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Leveraging Reasoning through Discourse: A Case Study in Secondary Mathematics Classrooms

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Abstract**—**Teaching and learning through the use of discourse support students' conceptual understanding by attending to key concepts and relationships. One discourse structure used in primary classrooms is number talks wherein students mentally calculate, discuss, and reason about the appropriateness and efficiency of their strategies. In the secondary mathematics classroom, the mathematics understudy does not often lend itself to mental calculations yet learning to reason, and articulate reasoning, is central to learning mathematics. This qualitative case study discusses how one secondary school in the Middle East adapted the number talk protocol for secondary mathematics classrooms. Several challenges in implementing 'reasoning talks' became apparent including shifting current discourse protocols and practices to a more student-centric model, accurately recording and probing student thinking, and specifically attending to reasoning rather than computations.

Keywords**—**Discourse, reasoning, secondary mathematics, teacher development.

I. Introduction

DISCOURSE in mathematics classrooms is often integral in teaching and learning mathematics. However, mathematical discourse is more than just talking about numbers or computations; it requires engaging in a complex social and dialogical process wherein students collectively make greater meaning of the mathematics. Helping students learn to engage and interact in discursive interactions can be difficult for many teachers as students often believe that mathematics is purely about obtaining correct solutions to calculations rather than effectively and efficiently reasoning about quantitative relationships [1]. Additionally, secondary mathematics teachers are often unsure how to effectively facilitate discourse in a whole class setting as they often believe there is insufficient time to engage in activities that are not well scripted and predictable [2]. One protocol that has been used successfully in primary grades to build a community of discourse by attending to students' ideas and ways of thinking, and the reasoning behind these ideas, is number talks [3]. However, the implementation of number talks in secondary settings, based on the protocol used in primary grades, is not always transferable due to the complexity of mathematics involved and the need to use appropriate pedagogical techniques for adolescents. This study examines the use of a modified number talk protocol, hereafter referred to as reasoning talks, to understand the extent to which secondary mathematics teachers could learn how to effectively implement discourse-based instructional strategies that help create student to student interactions around reasoning.

The National Council of Teachers of Mathematics (NCTM) [4] states that communicating is a central aspect of teaching and learning mathematics and discourse is one of these forms. According to Piccolo, Harbaugh, Carter, Capraro, and Capraro [5], mathematical discourse is an interactive, dynamic, and inclusive process by which students, with the support of the teacher, develop particular mathematical concepts or practices. Essentially, discourse serves to advance students' mathematical reasoning and conceptual understanding through intentional, focused, and shared social interactions. By using discourse as an instructional method for reasoning about, confirming, and challenging mathematical ideas, teachers and students alike can benefit.

II. Review of Literature

A. Benefits of Discourse

When students have opportunities to discuss mathematics, to have conversations about methods and procedures, they gain deeper insights and are better able to clarify their thinking [6]. This clarity then leads to more convincing arguments for solution processes and more complex connections; connections that stem from the open sharing of multiple perspectives [7], [8]. This process also adds to the credibility of the discussion at hand as peer reviews and critiques are important in formalizing and justifying content knowledge [4]. In mathematics, for a solution to be considered correct, the proof must be recognized and deemed acceptable by others. This is true in the professional mathematics community and a similar process of arriving at shared understandings should also exist within formal classroom settings. That is, when students collectively add to the mathematical discourse within the classroom, when they provide supporting justifications or pose clarifying questions to their fellow classmates, students become more engaged in their learning [9], [10].

For teachers, mathematical discourse provides opportunities to learn about and understand students' perspectives that may not be evident in other forms of assessment [11]. It can be common for students to struggle in communicating their understanding of mathematics in writing, especially when they encounter new vocabulary or problems that require methods for a solution that are not immediately apparent. During such situations, teachers can use discourse to accurately assess students' understandings or misunderstandings [11], [12].

Mathematical discourse also provides teachers with opportunities to support students' understanding by

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interjecting with probing questions that seek clarity in students' ideas and expand upon essential learning concepts that may otherwise be overlooked by the students [13]. By listening to how students construct meaning, posing probing questions, and helping students understand gaps in their reasoning, teachers can gain insights into the nuances in students' reasoning.

Discourse and conceptual understanding are linked in how they contribute to the development of students' thinking skills [7]. Mercer [14] supports this connection by asserting that discourse develops more creative and independent thinkers while simultaneously strengthening procedural knowledge, both of which support student learning. When the classroom discourse focuses on students sharing, critiquing, and evaluating ideas, it becomes a vehicle that operates within a social medium to develop, make more coherent, and advance all students' thinking. In turn, more advanced thinking means greater understanding, higher achievement, and more sophisticated learning [7]; "learning to communicate is learning to think" [6, p. 82].

