Limiting Fiber Extensibility as Parameter for Damage in Venous Wall

Lukas Horny, Rudolf Zitny, Hynek Chlup, Tomas Adamek, and Michal Sara

Abstract—An inflation–extension test with human vena cava inferior was performed with the aim to fit a material model. The vein was modeled as a thick–walled tube loaded by internal pressure and axial force. The material was assumed to be an incompressible hyperelastic fiber reinforced continuum. Fibers are supposed to be arranged in two families of anti–symmetric helices. Considered anisotropy corresponds to local orthotropy. Used strain energy density function was based on a concept of limiting strain extensibility. The pressurization was comprised by four pre–cycles under physiological venous loading (0 – 4kPa) and four cycles under nonphysiological loading (0 – 21kPa). Each overloading cycle was performed with different value of axial weight. Overloading data were used in regression analysis to fit material model. Considered model did not fit experimental data so good. Especially predictions of axial force failed. It was hypothesized that due to nonphysiological values of loading pressure and different values of axial weight the material was not preconditioned enough and some damage occurred inside the wall. A limiting fiber extensibility parameter J_m was assumed to be in relation to supposed damage. Each of overloading cycles was fitted separately with different values of J_m. Other parameters were held the same. This approach turned out to be successful. Variable value of J_m can describe changes in the axial force – axial stretch response and satisfy pressure – radius dependence simultaneously.

Keywords—Constitutive model, damage, fiber reinforced composite, limiting fiber extensibility, preconditioning, vena cava inferior.

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I. INTRODUCTION

BIOMECHANICAL literature which deals with veins is not so rich in comparison with number of papers focused on arterial walls. Constitutive models with identified material parameters useful in the context of nonlinear mechanics are rare. Authors usually describe venous response only in terms of displacements and forces, see e.g. [1]–[3]. Constitutive behavior is sometimes described in terms of incremental elastic moduli, see [4], [5]. However, latest papers consider venous wall in the framework of nonlinear continuum mechanics, see [6].

This study presents an attempt to contribute to research on venous wall mechanics. Our first idea was to fit limiting fiber extensibility model of a strain energy density function (SEF) on experimental data from an inflation–extension test. However, regression analysis revealed that considered model is not in good agreement with experimental data. This was probably caused by insufficient preconditioning within the test. Thus we adopted the hypothesis that damage occurred in the material during loading. This damage is represented by variable value of one material parameter. Such modification of the material model led to good agreement with data. At first the experimental method is briefly described. Consecutively the material model is introduced. Closing paragraph presents results of regression analysis.

II. EXPERIMENT

The sample of male vena cava inferior tissue was harvested within autopsy at the Institute of Forensic Medicine of the Faculty Hospital Kralovske Vinohrady in Prague. The sample was obtained from 31–year–old male donor who did not die in the link with cardiovascular diseases. No pathological changes were observed on the venous wall. After autopsy the sample was stored in the saline solution at temperature approximately 4°C. The inflation–extension test was finished 20 hours after death. All measurements were performed under room temperature. Afterwards the specimen was moved back to the Institute of Forensic Medicine to ethic removal. For the present study, use of autopsy material from human subjects was approved by the Ethics Committee of the Faculty Hospital Kralovske Vinohrady (Prague, Czech Republic).

Under reference configuration the sample had approximately tubular shape with following dimensions incorporated into the model: outer radius $R_o = 10.98$mm,

inner radius R_i = 10.38mm, thickness H = 0.6mm. Sizes were determined by an image analysis of photography. The length of the sample under reference configuration was 56mm. Residual strains were not observed. In order to prevent outflow through branches during pressurization a condom was placed inside the sample. After the preparation of final shape the sample was mounted into the experimental setup. This is illustrated in Fig. 1. The experimental configuration was vertical and the tube had closed bottom.
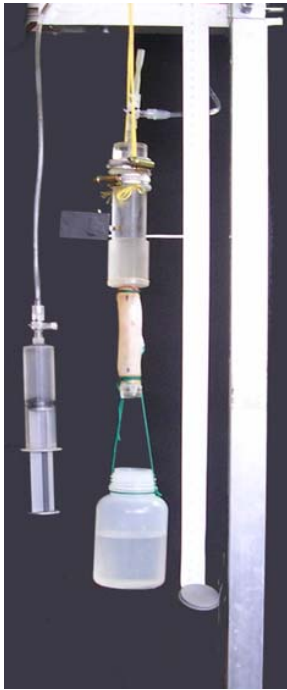

Fig. 1 The sample mounted in experimental setup

External loads had the form of internal pressure and axial force. The axial force is sum of a weight and the force originated by the pressure which is applied to the bottom of the tube (closed tube configuration).

