

# Longitudinal Vibration of a Micro-Beam in a Micro-Scale Fluid Media

M. Ghanbari, S. Hossainpour, G. Rezazadeh

**Abstract**—In this paper, longitudinal vibration of a micro-beam in micro-scale fluid media has been investigated. The proposed mathematical model for this study is made up of a micro-beam and a micro-plate at its free end. An AC voltage is applied to the pair of piezoelectric layers on the upper and lower surfaces of the micro-beam in order to actuate it longitudinally. The whole structure is bounded between two fixed plates on its upper and lower surfaces. The micro-gap between the structure and the fixed plates is filled with fluid. Fluids behave differently in micro-scale than macro, so the fluid field in the gap has been modeled based on micro-polar theory. The coupled governing equations of motion of the micro-beam and the micro-scale fluid field have been derived. Due to having non-homogenous boundary conditions, derived equations have been transformed to an enhanced form with homogenous boundary conditions. Using Galerkin-based reduced order model, the enhanced equations have been discretized over the beam and fluid domains and solve simultaneously in order to obtain force response of the micro-beam. Effects of micro-polar parameters of the fluid as characteristic length scale, coupling parameter and surface parameter on the response of the micro-beam have been studied.

**Keywords**—Micro-polar theory, Galerkin method, MEMS, micro-fluid.

## I. INTRODUCTION

THE solid-fluid interaction vibration is very important in the view point of structure performance and disaster prevention, for example in industries such as elevated water tanks and liquefied natural gas. Also, it is very important in the field of MEMS designing and manufacturing. Many works have been done on the behavior of liquid sloshing in circular cylindrical containers that are excited horizontally, considering the damping effect that liquid tanks have on the vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of liquid tank on the horizontal vibration of the cylinder [4], then experimental [5], [6] and nonlinear analyses have been reported [7]. Many studies have been done on the effect of fluid as an added mass on the vibration of the structure such as cylinders and spheres moving in fluid with acceleration. The added mass is a characteristic of fluid loading [8]; that concept of it for a supported plate is known from several theories as the slender wing theory [9], the traveling wave's solutions

[10], two-dimensional [11], or three dimensional [12], linear aerodynamic theory. Researches on the added mass of vibrating cylinders in water showed that the added mass of first mode fluid is equal to the fluid mass displaced by the cylinder [13], but it is not true for higher modes. Studying a cantilever plate in an axial flow also showed that the fluid loading has the effect of added mass and added damping and stiffness on the plate [14]. A formula was suggested by Sinha in 2003 for added mass of vibrating of perforated plate type structures submerged in fluids that designers could evaluate the structural dynamic of these structures without conducting a model test [15].

Recently, many researches have been done on micro-electro-mechanical systems (MEMS) in modern technologies and the focalization of researchers on this field is increasing rapidly. MEMS led to the reduced costs and energy consumptions and improved sensors performance. These micro-scale devices have found utility in several fields such as accelerometers [16], pressure sensors [17], micro-pumps [18], viscometers [19], micro-resonators [20]-[22] and so forth. Most of these micro-devices deal with a solid structure usually a micro-beam in interacting with a micro-scale fluid media. This causes the advance study on the behavior of micro-structures when immersed in a micro-fluid media. In one of the researches, dynamic characteristic and forced response of an electrostatically-actuated micro-beam subjected to fluid loading was investigated by Rezazadeh et al. [23]. They showed that fluid loading decreases the natural frequency of the micro-beam and because of higher electric coefficient and increment of electrical stiffness and decrement of total stiffness, maximum amplitude of the micro-beam decreases. Transversally vibrating micro-beams in micro-resonators have also been studied extensively. In micro-resonators, the air in the gap squeezes, causing the squeeze film damping phenomenon, which is the fundamental energy dissipation mechanism. Pandey and Pratap obtained damping characteristics of the resonator for the first three modes of vibration. In that work, static deflection due to DC load was neglected [24]. Younis and Nayfeh used perturbation method to obtain bias deflection of the micro-plate under different ambient pressures [25]. Chatterjee and Pohit obtained squeeze film characteristics of cantilever micro-resonators for higher modes of vibration under different ambient pressure conditions [26], [27].

In the researches mentioned above, the fluid in the micro-gap is modelled based on classical theories. Numerous experimental results indicate that, fluid flow moves differently in micro-scale than macro-scale. Hence, the Navier-Stokes

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equations become incapable of explaining the micro scale fluid transport phenomena in the study of micro and nano-scale fluid mechanics. The theory of micro-polar fluids was introduced by Eringen [28], [29], which contains the effect of and couple stresses and local rotary inertia, and provides a mathematical model for behavior of certain liquids such as blood, polymer and so forth. Today, researches show that in a micro-scale fluid field, micro-polar fluid theory is a useful tool for modeling the flow in micro-devices [30]-[32].

## II. GOVERNING EQUATIONS

The constitutive equations of the fluid field based on micro-polar theory considering the non-symmetric stress tensor  $\sigma^f = \{\sigma_{ij}^f\}$  and the couple stress tensor  $Co^f = \{Co_{ij}^f\}$ , are as following [31]:

$$\sigma_{ij}^f = (-p + \lambda^f V_{k,k})\delta_{ij} + \mu^f (V_{i,j} + V_{j,i}) + k^f (V_{j,i} - \varepsilon_{ijk} G_k^f) \quad (1)$$

$$Co_{ij}^f = \alpha^f G_{k,k}^f \delta_{ij} + \beta^f G_{i,j}^f + \gamma^f G_{j,i}^f \quad (2)$$

where the symbols denote;  $p$ : hydrodynamic pressure,  $\lambda^f$ : bulk viscosity coefficient,  $\mu^f$ : classical shear viscosity coefficient,  $k^f$ : vortex viscosity coefficient,  $\alpha^f, \beta^f, \gamma^f$ : spin gradient viscosity coefficients,  $\varepsilon_{ijk}$ : Levi-Civita symbol,  $\delta_{ij}$ : the Kronecker delta,  $V_i^f$ : velocity components and  $G_i^f$ : micro-rotation components.

