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Lattice Boltzmann Simulation of Natural Convection Heat Transfer in an Inclined Open Ended Cavity

M.Jafari, A.Naysari, K.Bodaghi

Abstract**—**In the present study, the lattice Boltzmann Method (LBM) is applied for simulating of Natural Convection in an inclined open ended cavity. The cavity horizontal walls are insulated while the west wall is maintained at a uniform temperature higher than the ambient. Prandtl number is fixed to 0.71 (air) while Rayligh numbers, aspect ratio of the cavity are changed in the range of 10^3 to 10^4 and of 1-4, respectively. The numerical code is validated for the previously results for open ended cavities, and then the results of an inclined open ended cavity for various angles of rotating open ended cavity are presented. Result shows by increasing of aspect ratio, the average Nusselt number on hot wall decreases for all rotation angles. When gravity acceleration direction is opposite of standard gravity direction the convection heat transfer has a manner same as conduction.

Keywords**—**Lattice Boltzmann Method- Open Ended Cavity- Natural Convection- Inclined Cavity

I. Introduction

THE Lattice Boltzmann Method in last decade has been used in the wide range of fluid flow and heat transfer problems in Mechanical engineering. Some of main reasons for using Lattice Boltzmann Method in most of researches are: easy implementation, ability to simulate multiphase flow and complex geometries, ability to use parallel computations and finally no need to use time-consuming Poisson equation for pressure. There are many researches using Lattice Boltzmann Method (LBM). [1-6] Natural convection in a square cavity and its fluid flow is a classical problem in Mechanical engineering. Mohamad et al. [7] presented a detailed analysis of Natural convection problem to use Lattice Boltzmann Method (LBM), they demonstrated the abilities of the LBM in simulating Natural convection. Open cavities are 2-D cavity that has an open side. These kind of cavities have special physics in open side because of outgoing of flow from this side. Some research papers have been published on studies of buoyant flows and their heat transfer in open cavities [8–10]. Javam et al [11], analysis stability of stratified natural convection flow in open cavities. Mohamad[12,13] et al presented natural convection in an open ended cavity and slots they analyzed the effect of aspect ratio of cavity on heat transfer rate. They presented a good procedure for simulating open boundaries in Lattice Boltzmann Method (LBM) [13]

M.Jafari is with the Faculty of Mechanical Engineering, Babol University of Technology, Babol, Mazandaran, Islamic Republic of Iran , P.O.Box 484
(Corresponding author:E-mail: M.Jafari177@ gmail.com.).
A.Naysari is with the Faculty of Mechanical Engineering, Khoramabad Azad University ,Khoramabad, Lorestan, Islamic Republic of Iran , (E-mail: Naysari.amir@gmail.com).
K.Bodaghi is with the Faculty of Medical Engineering, Dezful Azad University, Dezful, Khozestan, Islamic Republic of Iran, (E-mail: Kiarash_bodaghi@yahoo.com).

II. Lattice Boltzmann Method

The thermal LB model utilizes two distribution functions, f and g, for the flow and temperature fields, respectively. In this approach the fluid domain is discredited in uniform Cartesian cells. Each cell holds a fixed number of distribution functions. For this work the D2Q9 model has been used. This model is shown in Fig.1 and values of $w_0 = 4/9$ for $|c_0| = 0$, $w_{1-4} = 1/9$ for $|c_{1-4}| = 1$ and $w_{5-9} = 1/36$ for $|c_{5-9}| = \sqrt{2}$ are assigned.

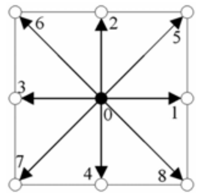

Fig.1

The f and g are calculated by solving the Lattice Boltzmann equation (LBE). By using BGK model, the general form of lattice Boltzmann equation with an added force term can be written as:

For the flow field:

$$f_i(x+c_i\Delta t, t+\Delta t) = f_i(x,t) + \frac{\Delta t}{\tau_v}[f_i^{eq}(x,t) - f_i(x,t)] + \Delta t c_i F_k \quad (1)$$

For the temperature field:

$$g_i(x+c_i\Delta t, t+\Delta t) = g_i(x,t) + \frac{\Delta t}{\tau_D}[g_i^{eq}(x,t) - g_i(x,t)] \quad (2)$$

