Mathematical Determination of Tall Square Building Height under Peak Wind Loads

Debojyoti Mitra

Abstract—The present study concentrates on solving the along wind oscillation problem of a tall square building from first principles and across wind oscillation problem of the same from empirical relations obtained by experiments. The criterion for human comfort at the worst condition at the top floor of the building is being considered and a limiting value of height of a building for a given cross section is predicted. Numerical integrations are carried out as and when required. The results show severeness of across wind oscillations in comparison to along wind oscillation. The comfort criterion is combined with across wind oscillation results to determine the maximum allowable height of a building for a given square cross-section.

Keywords—Tall Building, Along-wind Response, Across-wind Response, Human Comfort.

I. INTRODUCTION

THE present generation structures, unlike in the past, are remarkably flexible, low in damping and light in weight. These structures are results of the development of modern materials and construction techniques. Their enhanced susceptibility to the action of overall and local wind effects calls for development of newer methods of design. Such developments have augmented emergence of a new discipline called 'wind engineering'. Researchers in this discipline need a thorough knowledge in bluff body aerodynamics and their task is to ensure that the performance of structures subjected to the action of wind will be adequate during their anticipated life from the standpoint of both structural safety and serviceability. The designer can achieve this end if he/she has prior information on the wind environment, the relation between the environment and the forces it induces on the structure, and the behaviour of structure under the action of these forces. Information on the wind environment can be had from meteorology, micrometeorology and climatology. Estimation of aerodynamic forces like drag or along-wind force, lift or across-wind force and torsional moment can either be obtained using available results of aerodynamic theory or be found out by carrying out special wind tunnel tests. These forces and moments, in most cases, may be fluctuating with time and cause vibrations in earth-fixed structures, and structural response analysis becomes essential.

Debojyoti Mitra is with Sir Padampat Singhania University, Udaipur – 313601, Rajasthan, India (phone: +91-9602285716; fax: -+91-2957-226094; e-mail: debojyoti.mitra@spsu.ac.in).

The random character of this time-dependence calls for the elements of the theory of random vibrations be applied to the analysis. Thus, a study of the interaction between the aerodynamic and the inertial, damping and elastic forces is required with the purpose of investigating the aerodynamic stability of the structure.

A good number of researchers have contributed to this field of structural dynamics. Reference [6] first studied the effect of atmospheric turbulence on structural response back in 1952. Reference [3] suggested some procedure for estimation of along wind response of tall buildings in 1961. Reference [20] added more flexibility with respect to the choice of certain meteorological parameters. References [13] and [15] have significant contributions on prediction of building response. Gust buffeting is taken into consideration in along wind response analyses [16]. Reference [18] carried out significant researches on finding out maximum limits of 3-D responses of structures. 3-D responses in uncoupled manner are solved in closed form [7] and gust effect factors for slender vertical structures are estimated [8] as well. A general classification of vertical structures can also be arrived at [17] under wind loads. Reference {5] has significant contributions towards time frequency analysis of wind effects on structures. Very recently, Equivalent static wind actions on wind structures are analysed using gust factor technique as well as load combination technique and are solved in closed form [10]. A new method, referred to as the global loading technique, is also proposed here.

The present study attempts to estimate the along-wind and across-wind response of tall buildings not significantly affected by the presence of neighboring tall buildings. It may be approximately assumed that the interference effect is negligible if the distance between the two tall buildings exceeds about six to eight times the average of the horizontal dimensions of the buildings. Reference [8] noted that a square building located in urban terrain near a building with similar geometry and dimensions will show more or less the same effect which it shows in the absence of the neighboring structure. The present study also takes into account the factor of occupant comfort [2] and attempts to predict the maximum allowable height of a tall square building based on that comfort factor. This consideration, as the authors feel, is yet to be considered in any theoretical evaluation of building height subjected to wind loads till date, which makes the present study unique in that respect.

II. THEORETICAL ANALYSIS

A. Along wind response

The main assumptions on which the following theoretical analysis is based are as follows:

The terrain is approximately horizontal around the structure and its roughness is reasonably uniform over a sufficiently large fetch.

The mean wind speed is normal to the building face under consideration, which is endorsed by the highest values of along-wind response obtained in wind tunnel tests by [12].

The mean wind velocity profile is described by the relation

$$U(z) = 2.5u_* \ln\left(\frac{z - z_d}{z_o}\right), z \geq z_d + 10 \qquad (1)$$

$$U(z) = 2.5u_* \ln\left(\frac{10}{z_o}\right), z \leq z_d + 10 \qquad (2)$$

The mean velocity in (1) and (2) is averaged over a period of one hour.