Here, the coefficients satisfy following inequalities [31]:

$$k^f > 0, 3\lambda^f + k^f + 2\mu^f > 0, 2\mu^f + k^f > 0 \quad (3)$$

$$3\alpha^f + 2\gamma^f > 0, -\gamma^f < \beta^f < \gamma^f, \gamma^f > 0 \quad (4)$$

The balance equations of the fluid field based on micro-polar theory can be expressed as:

$$\sigma_{ij,j}^f + \rho^f f_i^f = \rho^f \dot{V}_i \quad (5)$$

$$Co_{ij,j}^f + \varepsilon_{ijk} \sigma_{kj}^f + \rho^f c_i^f = \rho^f J^f \dot{G}_i^f \quad (6)$$

Equation (5) presents the balance of linear momentum, while (6) presents the balance of angular momentum.  $f_i^f$  and  $c_i^f$  are the body forces and body couples per unit volume,  $\rho^f$  is the density of the fluid,  $V_i$  and  $G_i^f$  are the velocity and micro-rotation components and  $J^f$  denotes micro inertia density of the fluid, which is absent in the classical mechanics. By substituting the constitutive (1), (2) into the balance (5), (6), the conservation equations for the micro-polar fluid with constant physical properties can be obtained [31], [32]:

$$\frac{\partial \rho^f}{\partial t} + (\rho^f V)_{i,i} = 0 \quad (7)$$

$$(\lambda^f + \mu^f) V_{j,ji} + (\mu^f + k^f) V_{i,jj} + k^f \varepsilon_{ijk} G_{k,j}^f - p_{,i} + \rho^f f_i^f = \rho^f \dot{V}_i \quad (8)$$

$$(\alpha^f + \beta^f) G_{j,ji}^f + \gamma^f G_{i,jj}^f + k^f \varepsilon_{ijk} V_{k,j} - 2k^f G_i^f + \rho^f c_i^f = \rho^f J^f \dot{G}_i^f \quad (9)$$

with new non-dimensional parameters  $CP, L_f$  as [31]:

$$CP = \sqrt{\frac{k^f}{2\mu^f + k^f}}, L_f = \frac{L_c}{l}, l = \sqrt{\frac{\gamma^f}{4\mu^f + 2k^f}} \quad (10)$$

where  $L_c$  is a reference quantity of dimension,  $L_f$  characterizes the relationship between the geometric dimension of the flow  $L_c$  and the physical properties of the fluid. In the limiting case, for  $L_f \rightarrow \infty$ , the micro-rotation vector is equal to one half of the vector of the fluid stream vorticity: Parameter  $CP; 0 \leq CP \leq 1$ ; characterizes coupling between the vortex viscosity coefficient  $k^f$  and the shear viscosity coefficient  $\mu^f$ . In other words, it represents the dependency of micro-rotations to macro-rotation (classical rotation) in the fluid field. If the value of the coupling parameter approximates zero, then the equations of the linear and angular momentums become independent of each other and the linear momentum transforms into the classical Navier-Stocks equations for Newtonian fluids.

The constitutive equations for an isotropic elastic beam based on micro-polar elasticity theory considering the stress tensor  $\sigma^b = \{\sigma_{ij}^b\}$  and the couple stress tensor  $Co^b = \{Co_{ij}^b\}$  are as follows [33]:

$$\sigma_{ij}^b = (\lambda^b e_{kk}^b) \delta_{ij} + (2\mu^b + k^b) e_{ij}^b + k^b \varepsilon_{ijk} (r_k^b - G_k^b) \quad (11)$$

$$Co_{ij}^b = \alpha^b G_{k,k}^b \delta_{ij} + \beta^b G_{i,j}^b + \gamma^b G_{j,i}^b \quad (12)$$

And these equations for a piezoelectric material are as follows [33]-[35]:

$$\sigma_{ij}^{pi} = (\lambda^{pi} e_{kk}^{pi}) \delta_{ij} + (2\mu^{pi} + k^{pi}) e_{ij}^{pi} + k^{pi} \varepsilon_{ijk} (r_k^{pi} - G_k^{pi}) - \tilde{e}_{kij}^{pi} E_k^{pi} \quad (13)$$

$$Co_{ij}^{pi} = \alpha^{pi} G_{k,k}^{pi} \delta_{ij} + \beta^{pi} G_{i,j}^{pi} + \gamma^{pi} G_{j,i}^{pi} \quad (14)$$

where  $\lambda^b, \mu^b, \lambda^{pi}, \mu^{pi}$  are classical Lamé type constants of the micro-beam and piezoelectric material, respectively.  $\alpha^b, \beta^b, \gamma^b, k^b$  and  $\alpha^{pi}, \beta^{pi}, \gamma^{pi}, k^{pi}$  are new micro-polar

constants due to couple stresses.  $G_i^b, G_i^{pi}$  are micro-rotation components of the micro-beam and piezoelectric material.  $\tilde{e}_{kij}^{pi}$  are piezoelectric constants.  $E_k^{pi}$  is an electric field due to the voltage applied to the piezoelectric material.  $r_i^b, r_i^{pi}$  are macro-rotation components of the micro-beam and piezoelectric material while  $e_{ij}^b, e_{ij}^{pi}$  are the macro-strain tensors that have the following relations with the micro-beam displacement  $U^b$  and the piezoelectric material displacement  $U^{pi}$ :

$$r_i^b = \frac{1}{2} \varepsilon_{ijk} U_{k,j}^b, r_i^{pi} = \frac{1}{2} \varepsilon_{ijk} U_{k,j}^{pi} \quad (15)$$

$$e_{ij}^b = \frac{1}{2} (U_{i,j}^b + U_{j,i}^b), e_{ij}^{pi} = \frac{1}{2} (U_{i,j}^{pi} + U_{j,i}^{pi}) \quad (16)$$

where  $\varepsilon_{ijk}$  is permutation symbol. So, the conservation equations for the elastic beam and piezoelectric material based on micro-polar theory can be obtained as [33]-[36]:

$$(\lambda^b + \mu^b) U_{j,ji}^b + (\mu^b + k^b) U_{i,ji}^b + k^b \varepsilon_{ijk} G_{k,j}^b + \rho^b f_i^b = \rho^b \ddot{U}_i^b \quad (17)$$

$$(\alpha^b + \beta^b) G_{j,ji}^b + \gamma^b G_{i,ji}^b + k^b \varepsilon_{ijk} U_{j,k}^b - 2k^b G_i^b + \rho^b c_i^b = \rho^b J^b \ddot{G}_i^b \quad (18)$$

$$(\lambda^{pi} + \mu^{pi}) U_{j,ji}^{pi} + (\mu^{pi} + k^{pi}) U_{i,ji}^{pi} + k^{pi} \varepsilon_{ijk} G_{k,j}^{pi} + \rho^{pi} f_i^{pi} - \tilde{e}_{kij}^{pi} E_{k,j}^{pi} = \rho^{pi} \ddot{U}_i^{pi} \quad (19)$$

$$(\alpha^{pi} + \beta^{pi}) G_{j,ji}^{pi} + \gamma^{pi} G_{i,ji}^{pi} + k^{pi} \varepsilon_{ijk} U_{j,k}^{pi} - 2k^{pi} G_i^{pi} + \rho^{pi} c_i^{pi} = \rho^{pi} J^{pi} \ddot{G}_i^{pi} \quad (20)$$