Where Δt denotes lattice time step, c_k is the discrete lattice velocity in direction k , F_k is the external force in direction of lattice velocity, τ_v and τ_D denotes the lattice relaxation time for the flow and temperature fields. The kinetic viscosity

υ and the thermal diffusivity α, are defined in terms of their respective relaxation times, i.e. $\upsilon = c_s^2(\tau_v - 1/2)$ and $\alpha = c_s^2(\tau_D - 1/2)$, respectively. The local equilibrium distribution for flow and temperature fields is as follows respectively.

$$f_i^{eq} = w_i \rho[1 + \frac{c_i.u}{c_s^2} + \frac{1}{2}\frac{(c_i.u)^2}{c_s^4} - \frac{1}{2}\frac{u^2}{c_s^2}] \quad (3)$$

$$g_i^{eq} = w_i T[1 + \frac{c_i.u}{c_s^2}] \quad (4)$$

Where w_k is a weighting factor, ρ is the lattice fluid density. To model buoyancy force, the force term in the Eq. (1) need to be assumed as below in needed direction:

$$F = 3 w_i g_y \beta T \quad (5)$$

In lattice Boltzmann Method Eqs. (1) and (2) are solved in two important step that are called collision and streaming step. Collision step is as follows for flow field and temperature field respectively:

$$\tilde{f}_\alpha(x, t+\Delta t) = -\frac{1}{\tau_m}\left[f_\alpha(x,t) - f_\alpha^{eq}(x,t)\right] - f_\alpha(x,t) \quad (6)$$

$$\tilde{g}_\alpha(x, t+\Delta t) = -\frac{1}{\tau_t}\left[g_\alpha(x,t) - g_\alpha^{eq}(x,t)\right] - g_\alpha(x,t) \quad (7)$$

Streaming step can be written as follows:

$$f_\alpha(x + e_\alpha \Delta t, t+\Delta t) = \tilde{f}_\alpha(x, t+\Delta t) \quad (8)$$

$$g_\alpha(x + e_\alpha \Delta t, t+\Delta t) = \tilde{g}_\alpha(x, t+\Delta t) \quad (9)$$

Where $\tilde{f}_\alpha$ and $\tilde{g}_\alpha$ denotes the post-collision distribution function. Macroscopic variable can be calculated in terms of these variables, with the following formula.

Flow density**:**

$$\rho = \sum_i f_i$$

Momentum:

$$\rho u_j = \sum_i f_i c_{ij} \quad (10)$$

Temperature:

$$T = \sum_i g_i$$

III. PROBLEM DESCRIPTION

In this work the problem of Natural convection heat transfer in an inclined open ended cavity is simulated. The cavity has two horizontal insulated walls, a west wall with constant temperature and an open east side. The temperature of west wall is equal to unity that is more than the ambient zero temperature. One point must be noticed that this temperature is dimensionless in Lattice scale. In the present study to simulate Natural convection in inclined cavity, the gravity acceleration (g) is rotated counter clockwise. The negative values of rotating angel mean rotation is clockwise.

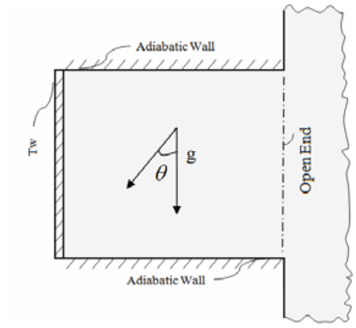

Fig.2

The rotation of g and the effect of it on buoyant force causes to have different Natural convection characteristics in the open ended cavity. For simulating of Natural convection in this geometry, the Rayligh number (Ra) and rotating angle (θ) are applied in the range of 10^3-10^5 and of -90 to 90 respectively. The effect of these parameters on the fluid flow and heat transfer rate are investigated when aspect ratio (A) of cavity change from 1 to 4. To validate the numerical simulation, the results for Natural convection flow in an open ended cavity with two vertical adiabatic walls and one horizontal hot wall are compared with those presented by Mohammad et al [13]. Streamline and temperature contours are presented in Fig.2 This comparison reveals good agreements between the present result and those published by Mohammad et al then the average Nusselt numbers (NU_{ave}) for different aspect ratio are compared with their results. (Figs.3 and 4.).

