

# Meta Model for Optimum Design Objective Function of Steel Frames Subjected to Seismic Loads

Salah R. Al Zaidee, Ali S. Mahdi

**Abstract**—Except for simple problems of statically determinate structures, optimum design problems in structural engineering have implicit objective functions where structural analysis and design are essential within each searching loop. With these implicit functions, the structural engineer is usually enforced to write his/her own computer code for analysis, design, and searching for optimum design among many feasible candidates and cannot take advantage of available software for structural analysis, design, and searching for the optimum solution. The meta-model is a regression model used to transform an implicit objective function into objective one and leads in turn to decouple the structural analysis and design processes from the optimum searching process. With the meta-model, well-known software for structural analysis and design can be used in sequence with optimum searching software. In this paper, the meta-model has been used to develop an explicit objective function for plane steel frames subjected to dead, live, and seismic forces. Frame topology is assumed as predefined based on architectural and functional requirements. Columns and beams sections and different connections details are the main design variables in this study. Columns and beams are grouped to reduce the number of design variables and to make the problem similar to that adopted in engineering practice. Data for the implicit objective function have been generated based on analysis and assessment for many design proposals with CSI SAP software. These data have been used later in SPSS software to develop a pure quadratic nonlinear regression model for the explicit objective function. Good correlations with a coefficient,  $R^2$ , in the range from 0.88 to 0.99 have been noted between the original implicit functions and the corresponding explicit functions generated with meta-model.

**Keywords**—Meta-modal, objective function, steel frames, seismic analysis, design.

## I. INTRODUCTION

**M**ETA-MODEL is a regression model that can be prepared with traditional regression techniques or with artificial neural network (ANN) techniques to decouple structural analysis process from those for checking and for optimum searching [1].

As structural analysis, design, and optimization search represent models for the physical world, the meta-model represents a model for the model.

The meta-model has many applications; some of them are presented below:

1. It can be used to correlate a specialized prepared design codes, written by MATLAB or Excel for example, with sophisticated finite element software that is prepared mainly for analysis, e.g. ABAQUS and ANSYS.
2. It can be used to save time where design checking for new proposed sections can be based on analysis results for other

design proposal. With this merit, adequacy of a lot design proposals can be checked without re-analysing for each proposal.

3. Finally, it can be used to decouple the analysis and design processes from the optimum searching process based on transformation of the implicit objective function to an explicit one. With this ability, a designer can use analysis and design software in sequence with optimum searching software.

This paper deals with the third one of aforementioned applications for the meta-model, where a nonlinear regression meta-model for explicit function of plane steel frames has been developed. Analysis and design data that are adopted in model building have been generated based on structural analysis and checking for different design proposals with CSI SAP.

## II. REVIEW OF LITERATURE

In general, structures are considered as statically determinate where static principles alone can determine external reactions and internal forces for statically indeterminate structures where compatibility conditions should be adopted in addition to equilibrium conditions to determine the structural response [2]. Even for statically determinate structures, compatibility condition and kinematic assumption are still essential for computing internal stresses and deformations [3]. This nature of structural behavior makes reactions, internal stresses, and deformations dependent in general on section parameters and material properties and leads to an iterative design process [4].

Objective function is a mathematical function which relates weight and/or cost of the structure as dependent variables to its topology, member's dimensions, and material properties as independent variables [5].

The objective functions are either unconstrained where optimization algorithm searches for a global optimum point or constrained where searching is within a specific feasible domain bounded by constraints depending on problem nature [6].

Most of optimization problems in structural engineering are constrained in nature, where searching process is guided with constraints related to dimensions of members, stresses in members, and deformations of the whole system. Constraints for member dimensions are usually imposed based on available sections, while stresses and deformations constraints are determined based on code recommendations for strength and serviceability aspects [7].

Stresses and deformations constraints, that are dependent on structure topology and member dimensions that are to be determined during searching process, make objective function implicit in nature and lead to couple the analysis and design process with the optimum searching process [1]. This computational problem could be overcome through adopting an explicit model instead of the original implicit model. As this explicit function is a model for the implicit model and is not a model for the original physical system, then it is usually called as meta-model, [1]. The meta-model can be developed based on traditional nonlinear regression, or can be developed with neural networks [8].

### III. META-MODEL

Many engineering applications, including structural engineering application, require sophisticated models to accurately analyse the systems for different inputs. Enormous calculations are usually required for these large and complex models. Optimization of such systems is challenging because determination of implicit and constrained objective function requires huge calculations [1].