With the following technical elastic properties [33]:

$$E^b = \frac{(2\mu^b + k^b)(3\lambda^b + 2\mu^b + k^b)}{(2\lambda^b + 2\mu^b + k^b)}, G^b = \frac{2\mu^b + k^b}{2}, \quad (21)$$

$$\nu^b = \frac{\lambda^b}{2\lambda^b + 2\mu^b + k^b}$$

$$l_t^b = \sqrt{\frac{\beta^b + \gamma^b}{2\mu^b + k^b}}, l_b^b = \sqrt{\frac{\gamma^b}{2(2\mu^b + k^b)}}, (CP)^b = \sqrt{\frac{k^b}{2\mu^b + k^b}} \quad (22)$$

where  $f_i^b, f_i^{pi}, c_i^b, c_i^{pi}$  are the body forces, body couples per unit volume of the micro-beam and piezoelectric material, respectively.  $\rho^b$  and  $\rho^{pi}$  are the micro-beam and piezoelectric material densities.  $U_i^b$  and  $U_i^{pi}$  are micro displacement components.  $J^b$  and  $J^{pi}$  denote micro inertia densities and

$E^b, G^b, \nu^b, l_t^b, l_b^b, (CP)^b$  are Young modulus, Shear modulus, Poisson's ratio, the characteristic length in torsion, the characteristic length in bending and coupling number of the micro-beam, respectively.

#### A. Case Study

For investigating the effect of a micro-scale fluid on the longitudinal vibration of a micro-beam, the mathematical model shown in Fig. 1 is proposed. It is made up of a micro-beam sandwiched between two piezoelectric layers and a micro-plate at its free end. The whole structure is assumed to be made of polycrystalline silicon. Two external forces act on the structure. The first one is a shear force that acts on the micro-plate due to the physical properties of the surrounding fluid and the second one is a longitudinal force created by an AC voltage applied to the piezoelectric layers. The surrounding fluid is bounded between two parallel-oriented surfaces in order to control the magnitude of the shear force. It is assumed that the both upper and lower surfaces of the micro-plate are smooth.

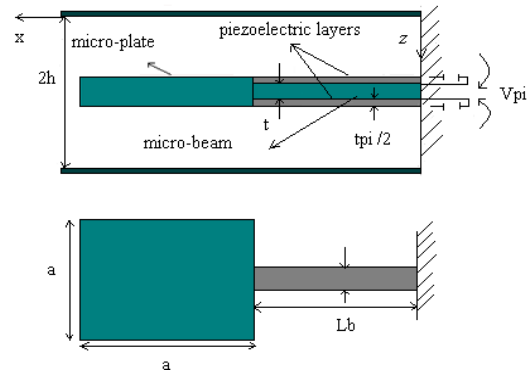


Fig. 1 Schematic of the proposed model with the piezoelectric layers immersed in micro-scale fluid bounded by two fixed plates

The following assumptions are considered in order to simplify (7)-(9) and (17)-(20):

1. The fluid is assumed to be incompressible.
2. There are no body forces and body couples acting along x direction.
3. The beam is considered an isotropic elastic beam with constant properties.
4. There are no couple stresses in longitudinal vibration of the beam
5. There is no Pressure gradient in x direction.

Consequently (7)-(9) and (17)-(20) are simplified to:

$$(\mu^f + k^f) V_{i,jj}^f + k^f \varepsilon_{ijk} G_{k,j}^f = \rho^f \dot{V}_i^f \quad (23)$$

$$\gamma^f G_{i,ji}^f + k^f \varepsilon_{ijk} V_{j,k}^f - 2k^f G_i^f = \rho^f J^f \dot{G}_i^f \quad (24)$$

$$(2\mu^b) U_{i,ji}^b = \rho^b \ddot{U}_i^b \quad (25)$$

$$(2\mu^{pi})U_{i,jj}^{pi} - \tilde{e}_{kj}^{pi}E_{k,j}^{pi} = \rho^{pi}\ddot{U}_i^{pi} \quad (26)$$

By supposing  $(E^{pi}A^{pi} + E^bA^b) \ll E_sA_s$  in which  $E^b, A^b$ ,  $E_s, A_s$  are the Young modulus of beam, cross section area of beam, Young modulus of micro-plate, and cross section area of the micro-plate, respectively, and the fluid field can be considered one-dimensional. Considering the Cartesian coordinate system shown in Fig. 1, and neglecting the effect of electric field on piezoelectric layers, equations of motion of the fluid field and also equation of motion governing longitudinal displacement of the micro-beam can be simplified to:

$$\rho^f \frac{\partial v}{\partial t} = (\mu^f + k^f) \frac{\partial^2 v}{\partial z^2} + k^f \frac{\partial g}{\partial z} \quad (27)$$

$$\rho^f J^f \frac{\partial g}{\partial t} = -2k^f g - k^f \frac{\partial v}{\partial z} + \gamma^f \frac{\partial^2 g}{\partial z^2} \quad (28)$$

$$(EA)_{eq} \frac{\partial^2 u}{\partial x^2} - \tilde{\tau} A^* \delta(x - L_b) = (\rho A)_{eq} \ddot{u} + \tilde{m}_0 \delta(x - L_b) \frac{\partial^2 u}{\partial t^2} \quad (29)$$

$$\int_0^{L_b} \tilde{\tau} A^* \delta(x - L_b) dx = \tau A^*; \int_0^{L_b} \tilde{m}_0 \delta(x - L_b) dx = m_0 \quad (30)$$

$$\rho^{pi} A^{pi} + \rho^b A^b = (\rho A)_{eq}; E^{pi} A^{pi} + E^b A^b = (EA)_{eq}$$

where  $A^{pi} = t^{pi}b$ ,  $A^b = tb$ .  $v$  and  $u$  are the velocity component of the fluid and displacement component of the micro-beam in  $x$  direction,  $g$  is the micro-rotation component of the fluid in  $y$  direction,  $\tau$  is the wall shear stress of the fluid acting on the lower and upper surfaces of the micro plate,  $m_0$  is the mass of the micro-plate,  $A^*$  is the area of the upper and lower surfaces of the micro-plate, and  $L_b$  is the length of the micro-beam, respectively. It should be noted that, for simplicity, the inertial force owing to the mass of the micro-plate and the shear force due to physical properties of the fluid are assumed to be singular distributed loads with zero load intensity through the whole length of the beam and infinite intensity at its end.