TABLE I
LOADING RANGE

Cycle	Internal pressure [kPa]	Axial force [N]
1	0.6 – 4.9	0.33 – 2.08
2	1.0 – 4.6	same
3	0.35 – 5	same
4	0.4 – 4.9	same
5	0.35 – 20.6	0.55 – 8.82
6	0.6 – 19.6	1.14 – 8.72
7	0.8 – 18.0	1.69 – 8.71
8	0.6 – 19.1	2.11 – 9.64

The weight was constant through one cycle. The vein underwent 8 measurement cycles. Loading conditions within 1st – 4th cycle ranged over 0.5 – 5kPa for internal pressure, and 0.33 – 2.08N for axial force (effect of closed configuration considered). Loading conditions within 5th – 8th cycle ranged

over 0.5 – 20kPa for internal pressure, and 0.5 – 9.6N for axial force. The value of the weight was the same for every cycle in physiological domain. Each overloading cycle has different value of the weight. Detailed information about loading sequence is in Table I. Several preconditioning cycles were performed before the measurement. Preconditioning loading corresponded to physiological range of pressures but no weight was applied.

Spatial configurations of the sample were photographed and displacements were consecutively determined by image analysis. The tube was pressurized manually by a syringe. Loading rate was very small and measurement is considered to be quasi–static. Internal pressure was measured by a pressure probe (KTS 438, Cressto, Czech Rep.) and recorded into PC by in-house software developed in LabView (National Instruments, USA).

III. CONSTITUTIVE MODELING AND REGRESSION

The material of venous wall was modeled as incompressible hyperelastic fiber reinforced continuum. Mechanical response of the hyperelastic material is described by a strain energy density function (SEF) and its derivatives. In the present study we performed the regression analysis for the model based on limiting fiber extensibility concept recently published by Horgan and Saccomandi [7], [8]. This is derived from a model proposed by Gent for macromolecular materials, [9]. The leading idea is that there exists a limiting value of a fiber stretch and toward this a stored energy increases unlimitedly. We consider this model in the form as follows

$$\psi = c\left(I_1 - 3\right) - \mu J_m \ln\left(1 - \frac{\left(I_4 - 1\right)^2}{J_m^2}\right). \qquad (1)$$

The form of (1) is additive split into isotropic part (Neo–Hook) related to energy stored in a matrix of a composite, and logarithmic part related to energy stored in fibers. Symbols in (1) have the following meaning: ψ – strain energy density function; c and μ – stress-like material parameters; J_m – non-dimensional material parameter; I_1 – first invariant of right Cauchy–Green strain tensor; I_4 – fourth (pseudo)invariant of right Cauchy–Green strain tensor related to local orthotropy of the material. I_4 can be expressed in the form

$$I_4 = \lambda_t^2 \cos^2 \beta + \lambda_z^2 \sin^2 \beta . \qquad (2)$$

Here λ_t, λ_z denote stretches in the circumferential and axial directions, respectively, in the thick walled–tube located into polar cylindrical coordinate system. Symbol β denotes helix angle of fiber helices measured to circumferential axes. I_4 is symmetric with respect to $\pm\beta$. Material parameter J_m is related to the limiting value of the stretch due to its incidence in the denominator. The limiting value of the fiber stretch is implicated by a requirement of positivity for the argument of the logarithm. Admissible fields of stretches must satisfy

$$(I_4 - 1)^2 < J_m^2. \qquad (3)$$

It should be noted that model (1) represents the locally orthotropic material. The local orthotropy arises from symmetry in (2) and mechanical equivalence of two families of fibers. The tube reinforced by such arranged mechanically equivalent fibers is illustrated in Fig. 2. Detailed information about local orthtropy can be found in [10]. It is necessary to remark the difference in the argument of logarithm in (1), where J_m^2 is used instead J_m, what is used in [8]. It is assumed that the matrix and fibers undergo the same deformation at each point of continuum. Another successful fitting of experimental data with the model (1) for human aorta can be found in [11].
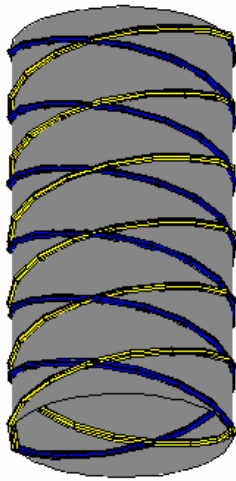

Fig. 2 The tube reinforced with two families of fibers

Regression analysis based on least square method gave the estimations for material parameters in the model (1). A system of nonlinear equations was solved by Levenberg – Marquardt algorithm using in-house software package FEMINA. Least square optimization was based on a comparison of measured and predicted values of internal pressure and axial force during inflation and extension of the cylindrical tube. Computational model of the thick–walled tube predicts the internal pressure p and the axial force F in the following forms

$$p = \int_{r_i}^{r_o} \lambda_t \frac{\partial \psi}{\partial \lambda_t} \frac{dr}{r}, \qquad (4)$$

$$F = \pi \int_{r_i}^{r_o} \left(2\lambda_z \frac{\partial \psi}{\partial \lambda_z} - \lambda_t \frac{\partial \psi}{\partial \lambda_t} \right) r \, dr. \qquad (5)$$

