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Lower Bounds of Some Small Ramsey Numbers

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Abstract—For positive integer s and t, the Ramsey number $R(s,t)$ is the least positive integer n such that for every graph G of order n, either G contains K_s as a subgraph or $\overline{G}$ contains K_t as a subgraph. We construct the circulant graphs and use them to obtain lower bounds of some small Ramsey numbers.

Keywords—Lower bound, Ramsey numbers, Graphs, Distance line.

I. Introduction

FOR positive integer s and t, the Ramsey number $R(s,t)$ is the least positive integer n such that for every graph G of order n, either G contains K_s as a subgraph or $\overline{G}$ contains K_t as a subgraph.

The problem of determining Ramsey numbers is known to be very difficult. The few known exact values and several bounds for different graphs are scattered among many technical paper [1]

	t								
s	3	4	5	6	7	8	9	10	11
3	6*	9*	14*	18*	23*	28*	36*	40	46
4		18*	25*	35	49	56	73	92	98
5			43	58	80	101	126	144	171
6				102	113	132	169	179	253

* *Exact Ramsey numbers*

Table 1. Known nontrivial values and some lower bounds for Ramsey numbers $R(s,t)$.

For small Ramsey numbers $R(s,t)$, the general method in establishing a lower bound is to construct a graph G which does not contain K_s and the $\overline{G}$ of G does not contain K_t. In this paper, we construct the circulant graphs and use them to obtain lower bounds for some small Ramsey numbers.

definition 1. *Let G be a circulant graph with n vertices and i, j be vertices in G. The line distance of line $\{i,j\}$, denoted by d_{ij}, is defined as*

$$d_{ij} = min\{|i-j|, n-|i-j|\}$$

and a line distance set is a set of the line distances.

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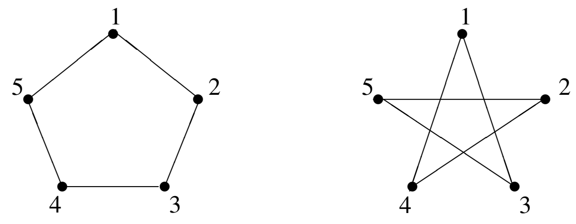
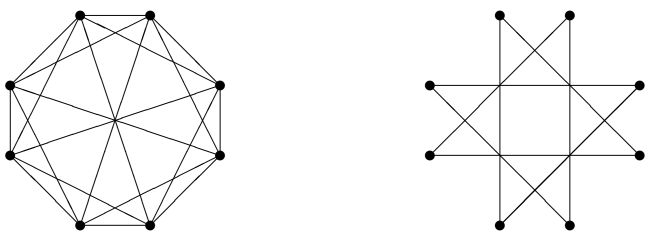


G $\overline{G}$

Figure 1

Figure 1, line distance of G is 1 and line distance of $\overline{G}$ is 2. In figure 2, line distances of G are $1, 2$ and 4. This is, line distance set of G is $\{1,2,4\}$ and line distance set of $\overline{G}$ is $\{3\}$.



G $\overline{G}$

Figure 2

In section II, we construct line distance sets in order to find lower bounds of some Ramsey numbers.

II. The main results

In this section, we find lower bounds of $R(3,10)$, $R(3,11)$, and $R(3,12)$ by constructing line distance sets of G and $\overline{G}$.

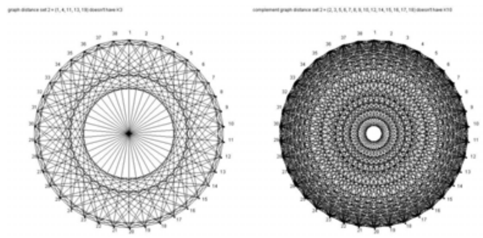
Since G and $\overline{G}$ have symmetric patterns, in verifying that G does not contain K_s and $\overline{G}$ does not contain K_t we can have one vertex fixed and only need to consider other $s-1$ vertices for the case of K_s and other $t-1$ vertices for the case of K_t.

Theorem 1. $R(3,10) \geq 39$.

Proof: The graph G of order 38 in Figure 3a has line distance set as $\{1,4,11,13,19\}$,and the graph $\overline{G}$ in Figure 3b has line distance set as $\{2,3,5,6,7,8,9,10,12,14,15,16,17,18\}$.

It can be verified that G contains no K_3 and $\overline{G}$ contains no K_{10}. According to the definition of Ramsey numbers, we have that $R(3,10) \geq 39$. ■

Next, we have a lower bound of $R(3,11)$.



(a) (b)

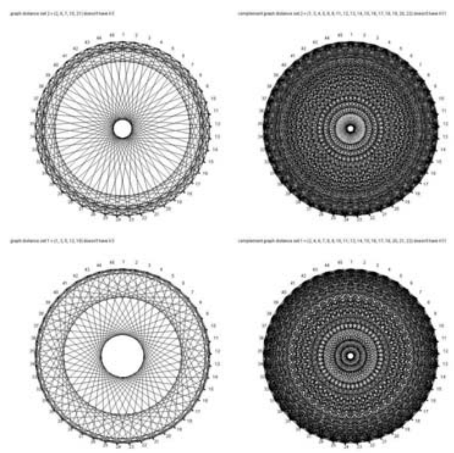
Figure 3. lower bound of Ramsey number $R(3,10) > 38$.