Accompany boundary conditions of (27) and (28) at  $z=0$  and  $z=h$  are:

$$v(0,t) = 0, v(h,t) = \frac{\partial u}{\partial t} \Big|_{(L_b,t)} = \alpha(t) \quad (31)$$

$$g(0,t) = -s \left( \frac{\partial v}{\partial z} \right)_{(z=0)} = A(t), g(h,t) = -s \left( \frac{\partial v}{\partial z} \right)_{(z=h)} = B(t) \quad (32)$$

where  $s$  is a constant such that  $0 \leq s \leq 1$  and is called surface parameter. In the case when  $s=0$  (strong concentration), the

microstructure does not rotate with respect to the surface. The case when  $s=0.5$  (weak concentration), the micro-structures spin is equal to fluid vorticity [37].  $\alpha(t)$ ,  $A(t)$  and  $B(t)$  are the time varying functions.

As the boundary conditions are time varying,  $v(z,t)$  and  $g(z,t)$  are introduced as follows:

$$v(z,t) = w(z,t) + \alpha(t) \frac{z}{h} \quad (33)$$

$$g(z,t) = \Re(z,t) + \left( \frac{B(t) - A(t)}{h} \right) z + A(t) \quad (34)$$

Using (33) and (34), the governing equations of motion of the fluid field can be rewritten as:

$$\Im(w(z,t)) = \frac{\partial w}{\partial t} - \left( \frac{\mu^f + k^f}{\rho^f} \right) \frac{\partial^2 w}{\partial z^2} \quad (35)$$

$$-\left( \frac{k^f}{\rho^f} \right) \frac{\partial \Re}{\partial z} + \dot{\alpha}(t) \frac{z}{h} + \left( \frac{k^f}{h\rho^f} \right) (A(t) - B(t))$$

$$\begin{aligned} \zeta(\Re(z,t)) &= \frac{\partial \Re}{\partial t} + \frac{2k^f}{\rho^f J^f} \left( \Re(z,t) + \left( \frac{B(t) - A(t)}{h} \right) z + A(t) \right) \\ &+ \left( \frac{k^f}{\rho^f J^f} \right) \left( \frac{\partial w}{\partial z} + \frac{\alpha(t)}{h} \right) - \left( \frac{\gamma^f}{\rho^f J^f} \right) \frac{\partial^2 \Re}{\partial z^2} + \left( \frac{\dot{B}(t) - \dot{A}(t)}{h} \right) z + \dot{A}(t) \end{aligned} \quad (36)$$

Therefore the accompanying boundary conditions of (31) and (32) take the following homogenous form:

$$w(0,t) = w(h,t) = 0 \quad (37)$$

$$\Re(0,t) = \Re(h,t) = 0 \quad (38)$$

Boundary conditions of (29) are:

$$u(0,t) = 0 \quad (39)$$

$$\begin{aligned} N(l,t) &= \left( E^p A^p + E^b A^b \right) \frac{\partial u}{\partial x} - A^p \tilde{e}_{31} E_z \\ &= (EA)_{eq} \frac{\partial u}{\partial x} - \tilde{f}(t) = 0 \Rightarrow \frac{\partial u}{\partial x} \Big|_{(l,t)} = \frac{\tilde{f}(t)}{(EA)_{eq}} = f(t) \end{aligned} \quad (40)$$

where  $E_z = \frac{2V_{pi}}{t_{pi}}$  is the uniform electric field in  $z$  direction

due to the voltage ( $V_{pi}$ ) applied to the piezoelectric layers, and  $\tilde{e}_{31}$  is the equivalent piezoelectric coefficient for the one-dimensional problem. Due to the time variability of the boundary conditions, by introducing  $u(x,t)$  as:

$$u(x, t) = \psi(x, t) + x f'(t) \quad (41)$$

Equation (29) can be rewritten as:

$$\begin{aligned} \tilde{\lambda}(\psi(x, t)) = & ((\rho A)_{eq} + \tilde{m}_0 \delta(x - L_b)) \left( \frac{\partial^2 \psi}{\partial t^2} + \ddot{f}(t)x \right) \\ & - (EA)_{eq} \frac{\partial^2 \psi}{\partial x^2} + \tilde{\tau} \tilde{A}^* \delta(x - L_b) \end{aligned} \quad (42)$$

With the corresponding homogenous boundary conditions:

$$\psi(0, t) = 0, \quad \frac{\partial \psi}{\partial x} \bigg|_{(L_b, t)} = 0 \quad (43)$$

### III. NUMERICAL SOLUTIONS

Many modern mathematical models pose challenges when used in numerical simulations, due to complexity and large dimension. Model order reduction reduces the computational complexity of these problems. In this work, we have applied Galerkin based reduce order model to solve the coupled equations of solid and fluid media. Galerkin methods are a class of methods for converting a continuous operator such as a differential equation to a discrete problem. In this method, we express an unknown function as a linear combination of a set shape functions. A Galerkin approximation precision depends on number and type of the shape functions. The accuracy of Galerkin method depends on number and type of the shape functions.