The response surface method is usually used to approximately generate the meta-models, where the original model is computed at many sample points, and then, the meta-model is formulated in terms of a linear or a non-linear function. Least squares method is used to minimize the error between original model and meta-model as presented in (1).

$$\epsilon(x) = f(x) - \hat{f}(x) \quad (1)$$

In a nonlinear regression model, a polynomial function which may be quadratic or cubic is usually assumed to correlate between dependent and independent variables with coefficients that are estimated based on the least square approach. A quadratic function approximation is adopted in this study [1].

Sample design points are either selected arbitrarily or based on orthogonal arrays technique [1].

The orthogonal arrays method represents a good approach for sample point's selection. Few methods are available to generate orthogonal arrays. The orthogonal array is called as such because the columns are orthogonal to each other when the dot product of any two columns is zero, that is performed by replacing the number of design variables with (-1) (0) (1). There are no specific rules to generate an orthogonal array [1].

In this study, W shape steel sections are used as design variables, where 273 W sections of current AISC manual are divided into following three types, [9]:

- Small type, with symbol of (S) or (-1), that includes 93 W section,
- Intermedia type, with symbol (I) or (0), that includes 90 W section,
- Large type, with symbol of (L) or (1), that includes 90 W section.

Flowchart indicated in Fig. 1 has been coded with MATLAB and adopted to random selection with uniform distribution from the population of all sections during the design process.

In this study, after analysis and design of several sample designs, sample points, coefficients for quadratic meta-model have been computed with SPSS software, where cross sectional area (A), sectional moment of inertia (I), elastic section modulus (S), and plastic section modulus (Z) have been considered as independent variable, while fitness value is a dependent variable.

As well-designed structure usually utilizes lower materials and requires low cost, structural weight represents an indication on design quality and can be adopted in objective function.

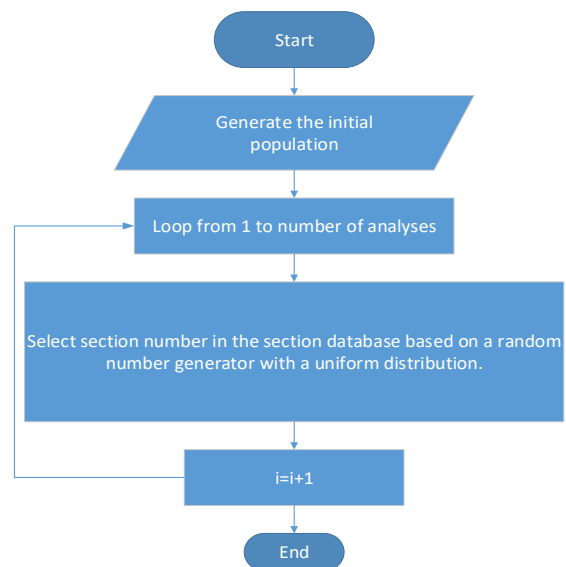


Fig. 1 Random selection of "W" sections

Weight objective function is derived based on assumption that unified column section and unified beam section are adopted for columns and beams within each story. After frame analysis and design, frame weight and demand per capacity ratios have been computed for each member, and the constrained objective function has been estimated as indicated in (2).

$$WFU = Wt. + (100W_{max}) \left( \sum_{i=1}^{No. of Stories} Ratio_{Columns} \times NC + Ratio_{Beams} \times NB \right) \quad (2)$$

where WFU: is the weight objective function,  $W_{max}$ : is the weight of heaviest member in the frame,  $Ratio_{Columns}$ : is the demand per capacity ratio for the most critical column in a specific story.

$Ratio_{Beams}$ : is the demand per capacity ratio for the most critical beam in a specific story.

The part of  $(100W_{max}) \left( \sum_{i=1}^{No. of Stories} Ratio_{Columns} \times NC + Ratio_{Beams} \times NB \right)$  represents penalty part in the equation. Finally, fitness function is determined by (3).

$$fitness = \frac{1}{1 + WFU} \quad (3)$$

Regarding the cost objective function, the weight objective

function of (2) has been firstly normalized based on relation

$$NWFU = \frac{W_{FU_i} - \min(WFU)}{\max(WFU) - \min(WFU)} \quad (4)$$

Then, costs for intermediate moment frame, IMF, and special moment frame, SMF, connections have been respectively assumed equal to 1.2, and 1.5 times the cost of ordinary moment frame, OMF, connection, while the cost of OMF connections has been taken equal to 5 percent of the frame weight. With these assumptions, the objective function for cost index takes the form presented in (5).

$$OFCI = NWFU + \mu \text{ Cost}_{\text{OMF Connection}} \quad (5)$$

where  $\mu$  is respectively equal to 1.0, 1.2, and 1.5 for connections of OMF, IMF, and SMF.

#### IV. SAP MODELING

Preparation of SAP model that is used in analysis and assessment of proposed sample designs or sample points is discussed in sub-articles below:

##### A. Elements Type

Space frame element is used to simulate beams and columns, whereas shell element is adopted in the simulations of slabs and shear walls. Hinge support is adopted to simulate spread footing on compressible soils, while raft and pile foundations have been simulated with fixed support.