B. Number Talks as Discourse

Number Talks are an instructional strategy often used in elementary schools to help students' develop number sense through focused discourse. During number talks, students learn to attend to logical aspects of other student's thinking, make connections to their own thinking, and develop more flexible and effective strategies for working with, and making sense of, number, shape, and space. In these short, non-instructional episodes of learning, the teacher writes a computational exercise on the board (i.e. 24 x 16) and asks students to mentally consider strategies for solving it and to provide a solution. After giving students time to independently think about the exercise, students then share their strategy and defend their reasoning to the other students in the class. While the student is sharing, the teacher's responsibility is to represent the student's thinking on the board and probe with questions to clarify key aspects within the mathematics if they are not sufficiently addressed by the student.

As students participate in number talks over time, they learn to connect various strategies of reasoning about numbers, operations, and other mathematical relationships and learn to accurately communicate these understandings. Teachers who regularly use number talks see incredible gains in the intensity of reasoning and justification by students [3]. Humphreys and Parker [15] state that the importance in number talks is not to have students understand the teacher's way of thinking about the mathematics but about students learning to make their own connections to the mathematics.

In secondary mathematics classrooms, attending to reasoning should be a primary focus [16] as making connections to, and reasoning about, mathematical relationships is central to learning mathematics. However, the mathematics under study in secondary classrooms rarely lends itself to mental calculations or simple computations and thus the current protocol for number talks is not well aligned for these classrooms. With that said, most of the benefits of number talks—including such things as making sense of mathematical contexts or situations, recognizing key elements in relationships, learning to reason about why these key elements matter, and articulating understandings in a clear and logical manner—can still happen as they are central elements of mathematical reasoning.

C. Developing Reasoning Talks

The purpose of reasoning talks, much like number talks, is to leverage students' ideas, ways of thinking, and generating preliminary understandings about a problem. Lannin, Elliott, and Ellis [16] indicate that reasoning is a primary area of study in school mathematics and is important for students to be able to logically reason about the mathematics they are learning. However, Lannin, Elliott, and Ellis go on to state that teaching reasoning is a challenge for many secondary teachers. Too frequently students excessively rely on teachers to tell them procedures, depend on text books to show them the algorithm needed without understanding why such an algorithm is appropriate, or begin working on problems before they fully understand the details involved. Reasoning talks focus on helping students learn to systematically think about and consider solution pathways, formulate questions, or attend to important elements within a presented task. Before students begin working on the task, teachers ask students to share what they have considered, recording the student's thinking on the board, and facilitating student to student discourse focused on defending and justifying these ideas in a non-evaluative manner. This helps students to attend to productive and efficient ways of thinking about the mathematics needed for the task.

Reasoning Talks look slightly different in secondary classrooms but they rely on many of the same principles [17]. Instead of framing exercises around methods for calculating numbers, such as decomposing or compensation, teachers focus students' attention to identifying mathematically important aspects in a problem and how this information will be used. Additionally, teachers might center the reasoning talk prompt on making meaning of symbolic representations, connecting mathematical relationships, developing potential solution pathways, developing questions to direct their explorations, or creating a counter argument. Essentially, the reasoning talk serves to help students learn to attend to reasoning needed to solve the mathematics before they begin working.

Since much of the current research in using number talks has been grounded in primary classrooms, and with the importance of developing students' reasoning in secondary mathematics classrooms, this study sought to better understand secondary mathematics teachers' perceptions of implementing a modified number talk protocol as a means of supporting all students' development of mathematical reasoning.

III. Methods

This multi-year qualitative case study is theoretically grounded in three distinct areas of research: supporting teachers' professional growth [18], leveraging dynamic mathematical discourse models [7], and attending to students' thinking through number talks [3]. The following research question was used to frame the study: 1) what challenges and opportunities became evident to secondary mathematics teachers during implementation of reasoning talks?

Data presented in this case study used a grounded theoretical approach [19] that allowed for the development of understandings based on multiple qualitative data sources including field notes, semi-structured interviews (post-lesson reflections), and observations. While this project is ongoing, the data and findings in this paper are only from the first year of implementation.

A. Participants and Setting

All participants were employed at the same American curriculum school in a small Middle Eastern country. Participants included seven secondary teachers all of whom had a minimum of three years of experience, with the median years teaching being 10 years. The maximum number of years teaching was 15. Additionally, all teachers held an education degree from an accredited university. At the start of this study, the school had only been in existence for one full academic year but was quickly growing its professional teaching staff and student population; to date, the school is in its fifth year of operation.

