# Lifetime Maximization in Wireless Ad Hoc Networks with Network Coding and Matrix Game

Jain-Shing Liu

*Abstract*—In this paper, we present a matrix game-theoretic cross-layer optimization formulation to maximize the network lifetime in wireless ad hoc networks with network coding. To this end, we introduce a cross-layer formulation of general NUM (Network Utility Maximization) that accommodates routing, scheduling, and stream control from different layers in the coded networks. Specifically, we develop a matrix game with the strategy sets of the players corresponding to hyperlinks and transmission modes, and design the payoffs specific to the lifetime. Given that, our cross-layer programming formulation can benefit from both game-based and NUM-based approaches at the same time by cooperating the programming model for the matrix game with that for the other layers in a consistent framework. Finally, our numerical example demonstrates its performance results on a well-known wireless butterfly network to verify the cross-layer optimization scheme.

*Keywords*—Cross-layer design, Lifetime maximization, Matrix game, Network coding.

## I. INTRODUCTION

WIRELESS networks with multihop transmissions such as wireless sensor networks (WSNs) are usually composed of a large number of stations to perform their tasks of sensing, computing and wireless communication. In such energy-limited networks, stations usually operate with small batteries that are difficult to replace in typical scenarios, and thus minimizing its energy consumption poses a considerable challenge to the engineers and maximizing network lifetime continuously intensify the interest of researchers in the development of energy-efficient wireless transmission schemes.

For increasing network lifetime, cross-layer optimization is mainly considered here because it can coordinate resources allocated to different layers to achieve globally optimal performance. In particular, network utility maximization (NUM) is usually regarded as a key tool to realize this aim. For example, the authors in [1] assume that the transmit power level can be adjusted to use the minimum energy required to reach the intended next hop receiver, and then the energy consumption problem can be reduced to that only depending on the routing decision.

Similarly, in [2], the authors study the problem of joint routing, link scheduling and power control to support high data rates on WSNs. To this end, they propose an algorithm for link scheduling and power control to minimize the total average

energy consumption in such networks. In [3], the authors consider a joint optimal design of physical, MAC, and routing layers to maximize the lifetime of WSNs. Specifically, they use TDMA as their MAC to formulate the optimization problem as a mixed integer convex problem, which can be solved with standard techniques such as interior point methods.

On the other hand, game theory is a useful tool to simplify problems in mathematics for a long time [4]. When considered with network coding [5] that can achieve the maximum multicast rate by information encoding at the relay nodes, the work in [6] shows that a generalized butterfly network can be analyzed as a two-source unicast coded network, and its robustness had been investigated by game theory with the desired solution to reach equilibrium. In addition, the work in [7] has modeled the multicast network switching in wired network by an equivalent matrix game. More recently, the authors in [8] jointly consider link, network component, and routing path with a nonlinear cubic game, which can be sequentially solved with a fictitious play (FP) technique.

In this work, we propose a cross-layer formulation of general NUM with network coding that can accommodate routing, scheduling and stream control from different layers in the coded wireless networks. Thanks to the nature of matrix game and the merit of NUM, we require no FP to converge to the solutions in two steps. Instead, our cross-layer programming formulation directly benefits from both game-based and NUM-based approaches at the same time by cooperating the programming model for matrix game with that for the other layers in a consistent programming framework. The maximum network lifetime for a coded network can be resulted by solving the cross-layer programming model once without stepped convergence, which significantly reduces the complexity of solving a nonlinear cubic game.

In the following, network coding is briefly summarized in Section II. Then, our cross-layer formulation for the network lifetime maximization problem is introduced in Section III. The cross-layer optimization is examined numerically in Section IV. Finally, conclusions are drawn in Section V.
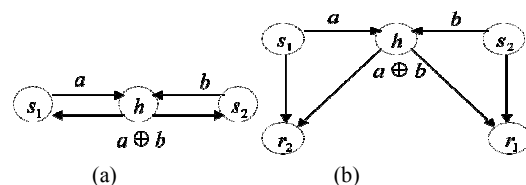
Jain-Shing Liu is with the Department of Computer Science and Information Engineering Providence University 200 Chung-Chi Rd., Shalu, Taichung, Taiwan, R.O.C (e-mail: chhliu@pu.edu.tw).