##### B. Gravity Load

A uniformly linear load has been adopted to simulate beams and columns weights, while uniformly distributed load per area is adopted to simulate the selfweight for slabs and shear walls.

Based on defined sections, thicknesses, and material densities, selfweight has been computed automatically by the SAP software.

Uniformly distributed loads with values of 2.5 kPa and 4.0 kPa have been adopted to simulate surfacing for floors and roofs, respectively, while uniformly distributed loads of 2.0 kPa and 1.5 kPa have been adopted to simulate live loads on floors and roofs, respectively (see Fig. 2). These values seem adequate for buildings with ordinary occupation and with accessible roofs.

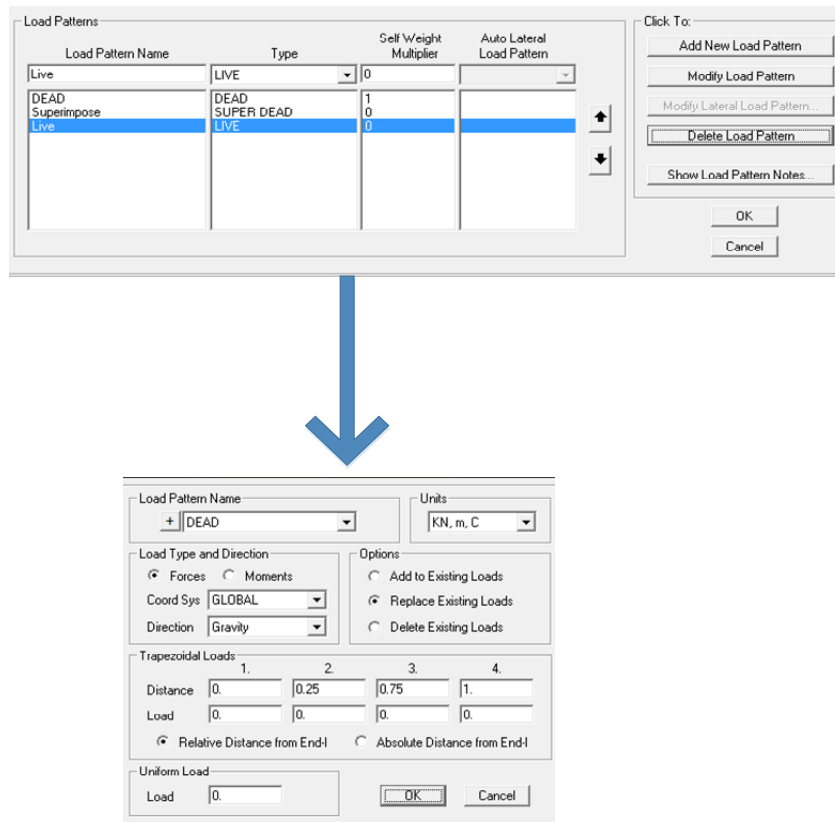


Fig. 2 Load pattern

**Load Patterns**

Load Pattern Name	Type	Self Weight Multiplier	Auto Lateral Load Pattern
EQX	QUAKE	0	IBC 2006
EQY	QUAKE	0	IBC 2006

Click To:  
 Add New Load Pattern  
 Modify Load Pattern  
 Modify Lateral Load Pattern...  
 Delete Load Pattern  
 Show Load Pattern Notes...  
 OK  
 Cancel

**Load Direction and Diaphragm Eccentricity**

☒ Global X Direction  
☐ Global Y Direction  
 Ecc. Ratio (All Diaph.) 0.0  
 Override Diaph. Eccen. Override...

**Time Period**

☒ Approx. Period Ct (R), x = 0.028, 0.8  
☐ Program Calc Ct (R), x =  
☐ User Defined T =

**Lateral Load Elevation Range**

☒ Program Calculated  
☐ User Specified  
 Max Z  
 Min Z

**Factors**

Response Modification, R 3.5  
 System Overstrength,  $\Omega_0$  3  
 Deflection Amplification,  $C_d$  3  
 Occupancy Importance, I 1

**Seismic Coefficients**

☐ Ss and S1 from USGS - by Lat./Long.  
☐ Ss and S1 from USGS - by Zip Code  
☒ Ss and S1 User Specified

Site Latitude (degrees) ?  
 Site Longitude (degrees) ?  
 Site Zip Code (5-Digits) ?  
 0.2 Sec Spectral Accel, Ss 1.24  
 1 Sec Spectral Accel, S1 0.56  
 Long-Period Transition Period 8