In this work approximate solutions for  $w(z, t)$ ,  $\Re(z, t)$  and  $\psi(x, t)$  are searched in the following form:

$$w(z, t) \cong w_M(z, t) = \sum_{j=1}^M a_j(t) \Omega_j(z) \quad (44)$$

$$\Re(z, t) \cong \Re_p(z, t) = \sum_{j=1}^p b_j(t) \varphi_j(z) \quad (45)$$

$$\psi(x, t) \cong \psi_N(x, t) = \sum_{j=1}^N q_j(t) \phi_j(x) \quad (46)$$

where,  $\Omega_j(z)$ ,  $\varphi_j(z)$  and  $\phi_j(x)$  are the shape functions which satisfy geometrical boundary conditions (37), (38), and (43), respectively. The shear stress of the fluid acting on the micro-plate is expressed in the following form:

$$\tau = ((\mu^f + k^f) \frac{\partial v}{\partial z} \bigg|_{(h, t)} + k^f g \bigg|_{(h, t)}) = (\mu^f + k^f) \left( \frac{\partial w}{\partial z} \bigg|_{(h, t)} + \frac{\alpha(t)}{h} \right) + k^f [\Re(h, t) + B(t)] \quad (47)$$

in which

$$\alpha(t) = \dot{f}(t)L_b + \sum_{j=1}^N \dot{q}_j(t) \phi_j(L_b) \quad (48)$$

$$A(t) = -s \left( \frac{\partial w}{\partial z} \bigg|_{(0, t)} + \frac{\alpha(t)}{h} \right), \quad B(t) = -s \left( \frac{\partial w}{\partial z} \bigg|_{(h, t)} + \frac{\alpha(t)}{h} \right) \quad (49)$$

Substituting the  $w_M(z, t)$ ,  $\Re_p(z, t)$ ,  $\alpha(t)$ ,  $A(t)$ , and  $B(t)$  into (47), it can be rewritten as:

$$\begin{aligned} \tau = & (\mu^f + k^f) \left( \sum_{j=1}^M a_j(t) \Omega_j'(h) + \frac{1}{h} \left( \dot{f}(t)L_b + \sum_{j=1}^N \dot{q}_j(t) \phi_j(L_b) \right) \right) \\ & + k^f \left( \sum_{j=1}^p b_j(t) \varphi_j(h) - s \left( \sum_{j=1}^M a_j(t) \Omega_j'(h) + \frac{1}{h} \left( \dot{f}(t)L_b + \sum_{j=1}^N \dot{q}_j(t) \phi_j(L_b) \right) \right) \right) \end{aligned} \quad (50)$$

Substituting (44)-(46) and (50) into (35), (36) and (42), and using the Galerkin method, the following reduced order models can be obtained:

$$\begin{aligned} \sum_{j=1}^N M_{ij}^{(1)} \ddot{q}_j + \sum_{j=1}^N C_{ij}^{(1)} \dot{q}_j + \sum_{j=1}^N K_{ij}^{(1)} q_j + \sum_{j=1}^M S_{ij}^{(1)} \dot{a}_j \\ + \sum_{j=1}^M D_{ij}^{(1)} a_j + \sum_{j=1}^p F_{ij}^{(1)} \dot{b}_j \sum_{j=1}^p E_{ij}^{(1)} b_j = P_i^{(1)}; i = 1, 2, \dots, M \end{aligned} \quad (51)$$

$$\begin{aligned} \sum_{j=1}^N M_{ij}^{(2)} \ddot{q}_j + \sum_{j=1}^N C_{ij}^{(2)} \dot{q}_j + \sum_{j=1}^N K_{ij}^{(2)} q_j + \sum_{j=1}^M S_{ij}^{(2)} \dot{a}_j \\ + \sum_{j=1}^M D_{ij}^{(2)} a_j + \sum_{j=1}^p F_{ij}^{(2)} \dot{b}_j \sum_{j=1}^p E_{ij}^{(2)} b_j = P_i^{(2)}; i = 1, \dots, P \end{aligned} \quad (52)$$

$$\begin{aligned} \sum_{j=1}^N M_{ij}^{(3)} \ddot{q}_j + \sum_{j=1}^N C_{ij}^{(3)} \dot{q}_j + \sum_{j=1}^N K_{ij}^{(3)} q_j + \sum_{j=1}^M S_{ij}^{(3)} \dot{a}_j \\ + \sum_{j=1}^M D_{ij}^{(3)} a_j + \sum_{j=1}^p F_{ij}^{(3)} \dot{b}_j \sum_{j=1}^p E_{ij}^{(3)} b_j = P_i^{(3)}; i = 1, 2, \dots, N \end{aligned} \quad (53)$$

where the corresponding coefficients are:

$$\begin{aligned} M_{ij}^{(1)} &= \frac{\phi_j(L_b)}{h} \int_0^h z \Omega_i(z) dz, \quad C_{ij}^{(1)} = 0, \quad K_{ij}^{(1)} = 0, \\ S_{ij}^{(1)} &= \int_0^h \Omega_j(z) \Omega_i(z) dz, \\ D_{ij}^{(1)} &= -(\frac{\mu^f + k^f}{\rho^f}) \int_0^h \Omega_i(z) \Omega_j''(z) dz + \frac{k^f s}{\rho^f h} (\Omega_j'(h) - \Omega_j'(0)) \int_0^h \Omega_j(z) dz, \end{aligned}$$

$$F_{ij}^{(1)} = 0, E_{ij}^{(1)} = -\frac{k^f}{\rho^f J^f} \int_0^h \varphi_j^I(z) \Omega_i(z) dz, \quad \Omega_i(z) = \sin\left(\frac{i\pi}{h} z\right) \quad (55)$$

$$p_i^{(1)} = -\frac{L_b \ddot{f}(t)}{h} \int_0^h z \Omega_i(z) dz, \quad M_{ij}^{(2)} = -\frac{s \phi_j(L_b)}{h} \int_0^h \varphi_i(z) dz, \quad \varphi_i(z) = \sin\left(\frac{i\pi}{h} z\right) \quad (56)$$

$$C_{ij}^{(2)} = \frac{k^f}{\rho^f J^f h} (1-2s) \phi_j(L_b) \int_0^h \varphi_i(z) dz, \quad K_{ij}^{(2)} = 0, \quad \varphi_i(x) = \sin\left(\frac{(2i-1)\pi}{2L_b} x\right) \quad (57)$$

$$S_{ij}^{(2)} = \left( -\frac{s}{h} (\Omega_j^I(h) - \Omega_j^I(0)) \right) \int_0^h z \varphi_i(z) dz - s \Omega_j^I(0) \int_0^h \varphi_i(z) dz,$$

$$D_{ij}^{(2)} = -\frac{2k^f s}{\rho^f J^f h} \left( (\Omega_j^I(h) - \Omega_j^I(0)) \int_0^h z \varphi_i(z) dz - \Omega_j^I(0) \int_0^h \varphi_i(z) dz \right)$$

$$+ \left( \frac{k^f}{\rho^f J^f} \right) \int_0^h \Omega_j^I(z) \varphi_i(z) dz$$