Fig. 1 Wireless network coding example: (a) coding without opportunistic listening (b) coding with opportunistic listening

## II. NETWORK CODING

Network coding gain in wireless network is mainly obtained by properly combining different packets before transmitting, and then transmitting the coded information to multiple neighboring nodes by a single transmission. To see this, we first consider an example of wireless information exchange shown in Fig. 1 (a), wherein two nodes $s_1$ and $s_2$, respectively, have two packets a and b, and they want to exchange these packets via a relay node h due to the communicating nodes being out of the transmission range of each other.

With the traditional store and forward method, i.e., routing, the source $s_1$ should first send packet a to the relay node $h$, and then h forwards this packet to the destination $s_2$, which takes two transmissions to deliver packet a from $s_1$ to $s_2$. Similarly, in order to send packet b from source $s_2$ to its destination $s_1$, we need two transmissions as well. Thus, there are 4 transmission required to exchange two data packets between $s_1$ and $s_2$. On the other hand, by using network coding, $s_1$ and $s_2$, respectively, send packets $a$ and $b$ to the relay node $h$, as usual. However, when the relay node receives the two packets, it generates a combined packet $a \oplus b$ and broadcasts it to both $s_1$ and $s_2$. Consequently, $s_1$ can recover $b$ by using the XOR operation of $a \oplus (a \oplus b)$, and $s_2$ can recover a by using that of $b \oplus (a \oplus b)$, respectively. Clearly, it takes only 3 transmissions to complete the information exchange.

Note that networking coding gain is not only obtained through the advantage of wireless broadcast as shown in the above, but also from that of opportunistic listening. To demonstrate this, we consider another example in Fig. 1 (b), where two sources $s_1$ and $s_2$ try to exchange their packets $a$ and $b$ via relay node $h$ to their destinations $r_1$ and $r_2$, respectively, which is similar to the scenario given previously in Fig. 1 (a). However, unlike the previous case, we can see additional arrows $s_1 \to r_2$ and $s_2 \to r_1$ existed to represent that $r_2$ is within the transmission range of $s_1$ and $r_1$ is within that of $s_2$. Now, with the wireless broadcast, $r_1$ and $r_2$ can opportunistically listen to the channel. That is, when $s_1$ sends packet a to relay node h, $r_2$ can overhear this transmission and store this packet $a$. Then, when $r_2$ receives the coded packet $a \oplus b$ from $h$, $r_2$ can decode it and thus obtain both packets $a$ and $b$. Similarly, $r_1$ can overhear packet $b$ from $s_2$ and decode the coded packet $a \oplus b$ received from h to obtain both packets $a$ and $b$. Obviously, this case and the previous exhibit the advantage of using network coding with or without opportunistic listening, we can save one transmission when compared with routing in these typical scenarios.

## III. CROSS-LAYER APPROACH FOR NETWORK LIFETIME MAXIMIZATION

Given the capability of network coding, we here target on the lifetime maximization problem by taking this capability into NUM and matrix game. For this aim, we first define the players in the game and the payoff matrix with regard to the lifetime. Then, we conduct the matrix game to solve the core problem of scheduling, revealing the relevant variables that can cooperate with other variables on routing and stream control in the coded

networks with a cross-layer programming model.

### A. Solving Scheduling Problem with Matrix Game

For the scheduling problem involved, we consider a two-person, zero-sum matrix game. In such a game, there are two players, namely Player I and Player II, and the relationship of payoff between these players can be represented by a $m \times n$ matrix $A$ of real numbers. In this work, we consider a wireless network to be modeled as a directed hypergraph $G = (N, L)$, where $N$ is the set of nodes and L is the set of hyperlinks, and the two players are hyperlink and transmission mode. Specifically, a hyperlink $(i, J) \in L$ for wireless network coding represents a one-hop boradcast transmission, wherein $i \in N$ is the transmitter and $J \subseteq N$ is the set of receivers due to the broadcast nature of the wireless channel. Clearly, when $J$ contains only one node j, the hypergraph resulted can reduce to a conventional graph model. Given that, a transmission mode $\xi \subset L$ is defined as a set of hyperlinks that can be concurrently activated. In addition, for concisely representing the game, we simply denote the hyperlinks by $l_1, l_2, \ldots, l_m$ under a fixed order whenever the notion of $(i, J)$ is not significant in the context, and denote the transmission modes by $\xi_1, \xi_2, \ldots \xi_n$, as well. Also, we replace each index such as $i$ or $j$ with $\hat{i}$ or $\hat{j}$ when the index is specific to the matrix game rather than the network graph considered at the beginning. With this notion, our payoff matrix particularly designed for the lifetime can be given by

$$a_{\hat{i}\hat{j}} = \begin{cases} \frac{E_{\hat{i}}}{e_{\hat{i}\hat{j}}}, & if \ l_{\hat{i}} \in \xi_{\hat{j}} \\ 0, & otherwise \end{cases} \quad (1)$$

where $E_{\hat{i}}$ denotes the initial energy in the transmitter of hyperlink $l_{\hat{i}}$ and $e_{\hat{i}\hat{j}}$ denotes the average energy spent by $l_{\hat{i}}$ when it is scheduled to be active in transmission mode $\xi_{\hat{j}}$.