Site Class B  
 Site Coefficient, Fa 1  
 Site Coefficient, Fv 1

**Calculated Coefficients**

SDS = (2/3) \* Fa \* Ss 0.8267  
 SD1 = (2/3) \* Fv \* S1 0.3733  
 Update Data  
 OK  
 Cancel

Fig. 3 Seismic load simulation in SAP software

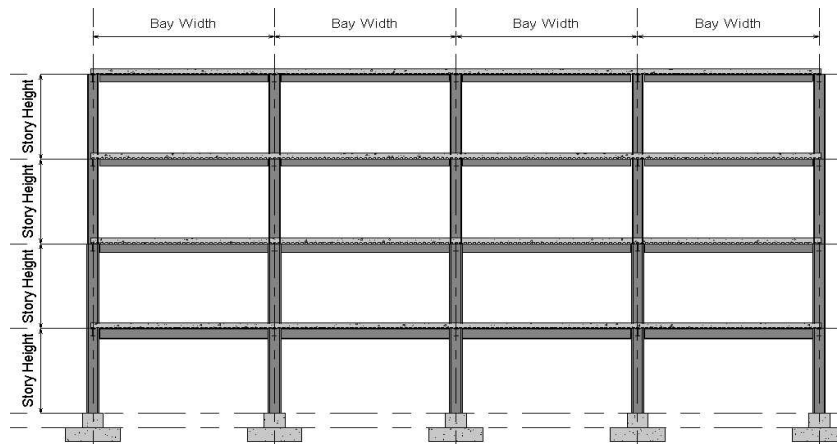


Fig. 4 Case studies frame notations

### C. Seismic Load

The seismic design provisions for ASCE 7 2010 [10] are adopted to simulate seismic effects on the structure. According to these provisions, a limited structural damage in the form of yielding or cracking is permitted during an earthquake; therefore, lateral seismic forces determined by an elastic method should be modified with response modification factor,  $R$ , the overstrength factor,  $\Omega_0$ , and the deflection amplification

coefficient,  $C_d$  to simulate building inelastic behavior during earthquake [11].

The response modification coefficient,  $R$ , is used to decrease the required lateral strength of a structure, and based on linear analysis, inelastic behavior to specific levels depends on characteristics of the structural system [11].

The deflection amplification coefficient,  $C_d$ , is used to predict the total elastic and inelastic lateral deformations of the building subjected to design earthquake ground motion [11].

This overstrength factor coefficient,  $\Omega_0$ , is adopted to determine the required strength to resist behavioral modes with limited inelastic capacity, such as buckling of columns or failure of connection in braced frames [11].

Definition load pattern and parameters that are adopted in SAP model to simulate seismic forces are presented in Fig. 3.

#### V. CASE STUDIES

The structural weight and structural cost meta-models are generated based on nonlinear regression analyses with SPSS software from the data of many alternative designs that have been prepared with CSI SAP software.

Cross sectional area (A), sectional moment of inertia (I), plastic section modulus (Z), and elastic section modulus (S) are considered as variables within the meta-model.

Notations shown in Fig. 4 are adopted to describe design variables in the case studies. Story height of 3 m and bay width of 6 m are adopted in all case studies.

In each case study, three different frame types, namely ordinary moment frame, OMF, intermediate moment frame, IMF, and special moment frame, SMF, are considered.

The following pure quadratic meta-model is assumed between explicit objective function and design variables of cross sectional area (A), section moment of inertia (I), plastic section modulus (Z), and elastic section modulus (S), as shown in (6).

$$\begin{aligned} \text{Explicit Function, } f = & \alpha_0 + \alpha_1 A_1 + \alpha_2 I_2 + \alpha_3 Z_3 \\ & + \alpha_4 S_4 + \dots + \alpha_n S_n + \alpha_{n+1} A_1^2 + \\ & \alpha_{n+2} I_1^2 + \alpha_{n+3} Z_1^2 + \alpha_{n+4} S_1^2 + \dots + \alpha_{n+n} S_n \end{aligned} \quad (6)$$

where  $\alpha_i$  are the coefficient parameters, and  $n$  is the number of variables that are computed as:

$$\text{Beam Variables} = (\text{No. of Stories})$$

where the same beam section is assumed within each story. Assuming that each two story to have same column section, the column variables are determined based on following relation:

$$\text{Column Variables} = \frac{1}{2} (\text{No. of Stories})$$

Finally, the total number of variable is computed based on:

$$n = \text{Beam Variables} + \text{Column Variables} \quad (7)$$

##### A. The First Case Study

This case study has a frame with four floors and two bays, as shown in Fig. 5. With grouping process, six design variables (four sections for beams and two sections for columns) have been adopted. 60 alternatives are designed and sample of these designs are presented in Table I are trialed in the preparing of the meta-model for this case study.