New symbols used in (4) and (5) are r_i what denotes internal radius in the current configuration, and r_o what denotes outer radius in the current configuration. The variable radius through the wall thickness in spatial configuration is denoted

r. Note that zero external pressure is supposed. Final term for least square optimization has the form of (6).

$$WSSQ = \sum_i w_i \left(p_i^{EXP} - p_i^{MOD} \right)^2 + \sum_j w_j \left(F_i^{EXP} - F_i^{MOD} \right)^2 \qquad (6)$$

In (6) $WSSQ$ means weighted sum of squares, p_i^{MOD} denotes internal pressure predicted by (4) and p_i^{EXP} means measured value of the internal pressure, respectively. Similarly F^{MOD} is the value of axial force predicted by (5), and F^{EXP} is the value of the total loading force. Weights are denoted w_i and w_j. Indices denote the number of the observation point.
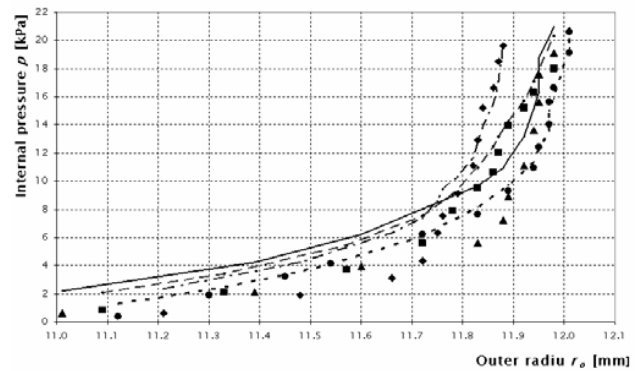

Fig. 3 Results of regression for non–physiological loading cycles for internal pressure vs. outer radius. Experimental data are represented by symbols: cycle no. 5 – circles; cycle no. 6 – diamonds; cycle no. 7 – boxes; cycle no. 8 – triangles. Model predictions are represented by curves: cycle no. 5 – doted; cycle no. 6 – dashed-doted; cycle no. 7 – dashed; cycle no. 8 – solid
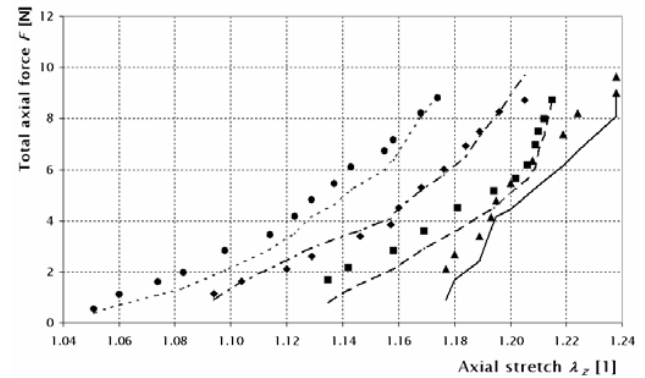

Fig. 4 Results of regression for non–physiological loading cycles for total axial force vs. axial stretch. Experimental data are represented by symbols: cycle no. 5 – circles; cycle no. 6 – diamonds; cycle no. 7 – boxes; cycle no. 8 – triangles. Model predictions are represented by curves: cycle no. 5 – doted; cycle no. 6 – dashed-doted; cycle no. 7 – dashed; cycle no. 8 – solid

IV. RESULTS AND CONCLUSION

The results of regression analysis are presented in the Fig. 4 and Fig. 5. Material parameters are listed in the Table II. Unfortunately histological analysis was not available and we

can not compare predicted value of coiling angle for collagen fibers with real material structure. The regression was performed for overloading cycles only. The value of limiting fiber extensibility parameter, J_m, was supposed to be variable through cycles. It is considered that J_m is related to any damage in material. This damage evolves through cycles in relation to a strain history. Important facts are that in each overloading cycle starting value of axial force was higher than in previous cycle, and that in each overloading cycle was maximal value of axial stretch higher than in previous one. This probably caused the evolution of the constitutive equation. An inelastic material which did not undergo any stress–strain state could not be seen as preconditioned for the first time at this state.

TABLE II
MATERIAL PARAMETERS

Cycle	c [kPa]	μ [kPa]	J_m [1]	β [°]
5th	0.2564	47.62	0.2993	27.08
6th	0.2564	47.62	0.2949	27.08
7th	0.2564	47.62	0.3188	27.08
8th	0.2564	47.62	0.331	27.08

Constitutive equations satisfy slightly better the axial equilibrium than radial equilibrium that is seen in comparison Fig. 3 with Fig. 4. Especially it is held for low pressures. But right–side shift for axial force and stretch is fitted well.

Mechanical response of un–preconditioned soft tissues is still opened problem. Biomedical engineers use Fung's concept of pseudoelasticity for decades. But physical models must cover all aspects of reality. Limiting strain extensibility or fiber extensibility models can successfully predict large strain stiffening of materials. There is direct relation of material parameter J_m to this stiffening. Relate this parameter to damage seems to be promising in our opinion.

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