In the sequel, we consider a mixed strategy of Player I or hyperlink as a probability distribution $p$ over the rows of payoff matrix $A = [a_{\hat{i}\hat{j}}]$, which can be represented by an element of the following set

$$P_m = \{p = (p_1, \ldots, p_m) \in R^m : p_{\hat{i}} \geq 0$$

$$\forall \hat{i} = 1, \ldots, m, \sum_{i=1}^{m} p_{\hat{i}} = 1\} \quad (2)$$

Similarly, a strategy of Player II or transmission mode is considered as a probability distribution $q$ over the columns of A, as represented by an element of the following set

$$Q_n = \{q = (q_1, \ldots, q_n) \in R^n : q_{\hat{j}} \geq 0$$

$$\forall \hat{j} = 1, \ldots, n, \sum_{j=1}^{n} q_{\hat{j}} = 1\} \quad (3)$$

Now, if Player I plays strategy p and Player II plays strategy $q$, then Player I receives the expected payoff

$$a(p, q) = p^T A q \quad (4)$$

where $p^T$ denotes the transpose of p. Given that, a strategy $p^*$ is called *maximin* strategy of Player I in the matrix game if

$$min\{ (p^*)^T Aq, q \in Q_n \}$$

$$\geq min\{ (p^T Aq, q \in Q_n \}, \forall p \in P_m \} \qquad (5)$$

Similarly, a strategy $q^*$ is called *minimax* strategy of Player II in the matrix game if

$$max\{ p^T Aq^*, p \in P_m \}$$

$$\leq max\{ p^T Aq, p \in P_m \}, \forall q \in Q_n \qquad (6)$$

That is to say, a maximin strategy of Player I maximizes the minimal payoff of Player I, while a minimax strategy of Player II minimizes the maximum that Player II has to pay to Player I. In the literature [9], it had been proved that for every matrix game there is a real number such that (i) a strategy $p$ of Player I guarantees a payoff of at least v to Player I if and only if $p$ is a maximin strategy, and (ii) a strategy $q$ of Player II guarantees a payment of at most v by player II to player I if and only if q is a minimax strategy.

In above, v is usually called the value of matrix game. To find this value, we consider the expectation given in [10].

$$E(P_i, Q) = \sum_{j=1}^{n} a_{ij} q_j \qquad (7)$$

and the expectation

$$E(P, Q_j) = \sum_{i=1}^{m} p_i a_{ij} \qquad (8)$$

where $P_i$ denotes a pure strategy, a special case of mixed strategy with $p_i = 1$ and $p_k = 0, \forall k \neq \hat{\imath}$, and $Q_j$ denotes a pure strategy having $q_j = 1$ and $q_k = 0, \forall k \neq \hat{\jmath}$. Given that, a necessary and sufficient condition that $v$ is the value of the game and that $P^*$ and $Q^*$ are the optimal strategies for Player I and Player II, respectively, is that, for $1 \leq \hat{\imath} \leq m$ and $1 \leq \hat{\jmath} \leq$ n, (see [10], p. 39, Theorem 2.9).

$$E(P_i, Q^*) \leq v \leq E(P^*, Q_j) \qquad (9)$$

### B. Cross-Layer Optimization with Network Coding

Instead of only considering the matrix game for the scheduling problem, in this work we aim to complete a cross-layer optimization scheme, which also accounts for routing and stream control in the network layer and the transport layer, respectively. To this end, we should introduce the other variables with respect to these layers, in addition to the hyperlink scheduling strategy $p_i$ and transmission mode scheduling strategy $q_j$ just introduced.