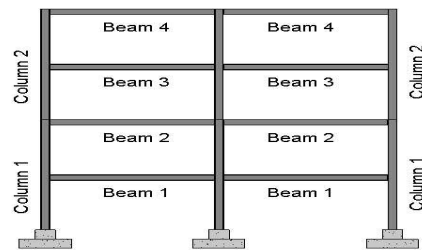


Fig. 5 Plane frame for first case study

TABLE I  
SAMPLE FROM ALTERNATIVE DESIGNS FOR FIRST CASE STUDY

Column 1	Column 2	Beam 1	Beam 2	Beam 3	Beam 4
W24X68	W16X36	W12X305	W16X67	W18X60	W16X67
W18X175	W40X235	W14X48	W5X16	W24X306	W6X20
W24X370	W10X39	W21X83	W24X68	W33X130	W36X302
W27X217	W14X38	W12X120	W14X99	W44X230	W12X19
W8X58	W12X170	W40X277	W24X162	W14X74	W14X38
W30X391	W30X116	W14X176	W21X57	W14X550	W30X292
W33X387	W12X30	W12X190	W8X35	W36X487	W33X318
W36X652	W18X311	W33X241	W8X40	W36X652	W40X277
W18X50	W12X170	W27X194	W33X354	W27X94	W33X152
W18X175	W10X17	W14X500	W36X529	W16X26	W40X199
W18X234	W21X166	W5X16	W12X53	W21X44	W10X30
W36X247	W24X117	W8X48	W8X67	W14X61	W14X665
W27X84	W8X31	W36X302	W14X34	W14X342	W12X26
W24X103	W12X305	W12X26	W27X146	W18X60	W6X16
W18X65	W27X114	W14X665	W24X250	W12X252	W24X62
W18X283	W14X99	W6X15	W27X129	W12X22	W14X26
W24X250	W14X605	W12X53	W40X503	W40X211	W6X15
W14X311	W30X326	W24X84	W36X330	W30X116	W30X148
W27X114	W36X210	W24X335	W36X330	W10X45	W40X149
W8X40	W24X94	W40X249	W27X84	W14X30	W12X87
W18X71	W14X120	W10X30	W40X199	W27X235	W40X249
W10X26	W16X50	W18X211	W14X730	W12X45	W18X97

### 1) The Structural Weight

- Ordinary Moment Frame (OMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 6 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9857$ .
- Intermediate Moment Frame (IMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 7 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9847$ .

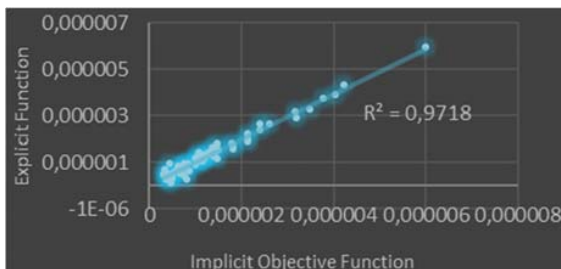


Fig. 6 Correlation between weight implicit and explicit objective functions for first case study with OMF

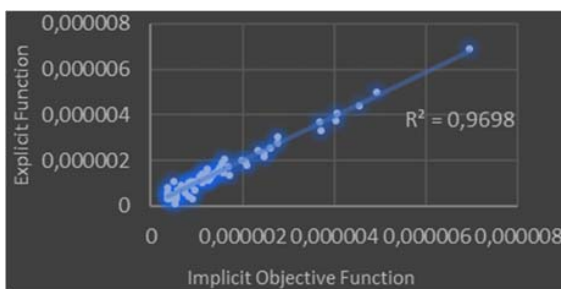


Fig. 7 Correlation between weight implicit and explicit objective functions for first case study with IMF

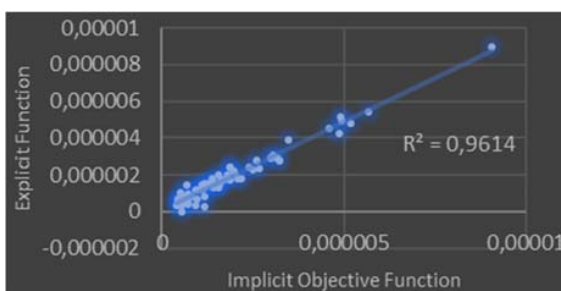


Fig. 8 Correlation between weight implicit and explicit objective functions for first case study with SMF

- Special Moment Frame (SMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 8 where it has been found that two sets of values are highly correlated with a correlation coefficient

of  $R^2 = 0.9805$ .

### 2) The Structural Relative Cost

- Ordinary Moment Frame, OMF: Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 9 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.8798$ .
- Intermediate Moment Frame (IMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 10 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9509$ .
- Special Moment Frame (SMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 11 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9634$ .