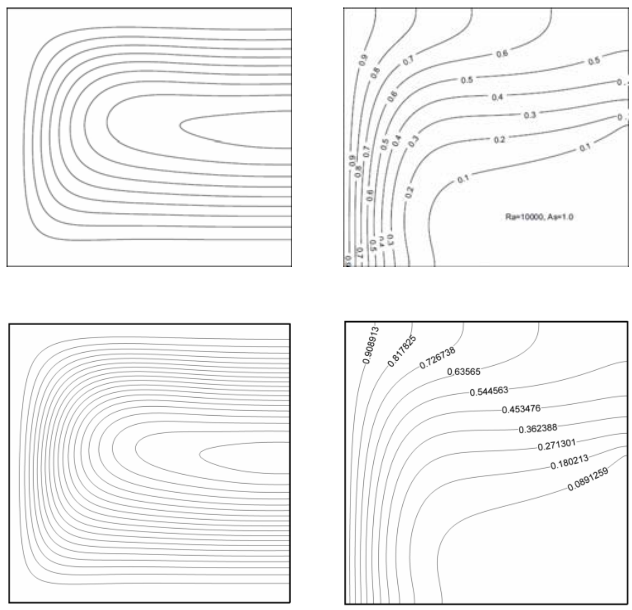

Fig.3 Streamline and Temperature contours for Ra=10000,A=1, Pr=0.71. Top: Mohammad[13] bottom: Present study

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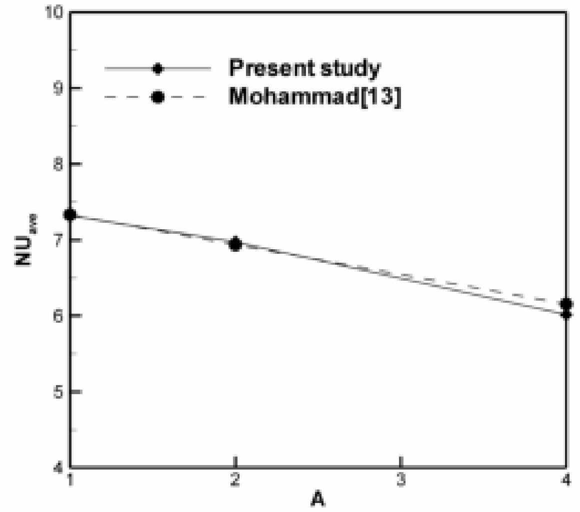


Fig.4 Nusselt local on west wall for different aspect ratio

To simulate the open ended boundary, applied procedure is same as those presented by Mohamad [13]In applied numerical code to model various aspect ratios of cavity 100 lattice nodes in y direction is used and number of nodes in x direction is of 100 to 400 for aspect ratio of 1 to 4 respectively.Nusselt number is defined on the west wall of the cavity.

$$NU = -\frac{\partial T}{\partial x} \tag{11}$$

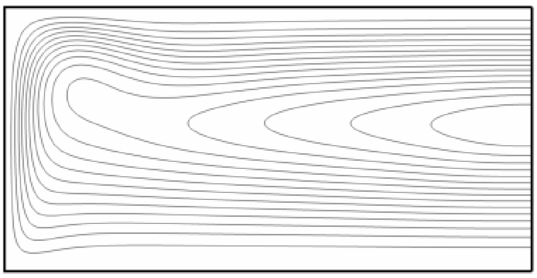
Average Nusselt number is calculated by integrating eq. (11) along the length of the west wall and dividing by number of lattices along the height.

$$NU_{ave} = \frac{1}{L}\int_{0}^{L} NU \ dx \tag{12}$$

Where, L is the length of west wall.

IV. Results and Discussion

The problem of Natural convection heat transfer in an inclined open cavity was solved for different Rayligh numbers, different angles and aspect ratios when Prantdl number (Pr) is fixed to 0.71. Streamline and Temperature contours are plotted and average Nusselt number on closed west wall is investigated in various conditions.