The longitudinal velocity fluctuations are described by

$$\overline{u^2} = \beta u_*^2 \qquad (3)$$

The values of β for different roughness terrains are given by [1].

Now, a tall vertical earth-fixed structure is considered in general, for which it may be assumed that the displacement in the horizontal direction x is the same for all points in the structure that have the same height z. it can be shown [11] that for small damping ratio the generalized co-ordinates $\xi_i(t)$ satisfy the equations

$$\ddot{\xi}_i(t) + 2\xi_i(2\pi n_i)\dot{\xi}_i(t) + (2\pi n_i)^2 \xi_i(t) = \frac{Q_i(t)}{M_i},$$

i = 1,2,3 $\qquad (4)$

where ξ_i, n_i, M_i and $Q_i(t)$ are the damping ratio, the natural frequency, the generalized mass and the generalized force in the i th mode and having expressions

$$M_i = \int_0^H [x_i(z)]^2 m(z)dz \qquad (5)$$

where m(z) is the mass of structure per unit length, and

$$Q_i(t) = \int_0^H p(z,t)x_i(z)dz \qquad (6)$$

where H is the height of the structure, and p(z,t) is the time-dependent load per unit length acting on the system.

If the load p(z,t) is such that

p(z,t)=F(t)δ(z-z1) $\qquad (7)$

where δ(z-z1) is defined in a manner δ(z-z1) = 0 for $z \neq z1$,

$$\lim_{\Delta z \to 0} \int_0^{\Delta z} (z - z_1)dz = 1 \qquad (8)$$

so that if the structure is subjected to a concentrated force F(t) acting at a point of co-ordinate z1, the generalised force Qi(t) will be

$$Q_i(t) = \lim_{\Delta z \to 0} \int_{z_1}^{z_1 + \Delta z} p(z,t)x_i(z)dz = x_i(z_1)F(t) \qquad (9)$$

Now, if the load p per unit length in (6) is independent of time, the corresponding mean along-wind deflection is given by

$$\overline{x}(z) = \sum_i \frac{\int_0^H \overline{p}(z)x_i(z)dz}{4\pi^2 n_i^2 M_i} x_i(z) \qquad (10)$$

where Mi is defined by (5) and $\overline{p}$ is the time-invariant load.

The mean wind load acting on a building of width B may be written as

$$\overline{p}(z) = \frac{1}{2}\rho(C_w + C_l)BU^2(z) \qquad (11)$$

where ρ is the density of air in kg/m³, C_w and C_l are the width-averaged values of mean pressure coefficient on the windward face and suction coefficient on the leeward face, respectively, and U(z) is the mean speed at elevation z in the undisturbed upstream flow, in m/sec.

A detailed mathematical treatment, already available in [14], ultimate leads to the following relations for the present case of along wind response.

The mean square value of the fluctuating along-wind deflection is given by

$$\sigma_x^2(z) = \int_0^\infty S'''(z,n)dn \qquad (12)$$

and the mean square value of the along-wind acceleration is

$$\sigma_{\ddot{x}}^2 = 16\pi^4 \int_0^\infty n^4 S'''(z,n)dn \qquad (13)$$

Here

$$S'''(z,n) = \frac{\rho^2}{16\pi^4} \sum \frac{x_i^2(z)\left[C_w^2 + 2C_w C_l N(n) + C_l^2\right]}{n_i^4 M_i^2 \left\{\left[1 - \left(\frac{n}{n_i}\right)^2\right]^2 + 4\xi_i^2\left(\frac{n}{n_i}\right)^2\right\}}$$

x

$$\int_0^B \int_0^B \int_0^H \int_0^H x_i(z_1)x_i(z_2)U(z_1)U(z_2)S''^{\frac{1}{2}}(z_1)S''^{\frac{1}{2}}(z_2)$$

$$xCoh(y_1,y_2,z_1,z_2,n)dy_1dy_2dz_1dz_2 \tag{14}$$

is the spectral density of the along wind fluctuating deflection and $S'^{\frac{1}{2}}(z_i,n)$ is the spectral density of the pressures at point $A_i (i = 1,2)$ and $Coh(y_1,y_2,z_1,z_2,n)$ and $N(n)$ are the across-wind and the along wind cross correlation coefficients, respectively.