Fig. 9 Correlation between relative cost implicit and explicit objective functions for first case study with OMF

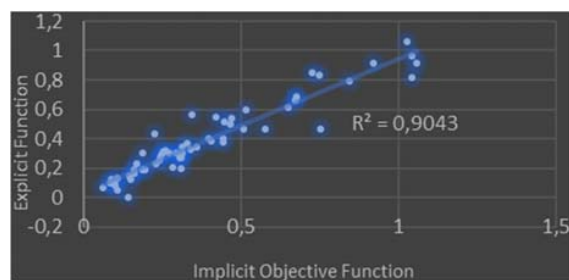


Fig. 10 Correlation between relative cost implicit and explicit objective functions for first case study with IMF

### B. The Second Case Study

This case study has a frame with six floors and four bays, as show in Fig. 12. With grouping process, nine design variables (six sections for beams and three sections for columns) and with eight alternative designs have been adopted.

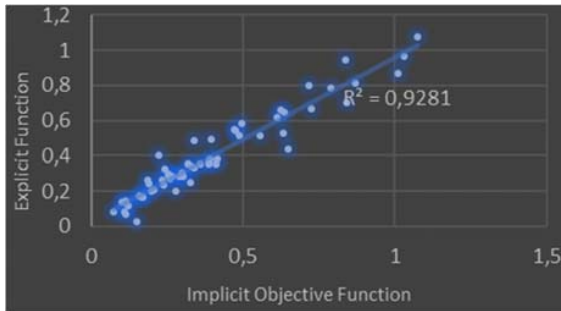


Fig. 11 Correlation between relative cost implicit and explicit objective functions for first case study with SMF

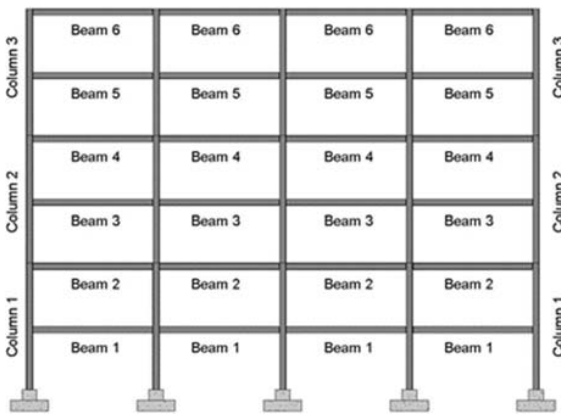


Fig. 12 Plane frame for second case study

#### 1) The Structural Weight

- Ordinary Moment Frame (OMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 13 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9826$ .

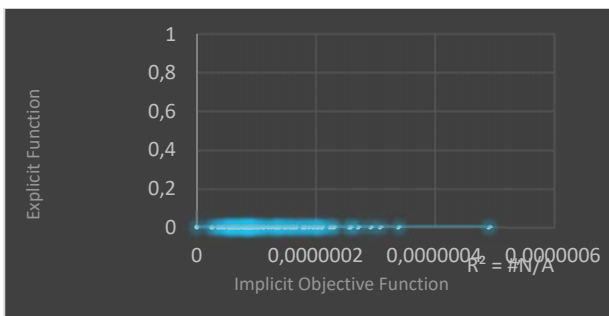


Fig. 13 Correlation between weight implicit and explicit objective functions for second case study with OMF

- Intermediate Moment Frame (IMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 14 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9842$ .

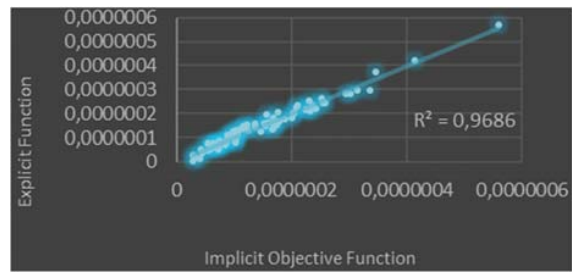


Fig. 14 Correlation between weight implicit and explicit objective functions for second case study with IMF

- Special Moment Frame (SMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 15 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9834$ .

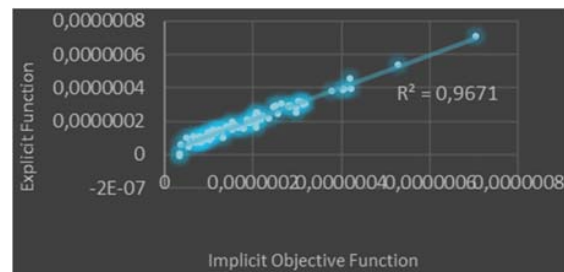


Fig. 15 Correlation between weight implicit and explicit objective functions for second case study with SMF

#### 2) The Structural Relative Cost

- Ordinary Moment Frame (OMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 16 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9906$ .
- Intermediate Moment Frame (IMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 17 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9835$ .