Participating teachers collaborated on developing a reasoning talk by first identifying a task [20] that would promote discourse, create divergent ideas on how best to mathematically approach it, or generate insightful questions that would assist in solving the task. The teachers and the researcher would then observe one teacher facilitate the reasoning talk followed by a shared post-lesson reflection in which evidence of student learning was discussed. Along with the observational field notes, additional data were captured from individual follow-up interviews with the facilitating teacher.

B. Data Analysis

Emphasis was placed on the interpretations and meaning that the researcher and the teachers had towards facilitating reasoning talks. Data were read to identify and confirm the key themes [21] and then themes, and sub-themes, were re-categorized as needed to better reflect the emergent patterns within the data set. However, not all categories were discrete, with some data segments illustrating more than one theme. In these cases, a data segment was coded using multiple codes.

C. Limitations

As with all qualitative studies, the findings from this study are not broadly generalizable. Firstly, the sample size within the selected case was relative low (n = 7) and was drawn from one American curriculum school in the Middle East. Given the uniqueness of the setting and the limited scope of the secondary mathematics context, settings in other locations will likely have somewhat varied findings. Additionally, cultural and/or geographic factors may have unknowable impacts on the findings of this study. Lastly, because of the job-embedded nature of this study, participants may have provided responses or feedback that is aligned to what their employer values thereby changing the nature of their responses. Finally, due to the multi-year duration of the project, it is possible that participants' perceptions will change meaning that data reported in this paper may no longer reflect the current beliefs for participating teachers. Given these limitations it is left to the reader to determine the extent to which this study aligns with their context.

IV. Findings

The initial findings from this project focus on three main aspects of implementing reasoning talks. First, shifting towards a student-centric discourse model to promote reasoning. Despite teachers' beliefs that discourse mattered, their lack of understanding in how to meaningfully facilitate reasoning talks ended up creating more teacher-centric instructional models. Next, teachers expressed that they liked how reasoning talks engaged students but they often struggled with recording students thinking and appropriately probing students to clarify students' thinking. Lastly, teachers' expressed a need to further develop reasoning talks as they came to understand that attention to reasoning as previously much of their efforts had been on helping students obtain correct solutions rather than understanding why their solutions were correct. Each of these three areas is elaborated on below.

A. Shifting to Student-Centric Discourse Structures

Participants in this study frequently talked about the importance of getting students to specifically and logically communicate their understandings to each other as well as to the teacher. However, the process of facilitating a reasoning talk proved to be challenging for some as their discourse style typically aligned with more traditional "initiation-response-evaluation" structures [22], which are highly teacher-centric. Shifting to a discourse structure that relied on students' ideas and ways of thinking were difficult for many to implement. Several teachers commented that they had not experienced a secondary mathematics classroom experience similar to reasoning talks so it was difficult for them to understand how to create such a structure. In essence, they had no previous scheme from which to relate this kind of learning and thus struggled to implement such learning structures in their classroom.

Additionally, and related to the initial struggle of implementing reasoning talks, a student-centric discourse model, teachers frequently reverted from a reasoning talk to an instructional moment. For example, during one reasoning talk students were working on identifying the "most important information" in a task involving ratios. When one student responded "you have to know what one is," the teacher turned to that student and said, "That is a good idea. So you need to know what the unit rate would be. Let's figure that out for a moment." Instead of leveraging the student's idea and asking them to clarify what "one is" actually means, the teacher funneled the student's statement and determined that the idea was good, instead of students generating this understanding;

imposed the correct mathematical vocabulary when it was not clear that the student was referring to unit rates, thereby telling the students what they should be thinking about; and then taking the idea and turning it into an instructional moment about identifying unit rates, instead of probing the student, or the class, to determine why this was mathematically important. Effectively, a student response that was ripe for discussing reasoning became a teacher-led instructional moment. Interactions of this nature were observed in other classrooms as well.

B. Recording and Probing Thinking

Additionally, the act of recording students thinking proved to be more challenging than the teachers initially anticipated. Many teachers expressed an uncertainty in how to capture this thinking. In a reasoning talk, students' ideas will be a mix of statements and symbolic notations. As in the example above, knowing how to record the statement "you have to know what one is" was important as it is grounded in a fundamental understanding of ratios, namely unit rates. In this instance, the teacher did not keep a record of this idea, although they did accept it verbally, despite the initial prompt of identifying the most important information in the task. During the post-lesson reflection that followed this attempt at a reasoning talk, the teacher indicated that "I knew this was a huge piece of information and it was totally where I wanted them to go but I just didn't know how to write this on the board. Should I have used sentences or just keep it abstract like one over x?" This statement exemplifies, and is typical of, the many other responses from participating teachers. They were able to correctly identify key mathematical moments in students' statements but understanding how to accurately capture these to create and preserve a record of student thinking was challenging.