The largest peak of the fluctuating response occurring in the time interval T is given by

$$x_{max}(z) = k_x(z)\sigma_x(z) \tag{15}$$

where the peak factor $k_x(z)$ can be expressed [14] approximately as

$$k_x(z) = \left[2\ln w(z)T\right]^{\frac{1}{2}} + \frac{0.577}{\left[2\ln w(z)T\right]^{\frac{1}{2}}} \tag{16}$$

$$\text{Here, } w(z) = \left[\frac{\int_0^\infty n^2 S'''(z,n)dn}{\int_0^\infty S'''(z,n)dn}\right]^{\frac{1}{2}} \tag{17}$$

Similarly, the largest peak of the fluctuating along-wind acceleration is, approximately,

$$\ddot{x}_{max}(z) = k_{\ddot{x}}(z)\sigma_{\ddot{x}}(z) \tag{18}$$

$$\text{where, } k_{\ddot{x}}(z) = \left[2\ln \ddot{w}(z)T\right]^{\frac{1}{2}} + \frac{0.577}{\left[2\ln \ddot{w}(z)T\right]^{\frac{1}{2}}} \tag{19}$$

$$\text{and } \ddot{w}(z) = \left[\frac{\int_0^\infty n^6 S'''(z,n)dn}{\int_0^\infty n^4 S'''(z,n)dn}\right]^{\frac{1}{2}} \tag{20}$$

Here, it is reasonable to assume

$$Coh(y_1,y_2,z_1,z_2,n) = \exp\left\{-\frac{n\left[C_z^2(z_1-z_2)^2 + C_y^2(y_1-y_2)^2\right]^{\frac{1}{2}}}{\frac{1}{2}[U(z_1)+U(Z_2)]}\right\} \tag{21}$$

$$\text{and } N(n) = \frac{1}{q} - \frac{1}{2q^2}(1 - e^{-2q}) \, [19] \tag{22}$$

where C_y and C_z are known as exponential decay parameters [20],

$$q = \frac{15.4n\Delta x}{\overline{U}} \tag{23}$$

$$\overline{U} = U(\frac{2}{3}H) \text{, and } \Delta x = \text{minimum of B, D and H.}$$

Hence, the maximum along wind deflection of the structure at elevation z may thus be written as

$$X_{max}(z) = \overline{x}(z) + x_{max}(z) \tag{24}$$

where $\overline{x}(z)$ and $X_{max}(z)$ are given by (10) and (15) respectively.

B. Across wind response

The across wind response, caused mainly by the asymmetrical wake flow behind the buildings and structures, does not have any expression based on first principles till date. However, empirical relation proposed on the basis of wind tunnel experiments can be used for obtaining satisfactory results in real life situations. A number of expressions are available for tall square cross-section buildings in cases where the root mean square value of the across wind oscillations at the tip of the building, σ_y, doesn't exceed a critical value σ_{ycr}. Vickery[3] proposed the expression

$$\frac{r_y\sigma_y(H)}{\sqrt{A}} = \alpha\left[\frac{U(H)}{n_1\sqrt{A}}\right]^n \frac{1}{\xi_1^{\frac{1}{2}}}\frac{\rho}{\rho_b} \tag{25}$$

where $\sigma_y(H)$ = rms of across wind oscillations at top of structure, r_y = peak factor expressing the ratio of the peak response to rms response ($r_y \approx 3.5$), H = height of the building in metres, A = cross-sectional area of the building in m^2, U(H) = mean wind speed at the top of the structure in m/s, n_1 = fundamental frequency of vibration in Hz, ξ_1 = damping ratio, ρ = air density in kg/m3, ρ_b = bulk mass of building per unit volume in kg/m3, n and α = constants determined empirically from wind tunnel experiments (n =

3.5, $\alpha = 0.0006 \pm 0.00025$). The rms of the accelerations at the top of the structure can be estimated using (25) and

$$\sigma_{\ddot{y}}(H) = (2\pi n_1)^2 \sigma_y(H) \tag{26}$$

However, (25) was obtained for a tall square building with an aspect ratio B/H = $\dfrac{1}{4.2}$, $\xi_1 = 0.01$ and $\rho_b \approx 200$ kg/m³. Hence, use of (25) should be restricted to buildings having characteristics that do not differ drastically from the values mentioned above.

