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Mechanical Structure Design Optimization by Blind Number Theory: Time-dependent Reliability

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Abstract—In a product development process, understanding the functional behavior of the system, the role of components in achieving functions and failure modes if components/subsystem fails its required function will help develop appropriate design validation and verification program for reliability assessment. The integration of these three issues will help design and reliability engineers in identifying weak spots in design and planning future actions and testing program.
This case study demonstrate the advantage of unascertained theory described in the subjective cognition uncertainty, and then applies blind number (BN) theory in describing the uncertainty of the mechanical system failure process and the same time used the same theory in bringing out another mechanical reliability system model. The practical calculations shows the BN Model embodied the characters of simply, small account of calculation but better forecasting capability, which had the value of macroscopic discussion to some extent.

Keywords—Mechanical structure Design, time-dependent stochastic process, unascertained information, blind number theory.

I. Introduction

THE development of mechanical design reliability theory, the reliability of design optimization and the stochastic time-dependent characteristics of mechanical system reliability is gaining more attention. In that theory, load process on the mechanical structure was considered to Poisson Process while the actual process is much complex and unpredictable [1-2]. The machine lifetime and the resistance of mechanical structure degenerate with the course of time [3-4], which is difficult to predict precisely due to the degradation of component characteristics. A more advanced stochastic process is employed in describing these mechanical load and resistance.
Due to several design, state and constraint variables, the reliability design optimization is difficult to achieve. While applying also the advanced stochastic process in describing time-dependent uncertain information in mechanical structure design, this complicates further the problem as it exist many distribution types of random variables including complex operations among them, in every single reliability optimization design model. The traditional stochastic method for the complex reliability optimization model is also very cumbersome and inadequate in solving the problem. Therefore, it is believed the mechanical design process is a kind of unascertained system, which the concern/research subject of unascertained mathematical theory.
Different from other kinds of uncertainties such as random, fuzzy, grey and so on, the blind number theory which is discrete and numerical tool for expressing all sorts of uncertainty variables from the perspective of microcosm will provide an effective solution to the above stated problem. Up to now the unascertained mathematical theories have been successfully applied to several research fields [6-7]. Therefore, the blind number, as the expansion of the unascertained mathematics, has more general description on unascertained problems [5]. The statistic values of uncertainty variables used by BN come from fitting special distribution and data of the actual measurement. The practical calculation shows that the BN model embodied the characters of simply, small account of calculation but better forecasting capability, which has the value microscopic discussion to some extent.

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II. BN Description

A. Unascertained Number and Unascertained Rational Number

When decision makers are faced with information that are certain and not to certain degree i.e. unascertained, then this information is called as unascertained information [8]. Unascertained mathematics and grey number prove consistent in dealing with this case and the main difference between the two is the certain amount of unascertained number is larger than the grey number. The most basic and simplest unascertained number is unascertained rational number [9] which is the expansion of real number. The blind number theory and its application were developed on the basis of unascertained mathematics [10-11]. It adopts the expression type of grey variable and unascertained variable to express uncertainty variables and thus unifying the two expression types. The BN describes objectively the unascertained information in details; avoid flaw of information omission and distortion which is produced by the single real number in expressing these amounts. Liu et al [11] have derived the mean and the variance of BN to describe the center point and the distribution characteristics of BN.
Let for arbitrary closed interval $[a,\ b]$ so that $a = x_1 < x_2 < \cdots < x_n = b$, and if function $\phi(x)$ satisfy

$$\phi(x) = \begin{cases} \alpha_i, & x = x_i \text{ where } i = 1, 2, ..., n \\ \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

and

$$\sum_{i=1}^{n} \alpha_i = \alpha \tag{2}$$

with $0 < \alpha \leq 1$ $[a,\ b]$ and $\phi(x)$ are called unascertained rational number with n order where $\{[a,\ \ b], \phi(x)\}$; $\alpha, [a,\ b]$; and $\phi(x)$ represent the total confidence degree; distribution interval and confidence distribution density function respectively.

B. BN Concept

Let $\mathbf{R}$ be a set of real number,and $\overline{\mathbf{R}}$ a is the set of unascertained rational number, $g(I)$ is the set of grey interval number, and let suppose that $x_i \in g(I)$, $\alpha_i \in [a,\ b]$; $i = 1, 2, \dots, n$ and if $f(x)$ represents a grey interval number defined on the $g(I)$,we get

$$f(x) = \begin{cases} \alpha_i, & x = x_i \text{ with } i = 1, 2, ..., n \\ \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

when $i \neq j$, $x_i \neq x_j$ and $\sum_{i=1}^{n} \alpha_i = \alpha$, $0 < \alpha \leq 1$ the function $f(x)$ is said to be blind number, α_i is the confidence degree of $f(x)$ for x to be x_i and α represents the overall confidence degree of $f(x)$.
Let a, b be real numbers so that $a \leq b$, and $\odot[a,\ b] = \frac{1}{2}(a+b)$, and let assumed that $\frac{1}{2}(a + b)$ is the "core" value of grey interval number $[a, b]$. The mean value of BN is given as follow:

$$\mathbf{E}(f(x)) = \begin{cases} \alpha_i, & x = \dfrac{1}{\alpha}\left(\odot \displaystyle\sum_{i=1}^{n} \alpha_i x_i\right) \\ \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