Along with accurately recording thinking, teachers frequently reverted back to questioning techniques that were comfortable to them despite having planned questions that were higher order and more complex before the lesson. As an example, the group of teachers who planned this reasoning talk together anticipated that some students might indicate that a unit rate would be helpful. The questions developed around unit rates included "how do you know a unit rate will be helpful here?" and "what information in the task made you think a unit rate would be appropriate to use?" but the follow-up responses where not questions to elicit reasoning or probe for clarity in thinking but funneled student thinking. Again, such practices are not pedagogically inappropriate but they do not serve the purpose of using reasoning talks, namely helping students learn to focus and build upon their reasoning and the reasoning of their peers.

C. Attending to Reasoning

Lastly, and addressed in some detail above, teachers attention to students' reasoning was not always realized despite careful planning in this area. That is, the purpose of the group pre-planning session was to identify an appropriate task, anticipate students' thinking that mattered in relation to the task, and develop higher-order probing questions that would elicit greater reasoning and sense making, but teachers often reverted back to more direct instruction and Socratic discourse models even though the focus was to be on developing students' reasoning. The purpose of attending to reasoning was clearly understood by all and many articulated that "students need to pay careful attention to details and need to be more clear with what they mean. Sometimes they just say things that don't make any sense." However, when moments arose that would have allowed for greater attention to details and clear articulation of ideas, the participating teachers reverted back to common teacher-centric instructional models because they "felt pressure to cover material" or because they "did not know where [the student] was going with what they were saying. I wanted to make sure they got it and I didn't know if they were going to get there." In essence, the purpose of the reasoning talk shifted from learning to reason to correctly understanding procedures needed to calculate correct solutions.

V. Discussion

The main purpose of reasoning talks is to help students understand mathematical relationships through a shared exchange of students' ideas; to focus on reasoning instead of just procedures. Reasoning talks can help teachers in supporting students' thinking by creating a space for reasoning and sense making in the classroom where deeper, more meaningful connections to mathematics can be made. Huang, Normandia, and Greer [23] state that when students are placed in situations where they are required to articulate methods, strategies, and the rationale for using these techniques to solve a problem, then students are also able to strengthen their conceptual understanding of more complex mathematical relationships. In mathematics, ideas that are required for one particular concept are often useful in other concepts as well [24], [25] and thus learning to reason about contexts, structure, and relationships through student to student discursive interactions makes mathematical concepts more accessible in future learning.

In this study, secondary mathematics teachers were in agreement that students needed more opportunities to focus on reasoning as this would help them develop sense-making skills that the teachers believed many students lacked. Teachers also agreed that engaging more students in discourse would further benefit all students' understanding of mathematics. However, making the transition to a more student-centric discourse model, as can be found in reasoning talks, was a challenge because of understandings in how to facilitate discourse in a model such as reasoning talks, accurately recording and probing student thinking, and attending to reasoning instead of procedures.

In all three challenges, a greater implication arises. Namely, perceived values, such as leveraging student to student discourse as a means of supporting all students, may not be actualized. Furthermore, intended outcomes, as when the teachers reverted to more familiar discourse patterns, may not be enacted [26]. This means that there was a discrepancy in the reality of learning experiences afforded to the students despite deliberate and targeted planning. However, during the post-lesson reflections, teachers became aware of their inhibitions in their practice and openly discussed how they might address them. Because these teachers were making their practice public and then collectively considering the nature of student learning, they were able to focus on how to improve the learning experiences for students. This means that engaging in meaningful reflections with instructional leaders within the school may help them move forward in realizing pedagogical values and learning outcomes. Such supports align with recent trends in schools to create specific positions for academic coaches [27].

Additionally, attending to students' thinking by accurately recording their thinking and probing with higher level questions is important in helping them understand the logical structure of their arguments or where they may have gaps in their reasoning [28]. Students should hold the intellectual authority of their learning, but teachers need to be able to capture this thinking as a means of creating a record of their ideas. Often students' thinking develops and becomes more precise as they talk, thus being able to "see" their thinking becomes an important method for helping students organize their thoughts. Writing only what teachers envision shifts the intellectual authority back to the teacher and limits how students make meaning of their own ideas.

The process of adapting number talks to reasoning talks was well received by teachers and they wanted to continue using them as part of their normal classroom practice. This is encouraging as it demonstrates that the teachers found value in creating or leveraging discourse structures that specifically focus on reasoning. Reasoning talks have potential for developing students' abilities to appropriately reason about mathematics but greater attention should be placed on understanding how best to support secondary mathematics teachers in this transition. Despite the best of intentions, the participating teachers often reverted back to more teacher-centric models of discourse, which restricted students' development of reasoning. Even with recent calls to action to support students' reasoning [8], this study demonstrates that more attention to this area is still needed.

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