If $\overline{S}(n)$ is the across wind modal force, and

$$\breve{Y}^2 \left[\frac{n_1 B}{U(H)} \right] = \frac{n_1 \overline{S} \left[\dfrac{n_1 B}{U(H)} \right]}{\left[\dfrac{1}{2} \rho U^2(H) BH \right]^2} \tag{27}$$

then

$$\sigma_y(H) \approx \frac{\pi^{\frac{1}{2}}}{2\xi_1^{\frac{1}{2}}} \frac{1}{(2\pi n_1)^2 M_1} \frac{1}{2} \rho U^2(H) BH \breve{Y} \tag{28}$$

One can safely assume that the mass is uniformly distributed over the building height. Then, if the building has a square cross-section and the fundamental modal shape is linear,

$$M_1 = \frac{1}{3} \rho_b B^2 H \tag{29}$$

and

$$\sigma_y(H) \approx 0.0337 \left[\frac{U(H)}{n_1 B} \right]^2 \frac{\rho}{\rho_b} \frac{1}{\xi^{\frac{1}{2}}} B \breve{Y} \tag{30}$$

where

$$\breve{Y} \approx (4.45 \pm 1.80) 10^{-3} \left[\frac{U(H)}{n_1 B} \right]^{1.5} \tag{31}$$

Then, the peak across wind response and acceleration are given respectively by

$$Y_{\max} = r_y \sigma_y(H) \tag{32}$$

and

$$\ddot{Y}_{\max} = r_{\ddot{y}} \sigma_{\ddot{y}}(H) \tag{33}$$

$$\left(r_{\ddot{y}} \approx 4.0 \right).$$

III. Solution Procedure

The computation of along wind response is carried out by evaluating the integrals in (10) through (20). The following data are assumed during the solution process:

a. Damping ratio ξ_1 is taken to be 0.016 [4].
b. The zero-plane displacement, $z_d \approx 0$ and z_0 is obtained from [1].
c. Exponential decay parameters, C_y and C_z are taken to be 16 and 10 respectively [20].
d. The friction velocity is obtained by

$$u_* = \frac{U(z_R)}{2.5 \ln\left[\dfrac{z_R - z_d}{z_0} \right]} \tag{34}$$

The reference height most commonly used is $z_R = 10$ m.

e. The hourly mean wind speed at height z_R is obtained from the Indian Standards [4]. The regional basic wind speed is taken to be 50 m/s for this particular case.
f. Mean pressure and suction coefficients are assumed to be $C_w = 0.8$ and $C_l = 0.5$.
g. Duration of storm T is assumed to be 3600 sec.
h. The bulk mass of the building per unit volume is taken to be 200 kg/m³.
i. $$x_1(z) = \frac{z}{H} \tag{35}$$
j. Contributions of higher vibration modes other than the fundamental mode are neglected.
k. The following expression [4] is used for determining the natural frequency of vibration of tall buildings in its fundamental mode when $H/B \geq 5.0$ or $n_1 \leq 1.0$ Hz.

$$n_1 = \frac{\sqrt{B}}{0.09H} \tag{36}$$

Equations are solved for a height range of 50 – 500 metres and for a width range of 25 – 150 metres.

Computation for across wind response is carried out with similar data set using (34) through (42). However, results are accepted for aspect ratio (B/H) of 0.2 to 0.3 only since (34) is obtained for B/H = $\dfrac{1}{4.2}$.

The serviceability of tall buildings or, human comfort criteria is imposed to determine a maximum height for a given width of the building located in a town. It is observed [2] that the degree of discomfort becomes annoying when the peak acceleration becomes equal to or more than 1.5% of acceleration due to gravity. This criterion is applied to determine maximum allowable dimensions for a tall square building.

IV. RESULTS AND DISCUSSION

Fig.1 shows variation of along wind peak response with base side length for different building heights. It is observed that the response is large for very high buildings with low base side length. This is prominent for base side ≤60 m for H = 600 m, and base side ≤ 45 m for H = 400 m.
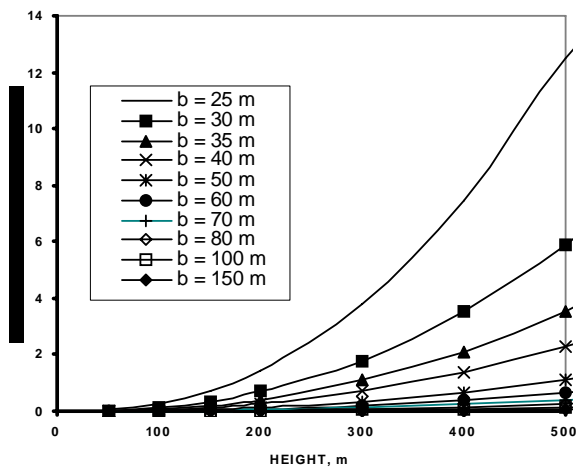
TABLE 1
MAXIMUM BUILDING HEIGHT SATISFYING HUMAN COMFORT FACTOR
(CONSIDERING ALONG WIND OSCILLATIONS)

Building Width (m)	Maximum Building Height (m)
25	120
30	185
35	310
40 and above	500 is safe

Fig.3 shows similar variations of across-wind response, while Fig.4 shows variations of across wind acceleration with building height. The across wind response is expectedly much greater than along wind response. Table 2 below depicts the height restrictions for some buildings from which results of buildings with aspect ratio between about 0.2 to 0.3 are accepted when the comfort criteria is imposed.