C. Processing unascertained information by BN

As the random variable represents usually a distribution of probability density on a certain open interval in mathematics, but in engineering practice the BN expression is to take the aggregate of probabilities on limited numbers of great-narrow interval (GNIs) as random variable's probability density distribution approximate [12]. Let consider X as a random variable in normal distribution for example, its probability of falling within $[\mu - 6\sigma,\ \mu + 6\sigma]$ reach 100% substantially. By dividing this interval into several GNIs (e.g. 200), each X distribution can be considered as certain typical distribution (usually equal distribution). Summing up those GNIs' probabilities can be view as random variable X's probability density distribution. The uncertainty of random variable is strictly limited to a "defined" range. Transferring strong uncertainty to weak uncertainty on the GNI by the result of dispersing method on the extensive interval, the random variable then can be numerically expressed and calculated easily. Under this principle, any uncertainty variable (e.g random variable or fuzzy variable) whose distribution type and parameters are known can be expressed by BN, the narrower the GNIs, the more approximate to original distribution function will be.
When the distribution type or parameters of uncertainty variable are not known, the uncertainty variable can also be expressed by BN based on statistical analysis of the known data, and to reduce the data fitting error experts engineering practice experience is used.

III. Time-dependent Characteristic of Mechanical Structure based on Stochastic Process

Material properties, manufacturing processes, operation conditions, physical environment and maintenance practices are some of the main factors determining the lifetime of mechanical structures or machines resulting to uncertainties in load on the mechanical structure and its structural strength. Through out their lifetime, the load and the strength of structure follow a stochastic process.
Let consider a probability space $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$ and let $\{z(t), t \geq 0\}$ be a standard 1-dimensional Brownian motion defined on this probability space. Assuming $\{\mathbf{F}, t \geq 0\}$ to be the natural filtration generated by Brownian motion: $\{z(t), t \geq 0\}$, i.e. $\mathbf{F} = \sigma\{z(t), t \geq 0\}$.
Let stochastic process $\mathbf{S}$ satisfy Itō lemma drift-diffusion process

$$d\mathbf{S} = \mu \mathbf{S} dt + \omega \mathbf{S} dz \tag{5}$$

And assuming that $\mathbf{G} = \ln \mathbf{S}$, then

$$d\mathbf{G} = \left(\frac{\partial \mathbf{G}}{\partial t} + \mu \frac{\partial \mathbf{G}}{\partial \mathbf{S}} \mathbf{S} + \frac{\omega^2}{2} \frac{\partial^2 \mathbf{G}}{\partial \mathbf{S}^2} \mathbf{S}^2\right) dt + \omega \frac{\partial \mathbf{G}}{\partial \mathbf{S}} \mathbf{S} dz \tag{6}$$

$$d\mathbf{G} = \left(\mu - \omega^2\right) dt + \omega dz \tag{7}$$

Where $\mathbf{G}$, $(\mathbf{G}_T - \mathbf{G}_t)$, $(\mu - \omega^2)(T - t)$ and $\omega\sqrt{(T - t)}$ represent a Generalized Wiener Process, a mean of normal distribution, and a standard deviation respectively.

$$\ln \mathbf{S}_T \sim \phi \left[\ln \mathbf{S}_t + \left(\mu - \omega^2\right)(T - t),\ \omega\sqrt{(T - t)}\right] \tag{8}$$

S_T and S_t are stochastic values of the future period $(T)$and the present period (t) respectively. μ is drift rate and measure of certainty while ω is a volatility and measure of uncertainty. They are estimated as follow from the historical data [13]

$$\mu = \overline{\eta} = \frac{1}{n} \sum_{i=1}^{n} \ln \frac{\mathbf{S}_i}{\mathbf{S}_{i-1}} \tag{9}$$

$$\omega = \left[\frac{1}{n-1} \sum_{i=1}^{n} (\eta_i - \overline{\eta})^2\right]^{\frac{1}{2}} \tag{10}$$

where $\eta_i = \ln(S_i / S_{i-1})$ and $\overline{\eta}_i$ is the mean value of η_i

IV. Time-dependent Design Optimization Model with BN

As it was assumed that the load on and the reaction of mechanical structures and the mechanical strength are subjected to the stochastic process described above (8). According to the same analysis e.g. at time $t = 0$, the logarithm of the load $\mathbf{W}_T$ and the strength δ_T are normally distributed to the next time T. We get:

$$\ln \mathbf{W}_T \sim \phi \left[\ln \mathbf{W}_0 + \left(\mu + \omega^2\right) T,\ \omega_W \sqrt{T} \right]$$

$$\ln \delta_T \sim \phi \left[\ln \delta_0 + \left(\mu + \omega^2\right) T,\ \omega_\delta \sqrt{T} \right]$$

V. The Mechanical Reliability Design Model based on BN Theory

A. Model Assumptions

In the model, let a BN x_i represents the time interval variable from $(i-1)^{th}$ time to i^{th} failure time, which means the average failure time interval t is $\mathbf{E}(x_i)$ where t is a random time between $(i-1)^{th}$ and i^{th} failure time. As the early failure data does play any important role in forecasting the future behavior of the mechanical structure, the current failure interval can be better in forecasting the future behavior than the early data. Therefore, $\mathbf{E}(x_i)$ can be obtained by a constant number A (in accordance with specific circumstances) from $(i-A)^{th}$ to $(i-1)^{th}$ failure time.