$$F_{ij}^{(2)} = \int_0^h \varphi_j(z) \varphi_i(z) dz,$$

$$E_{ij}^{(2)} = \frac{2k^f}{\rho^f J^f} \int_0^h \varphi_j(z) \varphi_i(z) dz - \frac{\gamma^f}{\rho^f J^f} \int_0^h \varphi_j^{II}(z) \varphi_i(z) dz,$$

$$p_i^{(2)} = \frac{k^f L_b}{\rho^f J^f h} (2s-1) \dot{f}(t) \int_0^h \varphi_i(z) dz + \frac{s L_b}{h} \ddot{f}(t) \int_0^h \varphi_i(z) dz$$

$$M_{ij}^{(3)} = m_0 \phi_i(L_b) \phi_j(L_b) + (\rho A)_{eq} \int_0^{L_b} \phi_i(x) \phi_j(x) dx,$$

$$C_{ij}^{(3)} = \left( \frac{A^* (\mu^f + k^f)}{h} - \frac{k^f A^* s}{h} \right) (\phi_i(L_b) \phi_j(L_b)),$$

$$K_{ij}^{(3)} = -(EA)_{eq} \int_0^{L_b} \phi_j^{II}(x) \phi_i(x) dx,$$

$$S_{ij}^{(3)} = 0, D_{ij}^{(3)} = \left( A^* (\mu^f + k^f) - k^f A^* s \right) \Omega_j^I(h) \phi_i(L_b),$$

$$F_{ij}^{(3)} = 0, E_{ij}^{(3)} = k^f A^* \varphi_j(h) \phi_i(L_b),$$

$$p_i^{(3)} = - \left( m_0 L_b \phi_i(L_b) + (\rho A)_{eq} \int_0^{L_b} x \phi_i(x) dx \right) \ddot{f}(t) - \left( \frac{A^* L_b}{h} (\mu^f + k^f - s k^f) \right) \phi_i(L_b) \dot{f}(t) \quad (54)$$

#### IV. NUMERICAL RESULTS AND DISCUSSIONS

As a case study, a Silicon beam with a pair of piezoelectric layers is considered. The geometric and material properties are listed in Table I.

The gap between the micro-plate and the parallel surfaces ( $h$ ) is considered to be  $30 \mu\text{m}$ . Shape functions which satisfy the boundary conditions (37), (38) and (43), respectively, are considered as following:

Frequency response of the micro-beam immersed in a micro-scale fluid is shown in Fig. 2. As illustrated, by increasing the number of the used shape functions ( $N$ ), the obtained results converge, and for  $N = 6$ , the obtained result is considered acceptable. As shown in Table II, for the case when the effect of the shear force is not considered, the first calculated natural frequency of the system for  $N = 6$  is the same as the first natural frequency of the cantilever beam having a concentrated mass at the free end with 1.1% error.

TABLE I  
GEOMETRICAL AND MATERIAL PROPERTIES OF THE MICRO-BERAM, PIEZOELECTRIC LAYERS AND MICRO-PLATE

properties	Micro-beam	Piezoelectric layer	Micro-plate
Width	4 $\mu\text{m}$	4 $\mu\text{m}$	400 $\mu\text{m}$
thickness	2 $\mu\text{m}$	1 $\mu\text{m}$	0.05 $\mu\text{m}$
length	100 $\mu\text{m}$	100 $\mu\text{m}$	400 $\mu\text{m}$
Young's modulus	169 Gpa	78.6 Gpa	169 Gpa
density	2331 kg/m <sup>3</sup>	7600 kg/m <sup>3</sup>	2331 kg/m <sup>3</sup>
$V_p$		40 volt	
$e_{31}$		-9.29	

TABLE II  
VALUES OF FIRST NATURAL FREQUENCY OF THE SYSTEM (MHZ)

$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$	Theoretical result [38]
4.96	4.752	4.683	4.660	4.629	4.615	4.564

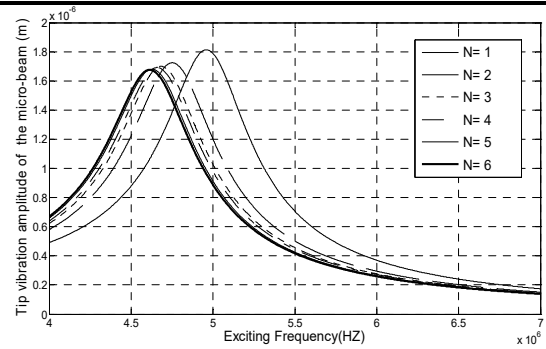


Fig. 2 Tip vibration amplitude of the micro-beam versus exciting frequency for different number of shape functions

Tip vibration amplitude of the micro-beam versus exciting frequency for different values of fluid coupling parameter is shown in Fig. 3. Also, tip vibration amplitude of the micro-

beam for different values of the voltage applied to the piezoelectric layers is shown in Fig. 4. As shown in Fig. 3, lower values of resonance frequency and amplitude are observed in fluids with higher values of coupling parameter. Analyzing the results, we can say that in fluids with higher value of coupling parameter, due to having higher value of vortex viscosity coefficient, micro-rotations are more dependent to macro-rotation. It led to higher damping and inertial effects on the longitudinal vibration of the micro-beam. In the case of classical fluid ( $CP=0$ ), the results are in good consistence with the results obtained in [12]. As shown in Fig. 4, the resonance amplitude of the micro-beam is also a function of voltage applied to the piezoelectric layers. It means that applying higher voltages to the piezoelectric layers the resonance amplitude of the micro-beam increases. It should be noted that the exciting frequency of the piezoelectric layers should be in the range of the first natural frequency of the micro-beam.

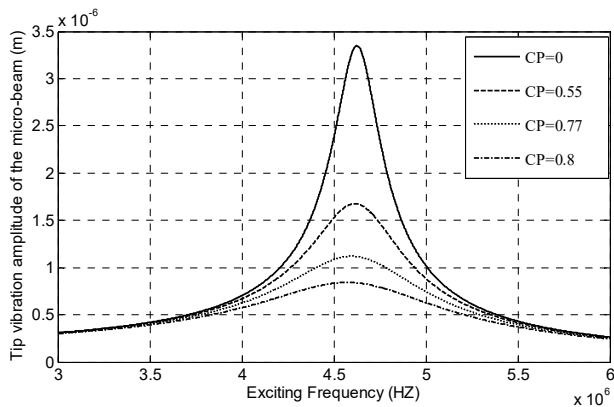


Fig. 3 Tip vibration amplitude of the micro-beam versus exciting frequency for different values of coupling parameter

Effects of the characteristic length scale and surface parameter of the micro-scale fluid, on dynamic response of the micro-beam are investigated and shown in Figs. 5 and 6. The results of Fig. 5 show that in fluids with lower value of length scale parameter, dependency of micro-rotations to macro-rotation is more considerable than in fluids with higher values of length scale. Consequently, fluids with lower values of length scale have more damping and inertial effects on vibration of the micro-beam than fluids with higher values of length scale. Fig. 6 shows that in the case of strong concentration damping effect of fluid is more considerable than in the case of weak concentration. As we know, slip boundary conditions decreases damping effect of fluid, consequently in the case of no-spin boundary condition, the fluid has more damping effect than in the case of spin boundary conditions.