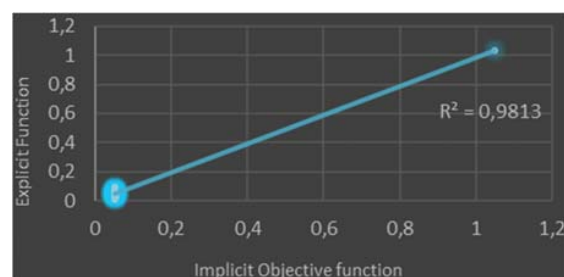


Fig. 16 Correlation between relative cost implicit and explicit objective functions for second case study with OMF



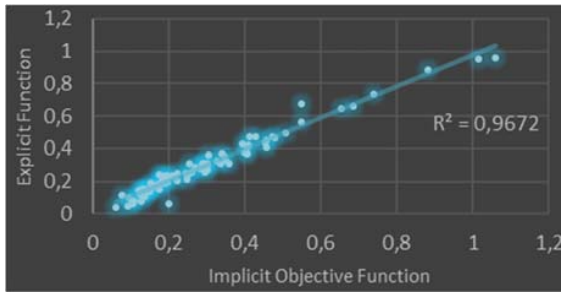


Fig. 17 Correlation between relative cost implicit and explicit objective functions for second case study with IMF

- Special Moment Frame (SMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 18 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.9850$ .

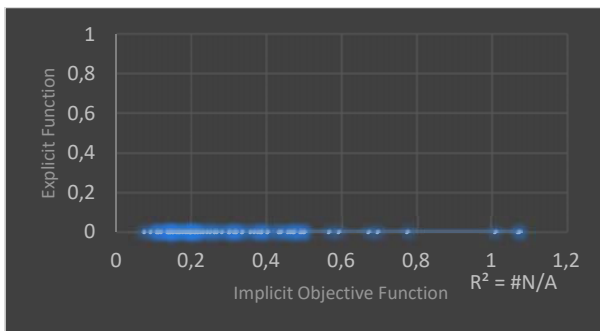


Fig. 18 Correlation between relative cost implicit and explicit objective functions for second case study with SMF

### C. The Third Case Study

This case study has a frame with ten floors, six bays, as show in Fig. 19. With grouping process, 16 design variables (ten sections for beams and six sections for columns) with 130 alternative designs have been adopted.

#### 1) The Structural Weight

- Ordinary Moment Frame (OMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 20 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.8974$ .
- Intermediate Moment Frame (IMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 21 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.8934$ .

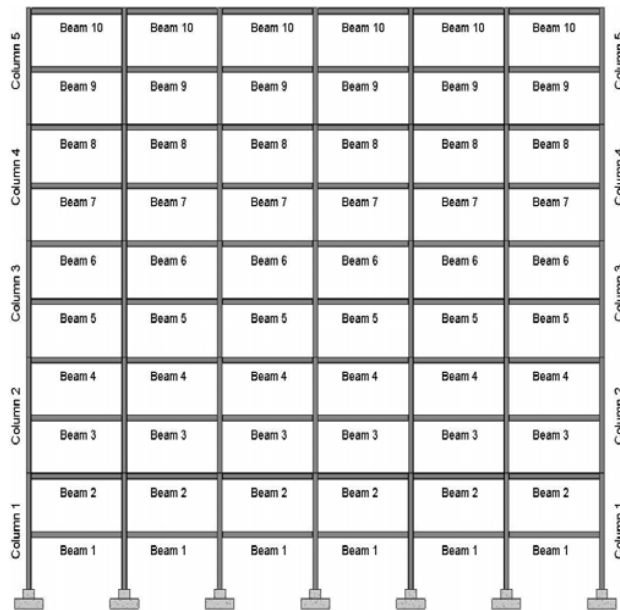


Fig. 19 Plane frame for third case study

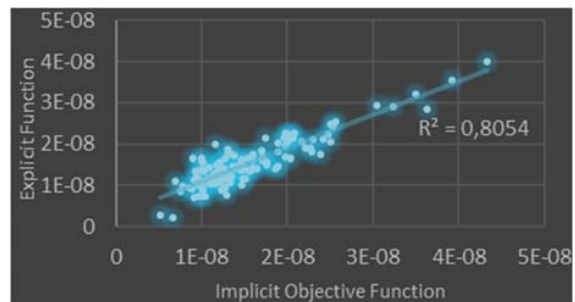


Fig. 20 Correlation between weight implicit and explicit objective functions for third case study with OMF

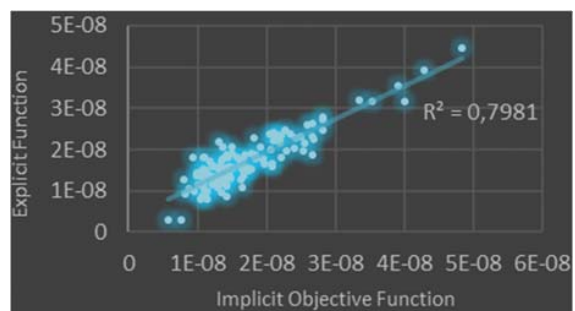


Fig. 21 Correlation between weight implicit and explicit objective functions for third case study with IMF

- Special Moment Frame (SMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 22 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.8840$ .