Theorem 2. $R(3,11) \geq 46$.

Proof: We have 6 line distance sets of G and $\overline{G}$ of order 45, see Figure 4 and Figure 5.

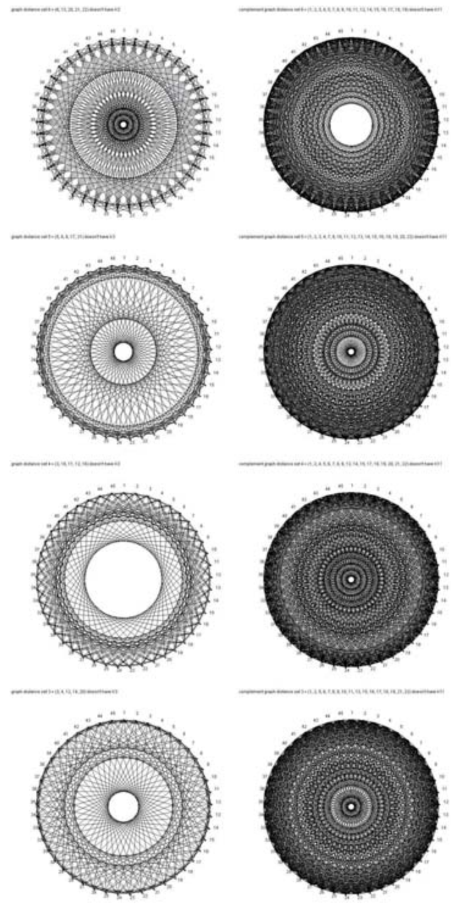
$$\{1,3,5,12,19\},$$
$$\{2,4,6,7,8,9,10,11,13,14,15,16,17,18,20,21,22\};$$
$$\{2,6,7,10,21\},$$
$$\{1,3,4,5,8,9,11,12,13,14,15,16,17,18]19,20,22\};$$
$$\{3,4,12,14,20\},$$
$$\{1,2,5,6,7,8,9,10,11,13,15,16,17,18,19,21,22\};$$
$$\{3,10,11,12,16\},$$
$$\{1,2,4,5,6,7,8,9,13,14,15,17,18,19,20,21,22\};$$
$$\{5,6,8,17,21\},$$
$$\{1,2,3,4,7,9,10,11,12,13,14,15,16,18,19,20,22\};$$
$$\{6,13,20,21,22\},$$
$$\{1,2,3,4,5,7,8,9,10,11,12,14,15,16,17,18,19\}.$$

It can be verified from each G and $\overline{G}$ that G does not contain K_3 and $\overline{G}$ does not contain K_{11}.
Hence $R(3,11) \geq 46$. ■



(a) (b)

Figure 4. lower bound of Ramsey number $R(3,11) > 45$.



(a) (b)

Figure 5. lower bound of Ramsey number $R(3,11) > 45$.

Next, we have a lower bound for $R(3,12)$.

Theorem 3. $R(3,12) \geq 49$

Proof: We have 12 line distance sets of G and $\overline{G}$ of order 48.

$$\{1,3,8,14,18,24\},$$
$$\{2,4,5,6,7,9,10,11,12,13,15,16,17,19,20,21,22,23\};$$
$$\{2,3,8,14,15,24\},$$
$$\{1,4,5,6,7,9,10,11,12,13,16,17,18,19,20,21,22,23\};$$
$$\{2,3,8,17,18,24\},$$
$$\{1,4,5,6,7,9,10,11,12,13,14,15,16,19,20,21,22,23\};$$
$$\{2,7,8,18,21,24\},$$
$$\{1,3,4,5,6,9,10,11,12,13,14,15,16,17,19,20,22,23\};$$
$$\{2,8,9,14,21,24\},$$
$$\{1,3,4,5,6,7,10,11,12,13,15,16,17,18,19,20,22,23\};$$
$$\{3,8,9,10,22,24\},$$
$$\{1,2,4,5,6,7,11,12,13,14,15,16,17,18,19,20,21,23\};$$
$$\{5,6,8,15,22,24\},$$
$$\{1,2,3,4,7,9,10,11,12,13,14,16,17,18,19,20,21,23\};$$
$$\{6,8,9,10,13,24\},$$
$$\{1,2,3,4,5,7,11,12,14,15,16,17,18,19,20,21,22,23\};$$
$$\{6,8,9,19,22,24\},$$

$$\{1,2,3,4,5,7,10,11,12,13,14,15,16,17,18,20,21,23\};$$
$$\{6,8,10,11,15,24\},$$
$$\{1,2,3,4,5,7,9,12,13,14,16,17,18,19,20,21,22,23\};$$
$$\{8,10,15,21,22,24\},$$
$$\{1,2,3,4,5,6,7,9,11,12,13,14,16,17,18,19,20,23\};$$
$$\{8,14,18,21,23,24\},$$
$$\{1,2,3,4,5,6,7,9,10,11,12,13,15,16,17,19,20,22\}.$$

It can be verified from each G and $\overline{G}$ that G does not contain K_3 and $\overline{G}$ does not contain K_{12}.
Hence $R(3,12) \geq 49$. ■

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