Fig. 1 Along Wind Peak Response of Tall Square Buildings

Fig.2 shows variation of along wind peak acceleration with base side length for different building heights. The comfort criterion of 1.5 % g, i.e., about 0.15 m/s2, is applied to find out the limiting values of building height for different base side lengths. These results are compiled in Table 1 below.
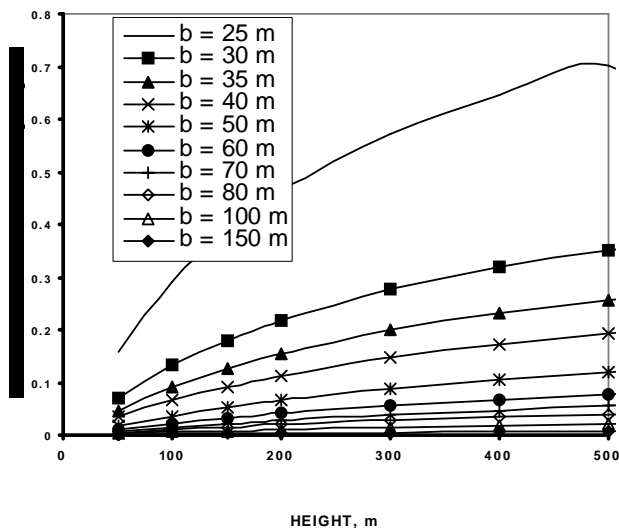

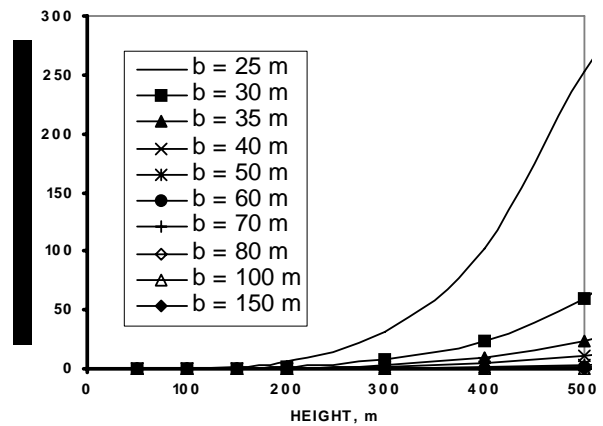Fig. 2 Along Wind Peak Acceleration of Tall Square Buildings

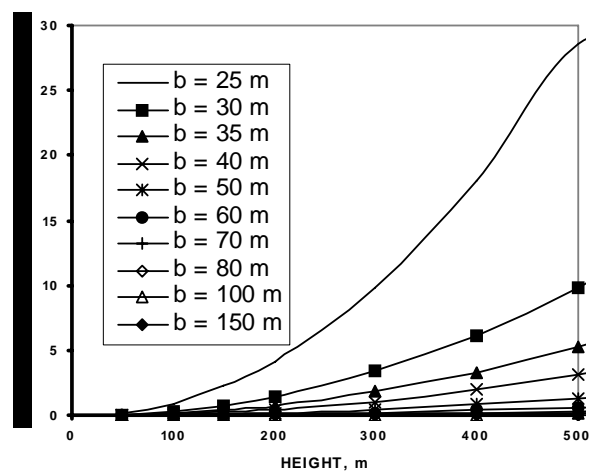Fig. 3 Across Wind Peak Response of Tall Square Buildings

Fig. 4 Across Wind Peak Acceleration of Tall Square Buildings

TABLE II
MAXIMUM BUILDING HEIGHT SATISFYING HUMAN COMFORT FACTOR
(CONSIDERING ACROSS WIND OSCILLATIONS)

Building Width (m)	Maximum Building Height (m)	Aspect Ratio (B/H)	Acceptable or not (Y/N)
25	Less that 50	More than 0.5	N
30	70	0.43	N
35	90	0.39	N
40	120	0.33	Y
50	180	0.28	Y
60	250	0.24	Y
70	340	0.21	Y
80	440	0.18	Y
100 and above	Much more than 500	Much less than 0.2	N

The above results depict that, across wind oscillation being more severe than along wind oscillation, the maximum building height has to be governed by across wind acceleration values.