For the transport layer in the coded network, we consider a set of multicast sessions to be transmitted through the network. Herein, a multicast session is denoted by its source node $s \in S \subset N$ multicasts packets to its destination node set $T_s$. Further, for the network layer, we let $f_{iJj}^{st}$ denote the information flow rate from source s to destination node $t \in T_s$ over hyperlink $(i, J)$ and being intended to node $j \in J$. Given that, for a multicast session where source s wants to transmit with a rate

of $x_s$ to its set of destination nodes $T_s$, we have the flow conservation law as follows:

$$\sum_{\{J|(i,J)\in L\}} \sum_{j \in J} f_{iJj}^{st} - \sum_{j \in N} \sum_{\{i|(j,I)\in L, i \in I\}} f_{jIi}^{st} = x_{i,s},$$
$$\forall i \in N, \forall s \in S, \forall t \in T_s \qquad (10)$$

where $x_{i,s}$ is

$$\begin{cases} x_s, & \text{if node i is the source of session s} \\ -x_s, & \text{if node i is the sink of session s} \\ 0, & \text{otherwise} \end{cases} \qquad (11)$$

Obviously, the session rate and then the flow rate on the upper layers should be realized by the hyperlink capacity to be scheduled in the MAC layer and the data rate in the physical layer. Thus, while giving the matrix game for the scheduling problem, we should proceed to establish the relationship between the upper layers and the lower layers. For this, let $g_{iJ}^s$ be the physical flow rate from source s to the set of destination nodes $T_s$ over $(i, J)$ Then, with the above, we can first show a relevant constraint that, with network coding, the sum of flow rate on hyperlink $(i, J)$ should not exceed the physical rate. That is,

$$\sum_{j \in J} f_{iJj}^{st} \leq g_{iJ}^s, \forall (i, J) \in L,$$
$$\forall s \in S, \forall t \in T_s \qquad (12)$$

Apart from this constraint, with the indices specific to the matrix game, we further let $r_{ij}$ be the capacity of $l_i$ scheduled by $\xi_j$ while using $g_i^s$ to correspond to $l_i$ in the matrix game. Then, we can derive another constraint that the physical flow accounting for all sessions $s \in S$ should be upper bounded by the physical capacity of $l_i$ scheduled by the hyperlink with $p_i$ and the transmission mode with $q_j$, which are the resulted strategies of the two players, as follows:

$$\sum_{s \in S} g_i^s \leq \sum_{j=1}^{n} p_i r_{ij} q_j, 1 \leq \hat{\imath} \leq m \qquad (13)$$

Finally, as implied by our payoff matrix, the lifetime of a node $k$ can be obtained by

$$\frac{E_k}{\sum_{\{i:tr(i)=k\}} \sum_{j=1}^{n} p_i e_{ij} q_j} \qquad (14)$$

where $tr(\hat{\imath})$ denotes the transmitter of hyperlink $l_i$, and $E_k$ specially denotes the initial energy of node $k$ to distinguish itself from $E_i$ that represents the initial energy of the transmitter of hyperlink $l_i$ shown in (1).

### C. Cross-Layer Programming Model

By taking all the above, we could now complete the cross-layer programming model for the lifetime maximization. However, before giving, we notice that without a traffic demand $TD_s$ on $x_s$ for each session s, maximizing lifetime will

make no sense tothe users because we can always forbid the traffic or sacrifice the throughput to increase such a lifetime. Hence, by enforcing a meaningful traffic demand along with all the above constraints and objective, we can finally formulate the lifetime maximization problem as follow:

$$\textbf{maximize}\, v \tag{15}$$

$$s.t. \sum_{\{J|(i,J)\in L\}} \sum_{j\in J} f_{iJj}^{st} - \sum_{j\in N} \sum_{\{i|(j,I)\in L, i\in I\}} f_{jIi}^{st} = x_{i,s}$$

$$\forall i \in N, \forall s \in S, \forall t \in T_s \tag{16}$$

$$\sum_{j=1}^{n} a_{\hat{i}j}q_j \leq v, \qquad 1 \leq \hat{i} \leq m \tag{17}$$

$$\sum_{i=1}^{m} p_i a_{i\hat{j}} \geq v, \qquad 1 \leq \hat{j} \leq n \tag{18}$$

$$\sum_{j\in J} f_{iJj}^{st} \leq g_{iJ}^{s}, \qquad \forall (i,J) \in L, \forall s \in S, \forall t \in T_s \tag{19}$$

$$\sum_{s\in S} g_{\hat{i}}^{s} \leq \sum_{j=1}^{n} p_i r_{\hat{i}j}q_j, \quad 1 \leq \hat{i} \leq m \tag{20}$$