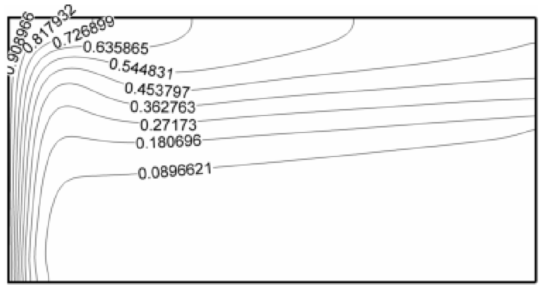


Fig.5 Streamline and Temperature contour for A=2,Ra=10^5,Pr=0.71, $\theta = 0$

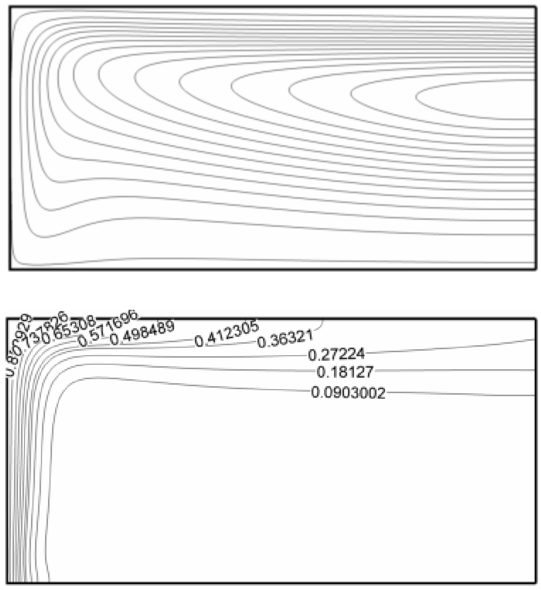


Fig.6 Streamline and Temperature contour for A=2,Ra=10^5,Pr=0.71, $\theta = 45$

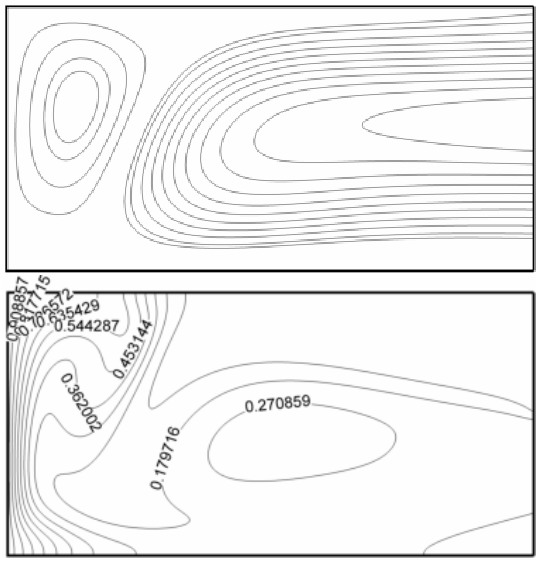


Fig.7 Streamline and Temperature contour for A=2,Ra=10^5,Pr=0.71, $\theta = 90$

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At Fig.(5-7) Streamline and Temperature contours are plotted for different rotation angle at $Ra=10^5$ and A=2 , the effect of rotation on Streamline and Temperature contours can be seen obviously.