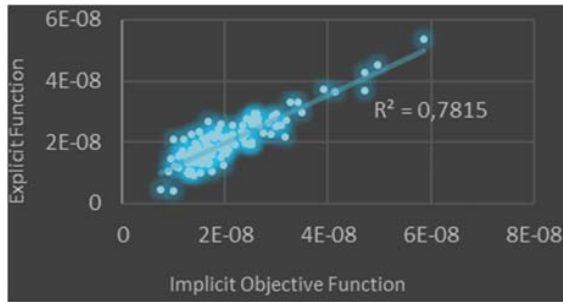


Fig. 22 Correlation between weight implicit and explicit objective functions for third case study with SMF

## 2) The Structural Relative Cost

- Ordinary Moment Frame (OMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 23 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.8930$ .
- Intermediate Moment Frame (IMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 24 where it has been found that two sets of values are highly correlated with a correlation coefficient of  $R^2 = 0.8881$ .

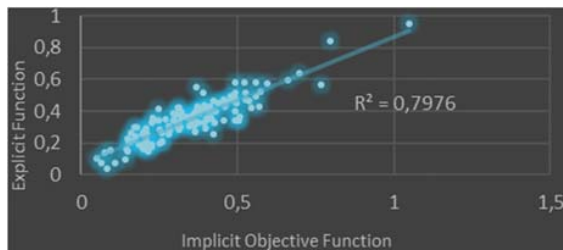


Fig. 23 Correlation between relative cost implicit and explicit objective functions for third case study with OMF

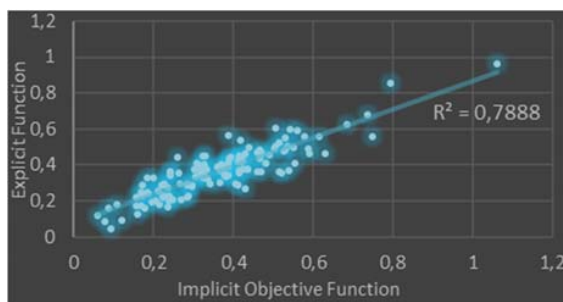


Fig. 24 Correlation between relative cost implicit and explicit objective functions for third case study with IMF

- Special Moment Frame (SMF): Correlation between implicit values of objective function with corresponding values predicated with meta-model explicit function is presented in Fig. 25 where it has been found that two sets of values are highly correlated with a correlation

coefficient of  $R^2 = 0.8690$ .

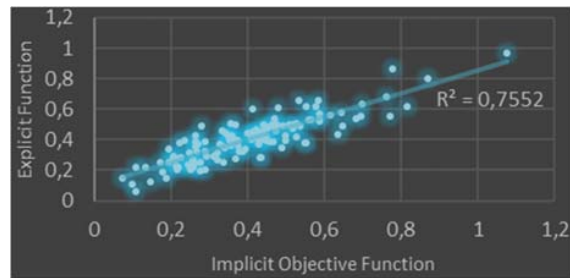


Fig. 25 Correlation between relative cost implicit and explicit objective functions for third case study with SMF

## VI. CONCLUSIONS

1. A powerful connection between structural analysis and design package, CSI SAP, and MATLAB optimization capabilities, has been executed in this study through using of a pure quadratic meta-model. With the meta-model concept, the cumbersome step of structural analysis and design has been excluded from optimization searching loop.
2. Explicit function formulation with the meta-model makes classical and metaheuristic optimization algorithms which have been developed and tested in other disciplines, directly applicable for structural optimization problems.
3. Good correlation with  $R^2$  in the range of 0.88 to 0.99 has been achieved between implicit objective functions and the corresponding explicit objective functions based on meta-model with pure quadratic relations.

## VII. RECOMMENDATIONS

1. A more powerful ANN nonlinear regression tool could be adopted to build a more accurate meta-model.
2. A more accurate structural simulation with including geometric and materials non-linearity could be adopted to provide excitation versus response data for the meta-model.
3. Instead of two-dimensional frame modeling, a more accurate three-dimensional model could be adopted.

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