 1, \quad j = 1, 2, ..., n$

$\quad\quad\quad\quad \sum\limits_{j=1}^{n} x_{ij1} = 1, \quad i = 1, 2, ..., n$

$\quad\quad\quad\quad \sum\limits_{j=1}^{n} x_{ij2} = 1, \quad i = 1, 2, ..., n$

$\quad\quad\quad\quad \sum\limits_{j=1}^{n} x_{ij3} = 1, \quad i = 1, 2, ..., n$

$x_{ij2} - x_{ij1}, x_{ij3} - x_{ij2}, x_{ij1}, x_{ij2}, x_{ij3}, \geq 0, \ \forall \ i, j$

Step 5 Find the optimal solution $x_{ij1}, x_{ij2}, x_{ij3}$ by solving the crisp linear programming problem, obtained in step 4.

Step 6 Find the fuzzy optimal solution $\tilde{x}_{ij}$ by putting the values of $x_{ij1}, x_{ij2}, x_{ij3}$ in $\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$.

Step 7 Find the minimum total fuzzy cost (payment) by putting the values of $\tilde{x}_{ij}$ in $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$.

A. Advantages of The Proposed Method

By using the proposed method a decision maker has the following advantages:

(i) The final results are fuzzy numbers.

(ii) It is easy to apply the proposed method, compare to the existing methods, to find the fuzzy optimal solution of fuzzy assignment problems occurring in real life situations.

V. Numerical Example

To illustrate the proposed method a fuzzy assignment problem is solved by using the proposed method.

Example 5.1 Three persons are available to do three different jobs. From past records, the cost (in dollars) that each person takes to do each job is known and are represented by triangular fuzzy numbers and are shown in following Table I:

TABLE I
Fuzzy costs (in dollars)

Person→ Job↓	A	B	C
1	(1,5,9)	(8,9,10)	(2,3,4)
2	(7,8,9)	(6,7,8)	(6,8,10)
3	(5,6,7)	(6,10,14)	(10,12,14)

Find the assignment of persons to jobs that will minimize the total fuzzy cost.

Solution: The fuzzy optimal solution of fuzzy assignment problem by using the proposed method can be obtained as follows:

Step 1 The given fuzzy assignment problem may be formulated into the following FLPP:

Minimize $((1,5,9) \otimes \tilde{x}_{11} \oplus (8,9,10) \otimes \tilde{x}_{12} \oplus (2,3,4) \otimes \tilde{x}_{13} \oplus$
$(7,8,9) \otimes \tilde{x}_{21} \oplus (6,7,8) \otimes \tilde{x}_{22} \oplus (6,8,10) \otimes \tilde{x}_{23} \oplus (5,6,7) \otimes$
$\tilde{x}_{31} \oplus (6,10,14) \otimes \tilde{x}_{32} \oplus (10,12,14) \otimes \tilde{x}_{33})$

subject to
$$\tilde{x}_{11} \oplus \tilde{x}_{12} \oplus \tilde{x}_{13} = (1,1,1)$$
$$\tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} = (1,1,1)$$
$$\tilde{x}_{31} \oplus \tilde{x}_{32} \oplus \tilde{x}_{33} = (1,1,1)$$
$$\tilde{x}_{11} \oplus \tilde{x}_{21} \oplus \tilde{x}_{31} = (1,1,1)$$
$$\tilde{x}_{12} \oplus \tilde{x}_{22} \oplus \tilde{x}_{32} = (1,1,1)$$
$$\tilde{x}_{13} \oplus \tilde{x}_{23} \oplus \tilde{x}_{33} = (1,1,1)$$

$\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}$ are non-negative triangular fuzzy numbers.

Step 2 Using step 2 to step 4 of proposed method, the formulated FLPP is converted into the following crisp linear programming problem:

Minimize $(\frac{1}{4}(x_{111}+10x_{112}+9x_{113}+8x_{121}+18x_{122}+10x_{123}+2x_{131}+6x_{132}+4x_{133}+7x_{211}+16x_{212}+9x_{213}+6x_{221}+14x_{222}+8x_{223}+6x_{231}+16x_{232}+10x_{233}+5x_{311}+12x_{312}+7x_{313}+6x_{321}+20x_{322}+14x_{323}+10x_{331}+24x_{332}+14x_{333}))$