$$\sum_{i=1}^{m} p_i = 1 \tag{21}$$

$$\sum_{j=1}^{n} q_j = 1 \tag{22}$$

$$0 \leq p_i \leq 1, \qquad 1 \leq \hat{i} \leq m \tag{23}$$

$$0 \leq q_j \leq 1, \qquad 1 \leq \hat{j} \leq n \tag{24}$$

$$x_s \geq TD_s, \qquad \forall s \in S \tag{25}$$

## IV. NUMERICAL RESULTS

In this section, we report on numerical results for our cross-layer optimization. As shown in Fig. 2 (a), a well-known wireless butterfly network for network coding in the literature [7], [8], [11] is adopted here as our simulation environment. Given this network, we consider six hyperlinks,(1,{2}), (1, {3}), (1, {2, 3}), (2, {4,5}), (3, {4,6}), and (4, {5,6}) and five transmission modes {(1,{2}), (3,{4,6})}, {(1,{2}), (4,{5,6})}, {(1, {3}), (2, {4,5})}, {(1, {3}), (4, {5,6})}, and {(1, {2, 3}), (4,{5,6})} as that given in [8]. Specifically, to focus on the correctness of this optimization framework, in the numerical example we do not consider a particular physical layer and its energy consumption, which can be included for a more realistic simulation afterward. Instead, we assume that each $l_i$ has one unit capacity and each$e_{ij}$has one unit energyconsumption, and each node i ∈ N has initial energy$E_i = 1$, which leads to the capacity matrix R with its element $r_{ij} = 1$ if $l_i \in \xi_j$ and 0 otherwise, as well as that for the payoff matrix.Then, a multicast session $s = 1$ is conducted with source node1 transmitting packets to its sink nodes 5 and 6, and requiring its rate $x_1$ to be at least 0.1 (i.e., traffic demand$TD_1$ of 0.1).
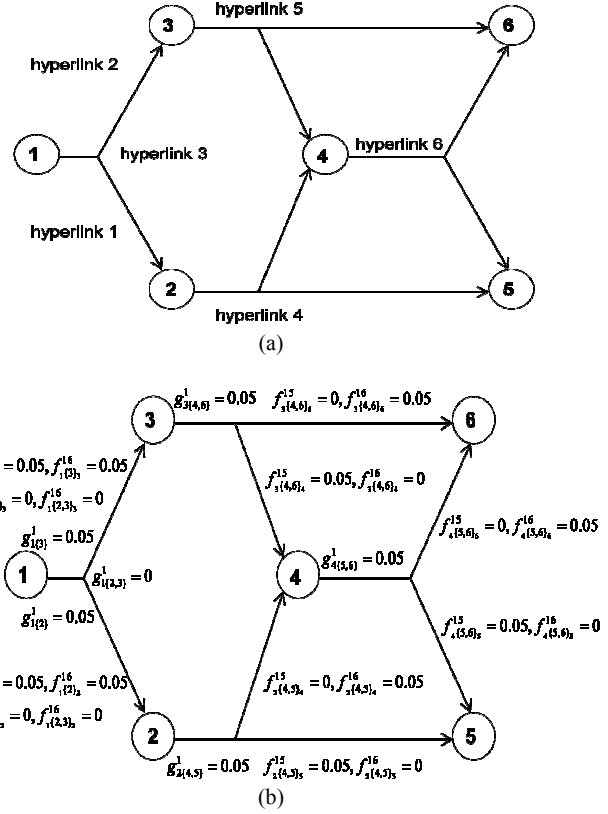


Fig. 2 Numerical example: (a) the wireless butterfly adopted, where the six hyperlinks under consideration are (1,{2}), (1, {3}), (1, {2, 3}), (2, {4,5}), (3, {4,6}) and (4,{5,6}) (b) the results of $g_{iJ}^{s}$ and$f_{iJj}^{st}$ in the example, where each hyerlink with multicast from $s = 1$to $t \in T_s = \{5,6\}$ is marked with $g_{iJ}^{1}$ and the pair $(f_{iJj}^{15}, f_{iJj}^{16})$

Now, given the traffic demand, the strategy of Player I or hyperlink$p = [0.18\bar{3}\ 0.18\bar{3}\ 0\ 0\ 0.15\ 0.15\ 0.\bar{3}]$ and that of Player II or transmission mode $q = [0.\bar{3}0\ 0.\bar{3}0\ 0.\bar{3}]$ can be resulted, which lead to the lifetimes of $8.\overline{18}$, 20, 20, and 9 for the first four nodes while revealing the infinity value for nodes 5 and 6 since they only receive packets and consume no transmit energy in question. In addition, these results (including also that for routing and stream control) comply with all the constraints imposed by the programming model. For example, the flow conservation law says that the total output rate should be equal to the corresponding input rate, and this can be seen, for instance, in the source node 1 that our result