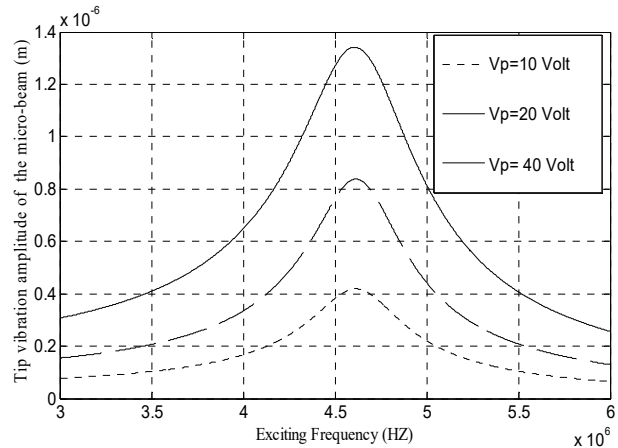


Fig. 4 Tip vibration amplitude of the micro-beam in the micro-polar fluid versus exciting frequency for different values of voltage applied to the piezoelectric layers

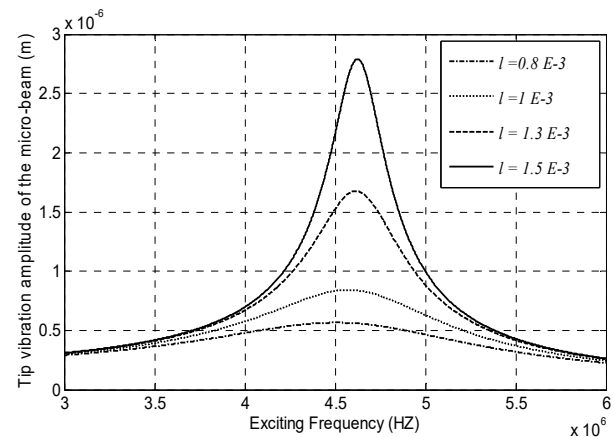


Fig. 5 Tip vibration amplitude of the micro-beam versus exciting frequency for different values of fluid characteristic length scale ( $l$ )

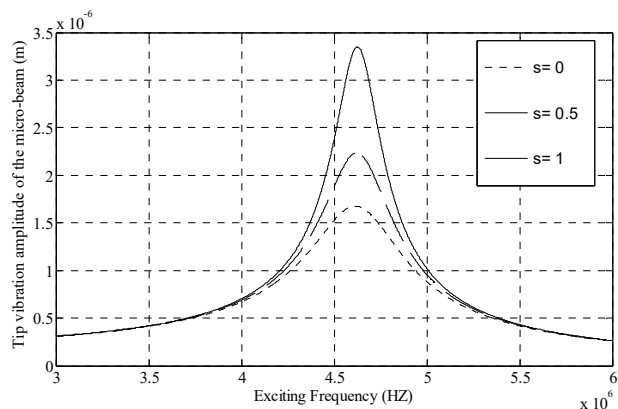


Fig. 6 Tip vibration amplitude of the micro-beam versus exciting frequency for different values of surface

## V. CONCLUSION

In this paper, the longitudinal vibration of a micro-beam in interacting with a micro-scale fluid media was investigated. A

mathematical model was proposed and the coupled governing equations of motion of fluid field and longitudinal vibration of the micro-beam were derived. The obtained equations were discretized and solved using a Galerkin-based reduced order model. It was shown that physical properties of the micro-scale fluid have dissipative and inertial effects on the dynamic response of the micro-beam. The effect of coupling parameter and length scale of the micro-scale fluid were studied and the results showed that in fluids with higher values of coupling parameter and lower values of length scale due to more dependence of micro-rotations to macro-rotations, inertial and damping effects of fluid are more considerable. The effect of surface parameter on dynamic response of the micro-beam was studied. We showed that lower values of resonance frequency and resonance amplitude and consequently more damping and inertial effects are observed in the case of weak concentration. Also, applying different voltages to piezoelectric layers showed that higher voltages result in higher amplitudes of vibration. It should be noted that, the presented mathematical model can be introduced as a micro-sensor for measurement of physical properties of micro-scale fluids.