V. CONCLUSIONS

The present study shows an analytical prediction method for ascertaining the maximum height of a tall square building when the base area is given at a certain terrain with certain meteorological conditions and various parameters of the building structure. The present analysis was confined to determination of height of a building, which was placed in a sparsely built area, i.e., a town. This analysis can easily be extended to cases at other terrains and with some other parameter sets of buildings. However, the present study revealed the severe ness of across wind oscillations compared to along wind oscillations and the comfort criteria of humanity at the top of the building played a major role in determining the maximum height of a tall square building.

REFERENCES

[1] Bietry J, Simiu E and Sacre C.1978. Mean wind profiles and changes of terrain roughness. *J. Struct. Div., ASCE*, vol.104, pp 1585-1593.
[2] Chang F K. 1973. Human response to motions in tall buildings. *J. Struct. Div., ASCE*, vol. 98 No ST6, pp 1259-1272.
[3] Davenport A G. 1961. The application of statistical concepts to the wind loading of structures. *Proc. Inst. Civ. Eng.*, vol. 19, pp 449-472.
[4] IS: 875 (Part 3) – 1987,.Code of practice for design loads for buildings and structures (Wind loads), (Reaffirmed 1997).
[5] Kareem A and Kijewski K. 2002. Time-frequency analysis of wind effects on structures. *J. Wind Engg. Ind. Aerodyn.* Vol. 90, pp. 1435.
[6] Liepmann H W. 1952. On the application of statistical concepts to the buffeting problem. *J. Struct. Div., ASCE*, vol. 19, pp 793-800,822.
[7] Piccardo G and Solari G. 2000. 3D wind excited response of slender structures: closed form solution, *J. Struct. Engg., ASCE*, vol. 126, pp 936-943.
[8] Piccardo G and Solari G. 2002. 3D gust effect factor for slender vertical structures. *Prob. Eng. Mech.*, vol. 17, pp 143-155.
[9] Reinhold T A. 1979. Mean and fluctuating forces and torques on a tall building model of square cross section as a single model, in the wake of a similar model, and in the wake of a rectangular model, *Report VP1-E-79-11*, Dept. of Engg. Sc. & Mechanics, Virginia Polytechnic Institute, Blacksburg, VA.
[10] Repetto M P and Solari G. 2002. General tendencies and classification of vertical structures under wind loads, *J. Wind Engg. Ind. Aerodyn*. Vol. 90, pp 1535-1545.
[11] Robson J D. 1964. *An Introduction to Random Vibration.* Elsevier, New York.
[12] Rosati P A. 1968. An experimental study of the response of a square prism to wind load, BLWT II-68, *Faculty of Graduate Studies, university of Western Ontario, London, Ontario, Canada.*
[13] Simiu E. 1980. Revised procedure for estimating along wind response, *J. Struct. Div., ASCE*, vol. 106, pp 1-10.
[14] Simiu, E., Scanlan, R.H. (1986) Wind Effects on Structures, *John-Wiley & Sons,* New York.
[15] Solari G. 1982. Along wind response estimation: closed form solution, *J. Struct. Div., ASCE*, vol. 108, pp 225-244.
[16] Solari G. 1993. Gust buffeting I: peak wind velocity and equivalent pressure, II: dynamic along wind response, *J. Struct. Engg.., ASCE*, vol. 119, pp 365-382.
[17] Solari G and Repetto M P. 2002. General tendencies and classification of vertical structures under gust buffeting. *J. Wind Engg. Ind. Aerodyn,.* vol. 90, pp. 1299-1319.
[18] Tamura Y, Kawai H, Uematsu Y, marukawa H, Fujii K and Taniike Y. 1996. Wind loads and wind induced response estimations in the recommendations for loads on buildings AU 1993. *Engg. Struct.*, vol. 18, No. 6, pp 399-411.
[19] Vellozzi J and Cohen E. 1968. Gust response factors. *J. Struct. Div., ASCE*, vol. 94, pp 1295-1313.
[20] Vickery B J. 1970. On the reliability of gust loading factors. *Proc. Technical Meeting Concerning Wind Loads on Buildings and Structures.* Building Science Series, National Bureau of Standards, Washington DC.