$$f^{15}_{1\{2\}_2} + f^{15}_{1\{3\}_3} + f^{15}_{1\{2,3\}_2} + f^{15}_{1\{2,3\}_3}$$
$$= 0.05 + 0.05 + 0 + 0 = 0.1$$

actually satisfies the constraint (16) with the traffic demand $TD_1 = 0.1$. Besides, it can be also seen that with network coding, the sum of flow rates for a specific source-destination pair over a hyperlink $(i,J)$ to its different intended nodes $j \in J$ is upper bounded by the physical flow rate over this hyperlink $g_{i,J}^{s}$. Taking node 2 as an example, we have

$$f^{15}_{2\{4,5\}_4}(=0) + f^{15}_{2\{4,5\}_5}(=0.05) \leq g^1_{2\{4,5\}} = 0.05$$
$$f^{16}_{2\{4,5\}_4}(=0.05) + f^{16}_{2\{4,5\}_5}(=0) \leq g^1_{2\{4,5\}} = 0.05$$

which satisfy the constraint (19). Taking hyperlink $(2, \{4,5\})$ as a further example, we have

$$g^1_{2\{4,5\}}(=0.05) \leq p_4(=0.15) * r_{4,1}(=0) * q_1(=0.\bar{3}) +$$
$$p_4(=0.15) * r_{4,2}(=0) * q_2(=0) + p_4(=0.15) * r_{4,3}(=1)$$
$$* q_3(=0.\bar{3}) + p_4(=0.15) * r_{4,4}(=0)$$
$$* q_4(=0) + p_4(=0.15) * r_{4,5}(=0)$$
$$* q_5(=0.\bar{3}) = 0.05$$

which satisfy the constraint (20). Apart from these examples, in Fig. 2 (b) we mark also each hyperlink with $g^1_{ij}$ and $(f^{15}_{ijj}, f^{16}_{ijj})$ to help denomstrating how the flow rates can satisfy the flow-sharing network coding property, in addition to that exhibited by the above exemplified constraints in (16), (19) and (20).

## V. Conclusion

In this work, we have introduced a mathematical programming model for the lifetime maximization with matrix game. In particular, with the inherit merit that matrix game can be solved with linear programming, we have constructed a mathematical programming model and resolved it without stepped convergence, avoiding the complexity of formulating such a cross-layer problem with a relevant nonlinear cubic game. The numerical example readily exhibits the correctness of this programming model and contributes valuable viewpoints on the network optimization problem using matrix game.

## Acknowledgement

## References

[1] Jae-Hwan Chang and L. Tassiulas. Maximum lifetime routing in wireless sensor networks. *IEEE/ACM Transactions on Networking*, 12(4):609 – 619, aug. 2004.

[2] R.L. Cruz and A.V. Santhanam. Optimal routing, link scheduling and power control in multihop wireless networks. In *Proceedings of IEEE INFOCOM 2003*, volume 1, pages 702 – 711 vol.1, march, 2003.

[3] R. Madan, S. Cui, S. Lal, and A. Goldsmith. Cross-layer design for lifetime maximization in interference-limited wireless sensor networks. *IEEE Transactions on Wireless Communications*, 5(11):3142 –3152, november 2006.

[4] R. Banner and A. Orda. Bottleneck routing games in communication networks. *IEEE Journal on Selected Areas in Communications*, 25(6):1173–1179, Aug. 2007.

[5] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung. Network information flow. *IEEE Transactions on Information Theory*, 46:1204–1216, 2000.

[6] J. Price and T. Javidi. Network coding games with unicast flows. *IEEE Journal on Selected Areas in Communications*, 26(7):1302–1316, Sep. 2008.

[7] X. B. Liang. Matrix games in the multicast networks: maximum information flows with network switching. *IEEE Transactions on Infromation Theory*, 52(6):2433–2466, June 2006.

[8] E. Karami and S. Glisic. Joint optimization of scheduling and routing in multicast wireless ad hoc networks using soft graph coloring and nonlinear cubic games. *IEEE Transactions on Vehicular Technology*, 60(7):3350–3359, Sept. 2011.

[9] J. von Neumann and O. Morgenstern. *Theory of games and economic behavior*. Princeton University Press, 1944.

[10] J. C. C. McKinsey. *Introduction to the theory of game*. RAND Corporation, 1952.

[11] T. Ho and D. S. Lun. *Network coding: an introduction*. Cambridge University Press, 2008.