## REFERENCES

- [1] H. N. Abramson, W. H. Chu, D.D. Kana "Some studies of nonlinear lateral sloshing in rigid containers" *ASME J. App. Mech.*, vol. 33, Dec.1984, pp.777-784.
- [2] J. W. Miles, "Resonantly forced surface waves in a circular cylinder" *J. Fluid Mech.*, vol. 149, Dec.1984, pp.15-31.
- [3] F. G. Dodge, D. D. Kana, H. N. Abramson, "Liquid surface oscillations in a longitudinal excited rigid cylindrical container" *J. AIAA*, vol.3, Apr. 1965, pp. 685-695.
- [4] K. Senda, N. Nakagawa, "On the vibration of an elevated water tank" Tech. Rep. Osaka University, No.193, 1954, pp. 247-264.
- [5] H. Hagiuda, "Oscillating control system exploiting fluid force generated by water sloshing," *Mitsui Zosen Tech. Rev.*, Issue.137, Jun.1989, pp. 13-20.
- [6] Y. Fujino, B. Pachecco, C. Piyawat, K. Fujii, "An experimental study on tuned liquid damper using circular containers" *J. JSCE*, vol. 34, Oct.1988, pp. 603-616.
- [7] T. Ikeda, N. Nakagawa, "Non-linear vibrations of a structure caused by water sloshing in a rectangular tank" *J. Sound Vib.*, vol. 201, Mar. 1977, pp. 23-41.
- [8] H. Minami, "Added mass of a membrane vibrating at finite amplitude" *J. Fluids. Struct.*, vol.12, Oct.1998, pp. 919-932.
- [9] R. T. Jones, "Properties of low-aspect ratio pointed wings at speeds below and above the speed of sound" *NASA Tech Rep.*, 1954, No. 834.
- [10] A. J. Pretlove, "Note on the virtual mass for a panel in an infinite baffle" *J. Acoustic. Soc.*, vol. 38, Aug. Jul.2005, pp. 266-270.
- [11] A. Kornecki, E. H. Dowell, J. Obrien, "On the aeroelastic instability of two-dimensional panels in uniform incompressible flow" *J. Sound. Vib.*, vol. 43, Jul.1976 pp.163-178.
- [12] A. D. Lucey, P. W. Carpenter, P. W., "The hydroelastic stability of three-dimensional disturbances of a finite compliant wall," *J. Sound. Vib.*, vol. 165, Aug.1993, pp.527-552.
- [13] M. R. Maheri, R. T. Severn, "Experimental added- mass in modal vibration of cylindrical structures" *J. Eng. Struct.*, vol. 14, Dec.1992, pp.163-175.
- [14] L. Huang, "Flutter of cantilevered plates in axial flow" *J. Fluids. Struct.*, vol. 9, Feb.1995, pp. 127-147.
- [15] J.K. Sinha, S. Sandeep, "Added mass and damping of submerged perforated plates" *J. Sound. Vib.*, vol. 260, Feb.2003, pp. 549-564.
- [16] M. Boa, W. Wang, W. "Future of micromechanical systems (MEMS)" *J. Sens. Actuators.*, vol.56, Aug. 1996, pp.135-141.
- [17] J.M. Sallese, W. Grabinski, V. Meyer, C. Bassin, P. Fazan, "Electrical modeling of a pressure sensor MOSFET" *J. Sens. Actuators*, vol. 94, Oct.2001, pp.53-58.
- [18] MTA. Saif, BE. Alaca, H. Sehitoglu, H, "Analytical modeling of electrostatic membrane actuator for micro pumps" *JMEMS*, vol.8, Sep.1999, pp.335-345.
- [19] G. Rezazadeh, M. Ghanbari, I. Mirzaee, A. Keyvani, "On the modeling of a piezoelectrically actuated microsensor for simultaneous measurement of fluids viscosity and density" *Measurement*, vol.43, Dec.2010, pp. 1516-1524.
- [20] C.Y. Wang, "The squeezing of a fluid between two plates" *ASME J. Appl. Mech.*, vol.43, Dec. 1976, pp. 579-582.
- [21] H. Hashimoto, "Viscoelastic squeeze film characteristics with inertia effects between two parallel circular plates under sinusoidal motion" *ASME J. Tribol.*, vol. 116, Jan. 1994, pp. 110-117.
- [22] M. Ghanbari, S. Hossainpour, G. Rezazadeh "Study of squeeze-film damping in a micro-beam resonator based on micropolar theory" *LAISS*. Vol.12, Oct.2014, pp. 77-91.
- [23] G. Rezazadeh, M. Fathalilou, R. Shabani, S. Tarverdiloou, "Dynamic characteristics and forced response of an electrostatically- actuated micro-beam subjected to fluid loading" *J. Microsyst. Technol.*, vol.15, Jul.2009, pp. 1355-1363.
- [24] A. K. Pandey, R. Pratap, "Effect of flexural modes on squeeze film damping in MEMS cantilever resonators" *J. Micromech. Microeng.*, vol. 17, Nov. 2007, pp. 2475-2484.
- [25] M. I. Younis, A. H. Nayfeh, "Simulation of squeeze-film damping of microplates actuated by large electrostatic load" *ASME. J. Comput. Nonlinear Dynam.* Vol.2, Jan. 2007, pp. 101-112.
- [26] S. Chatrejee, G. Pohit, "A large deflection model for the pull-in analysis of electrostatically actuated micro cantilever beams" *J. Sound and Vib.*, vol. 322, May. 2009, PP. 969-986.
- [27] S. Chatrejee, G. Pohit, G, "Squeeze- film characteristics of cantilever micro-resonators for higher modes of flexural vibration" *Int. J. Eng. Sci and Technol.*, vol. 2, Sep.2010, pp. 187-199.
- [28] A. C. Eringen, "Theory of micro-polar fluids" *J. Math. Mech.*, vol. 16, Jul.1965, pp.1-18.
- [29] A. C. Eringen, "Theory of thermo micro-polar fluids" *J. Math anal. Appl.*, vol.38, May.1972, pp. 480-496.
- [30] A. Kucaba-Pietal, "Applicability of the micropolar fluid theory in solving microfluidics problems" in *Proc. 1th Annu. European Conference on Microfluidics*, Bologna, 2008.
- [31] A. Kucaba-Pietal, "Micro channels flow modelling with the micropolar fluid theory" *Bull. Pol. Acad. Sci.*, vol.52, Dec.2004, pp. 209-213.
- [32] J. Chen, C. Liang, J. D. Lee, 2011, "Theory and simulation of micropolar fluid dynamics" *J. Nanoeng. Nanosyst.*, vol. 224, Jan.2011, pp. 31-39.
- [33] A. Yavari, S. Sarkani, E. T. Moyer, "On fractural cracks in micro-polar elastic solids" *ASME J. Appl.Mec.*, vol. 69, Jan.2002, pp. 45-54.
- [34] G. Rezazadeh, A. Tahmasebi, 2006, "Application of piezoelectric layers in electrostatic MEM actuators: controlling of pull-in voltage" *J. Microsyst. Techno.*, vol. 12, Aug.2006, pp. 1163-1170.
- [35] E. Crawley, J. Luis, "Use of piezoelectric actuators as elements of intelligent structures" *J. AIAA*, vol. 25, Nov.1987, pp. 1373-1385.
- [36] T. Sree Lakshmi, A. Chandulal, K. Sambaiah, "Reflection and transmission of P- Waves at an interface of two micro-polar solid half-spaces" *Int. J. Pure Appl. Sci. Technol.*, vol. 2, Feb.2011, pp. 19-28.
- [37] G. Ahmadi, "Self-similar solution of incompressible micro-polar boundary layer flow over a semi-infinite plate" *Int. J. Eng. Sci.*, vol. 14, Dec. 1976, pp.639-646.
- [38] F. S. Tse, I. E. Morse, R. T. Hinkle, R. T, *Mechanical Vibrations: Theory and Applications*, Boston, 2004, Ch.7.