subject to

$x_{111} + x_{121} + x_{131} = 1, \quad x_{112} + x_{122} + x_{132} = 1,$
$x_{113} + x_{123} + x_{133} = 1, \quad x_{211} + x_{221} + x_{231} = 1,$
$x_{212} + x_{222} + x_{232} = 1, \quad x_{213} + x_{223} + x_{233} = 1,$
$x_{311} + x_{321} + x_{331} = 1, \quad x_{312} + x_{322} + x_{332} = 1,$
$x_{313} + x_{323} + x_{333} = 1, \quad x_{111} + x_{211} + x_{311} = 1,$
$x_{112} + x_{212} + x_{312} = 1, \quad x_{113} + x_{213} + x_{313} = 1,$
$x_{121} + x_{221} + x_{321} = 1, \quad x_{122} + x_{222} + x_{322} = 1,$
$x_{123} + x_{223} + x_{323} = 1, \quad x_{131} + x_{231} + x_{331} = 1,$
$x_{132} + x_{232} + x_{332} = 1, \quad x_{133} + x_{233} + x_{333} = 1,$
$x_{112} - x_{111} \geq 0, \ x_{113} - x_{112} \geq 0, \ x_{122} - x_{121} \geq 0,$
$x_{123} - x_{122} \geq 0, \ x_{132} - x_{131} \geq 0, \ x_{133} - x_{132} \geq 0,$
$x_{212} - x_{211} \geq 0, \ x_{213} - x_{212} \geq 0, \ x_{222} - x_{221} \geq 0,$
$x_{223} - x_{222} \geq 0, \ x_{232} - x_{231} \geq 0, \ x_{233} - x_{232} \geq 0,$
$x_{312} - x_{311} \geq 0, \ x_{313} - x_{312} \geq 0, \ x_{322} - x_{321} \geq 0,$
$x_{323} - x_{322} \geq 0, \ x_{332} - x_{331} \geq 0, \ x_{333} - x_{332} \geq 0$
$x_{111}, x_{112}, x_{113}, x_{121}, x_{122}, x_{123}, x_{131}, x_{132}, x_{133}, x_{211}, x_{212},$
$x_{213}, x_{221}, x_{222}, x_{223}, x_{231}, x_{232}, x_{233}, x_{311}, x_{312}, x_{313}, x_{321},$
$x_{322}, x_{323}, x_{331}, x_{332}, x_{333}, \geq 0.$

Step 3 The optimal solution of the crisp linear programming problem, obtained in step 2, is $x_{131} = 1, x_{132} = 1, x_{133} = 1,$ $x_{221} = 1, x_{222} = 1, x_{223} = 1, x_{311} = 1, x_{312} = 1, x_{313} = 1$ and the remaining variables are zero.

Step 4 Putting the values of x_{ij1}, x_{ij2} and x_{ij3} in $x_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$, the fuzzy optimal solution is $\tilde{x}_{11} = (0,0,0),$ $\tilde{x}_{12} = (0,0,0), \ \tilde{x}_{13} = (1,1,1), \ \tilde{x}_{21} = (0,0,0), \ \tilde{x}_{22} = (1,1,1), \ \tilde{x}_{23} = (0,0,0), \ \tilde{x}_{31} = (1,1,1), \ \tilde{x}_{32} = (0,0,0),$ $\tilde{x}_{33} = (0,0,0).$

Step 5 Putting the values of $\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{31},$ $\tilde{x}_{32}, \tilde{x}_{33}$ in $((1,5,9) \otimes \tilde{x}_{11} \oplus (8,9,10) \otimes \tilde{x}_{12} \oplus (2,3,4) \otimes \tilde{x}_{13} \oplus$ $(7,8,9) \otimes \tilde{x}_{21} \oplus (6,7,8) \otimes \tilde{x}_{22} \oplus (6,8,10) \otimes \tilde{x}_{23} \oplus (5,6,7) \otimes$ $\tilde{x}_{31} \oplus (6,10,14) \otimes \tilde{x}_{32} \oplus (10,12,14) \otimes \tilde{x}_{33})$ the minimum total fuzzy cost is $(13,16,19)$ dollars.

VI. Result and Discussions

The obtained result can be explained as follows:

1) The total cost is greater than 13 and less than 19 dollars.
2) Maximum number of persons are in favour that total cost will be 16 dollars.

3) The percentage of favourness for remaining values of total cost can be obtained as follows:

Let x represents the total cost, then the percentage of the favourness for x is given by $\mu_{\tilde{A}}(x) \times 100,$

where, $\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-13)}{3}, & 13 \leq x \leq 16 \\ 1, & x = 16 \\ \frac{(19-x)}{3}, & 16 \leq x \leq 19 \end{cases}$

VII. Conclusion

In this paper, a new method is proposed to solve the fuzzy assignment problems, occurring in real life situations. To illustrate the proposed method a numerical example is solved and obtained results are explained. If there is no uncertainty about the cost, then the proposed method gives the same result as in crisp assignment problem.

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