The followings basics can be assumed for the model

1) The test and run environment of the procedure are identical.
2) The failure rate of the procedure within each failure interval is a constant λ whose value is $1/\mathbf{E}(xi)$, where x_i is the time variable from $(i-1)^{th}$ to i^{th} failure time.

B. Model Formula

As the mathematical expectation of x_i; $\mathbf{E}(x_i)$ is mean time between failures at t, where t is random time between $(i-1)^{th}$ failure time and i^{th} failure time. From the above assumption (2) we get:

$$\mathbf{E}(x_i) = MTBF = 1/\lambda \tag{11}$$

where $\mathbf{E}(x_i)$ is the mathematical expectation of BN

VI. Model Validation and Simulation

Certainties and uncertainties that may result from variations in material properties, manufacture quality, operating conditions, inspection procedures and maintenance practices always exist during machine or mechanical structure lifetime. Such factors give rise to uncertainties in load (or reaction) on the mechanical structure and the structural strength. Throughout service life of machine, load and strength each is obedient to a stochastic process. Its evolution can be simulated by the follow stochastic process. For example, the tensile strength of a batch of steel is tested 50 times. The yield limits is between say $a = 398.5$ and $b = 412.3$ MPa, the number of times that the yielding points (in MPa) arise at every interval $[x_i,\ x_i+1.5]$ is shown in table I.

The number of the intervals is determined as required, and the reliability that the yielding limit value arises in each interval can be given in accordance with the number of times that the yielding limit value arise at the interval and with engineering experience. Thus, subjective engineering experience can be brought into the expression of the uncertain variables

$$\phi(x) = \begin{cases} 0.007, & [398,\ 399.5] \\ 0.028, & [399.5,\ 401] \\ 0.079, & [401,\ 402.5] \\ 0.159, & [402.5,\ 404] \\ 0.226, & [404,\ 405.5] \\ 0.226, & [405.5,\ 407] \\ 0.159, & [407,\ 408.5] \\ 0.079, & [408.5,\ 410] \\ 0.028, & [410,\ 411.5] \\ 0.007, & [411.5,\ 413] \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

When there is a lack of engineering experience, the reliability that the yielding limit value arises in each interval can be determined strictly in accordance with objective data in the expression below:

$$\phi(x) = \begin{cases} 0.02, & [398,\ 399.5] \\ 0.02, & [399.5,\ 401] \\ 0.06, & [401,\ 402.5] \\ 0.14, & [402.5,\ 404] \\ 0.24, & [404,\ 405.5] \\ 0.22, & [405.5,\ 407] \\ 0.16, & [407,\ 408.5] \\ 0.08, & [408.5,\ 410] \\ 0.04, & [410,\ 411.5] \\ 0.02, & [411.5,\ 413] \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

We take MTBF as an example to explain the algorithm of calculating the mechanical structure reliability parameters by the BN model, in the following example (the tensile strength of a batch of steel is tested 50 times): where $x_i = [x_i,\ x_i+1.5]$ and $x_0 = 398$ MPa. Finally $\mathbf{E}(x_i) = 405.71$ and 405.50 MPa respectively. The namely obtained MTBF of tensile strength of a batch of steel failure after tested 50 times is 405.71 or 405.50 MPa according to subjective engineering or the lack or engineering experience respectively.

TABLE I
MEASURED DATA OF YIELD LIMIT

No. of Occurrences	1	1	3	7	12
Yield Limit (MPa)	x_1	x_2	x_3	x_4	x_5
No. of Occurrences	11	8	4	2	1
Yield Limit (MPa)	x_6	x_7	x_8	x_9	x_{10}

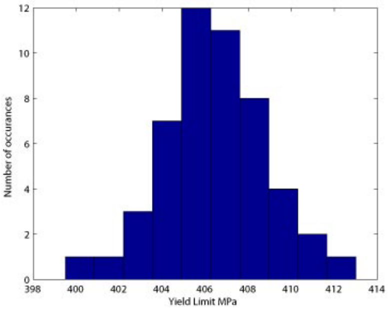

Fig. 1. Reliability Histogram

VII. CONCLUSION

After a brief introduction of unascertained information and BN theory; in this paper; a time-dependent stochastic process analysis is applied to a mechanical structure design. Based on this time-dependent stochastic process theory, reliability optimization theory and BN theory, a time-dependent reliability optimization model is built. The result shows that BN prove to be an important numerical tool which is convenient and effective in solving complex optimization design problems. To achieve a target reliability and to ensure BN performance optimization in the future, other issues regarding mechanical design based on BN have to be carried on in providing accurate forecasting capabilities on the change point.

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