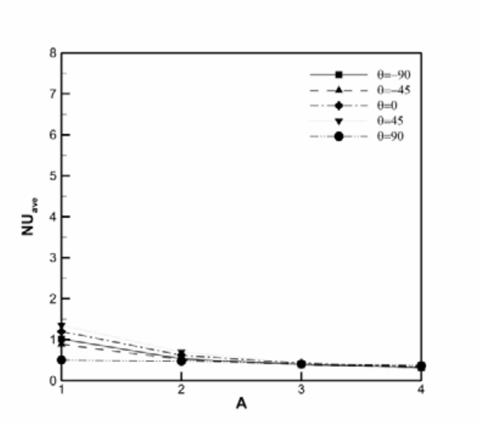

Fig.8 Average Nusselt for $Ra=10^3$,Pr=0.71

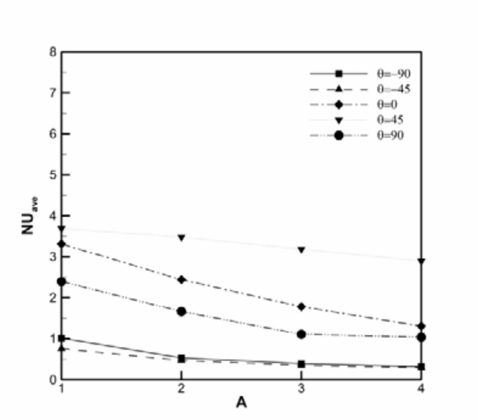

Fig.9 Average Nusselt for $Ra=10^4$,Pr=0.71

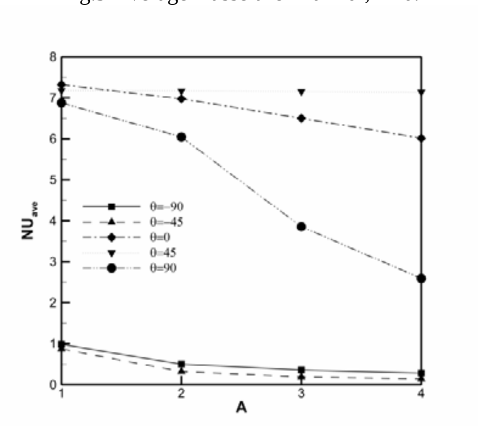

Fig.10 Average Nusselt for $Re=10^5$,Pr=0.71

Fig.8 shows, aspect ratio and rotation angle have low effect on average Nusselt in the low Rayligh number ($Ra=10^3$.).By increasing of Rayligh number the positive rotation angle become a more important parameter on average Nusselt number but negative the effect of rotation angle does not change clearly.(Figs.9 and 10.)Increasing of aspect ratio has more important role on heat transfer rate for $\theta = 0$ at comparison of other rotation angles.(Fig10)

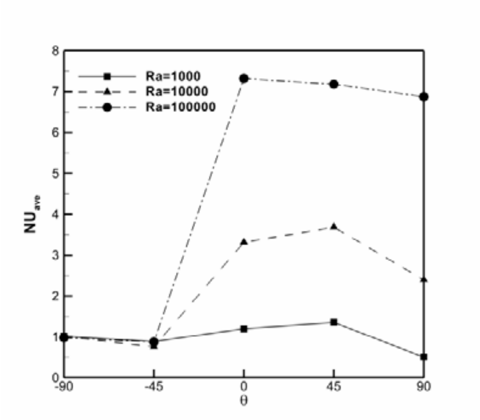

Fig.11 Average Nusselt for A=1,Pr=0.71

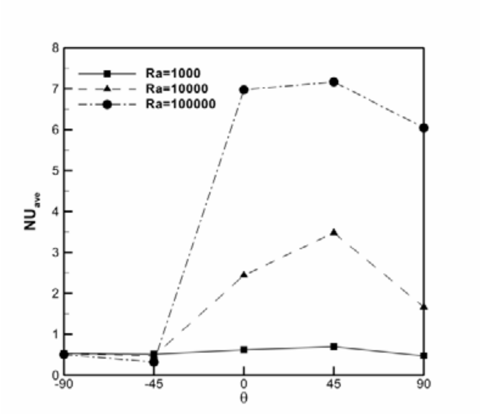

Fig.12 Average Nusselt for A=2,Pr=0.71

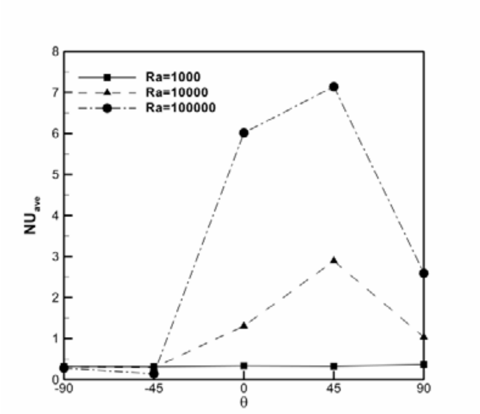

Fig.13 Average Nusselt for A=4,Pr=0.71

The rate of increasing average Nusselt number aspect to the increasing of Rayligh number for $\theta = 45$ is more than other angles.(see Figs.11-13)At $\theta = -90$ the convection heat transfer will change to a condition same as Conduction heat transfer because of the g direction, for this condition Rayligh number will lose its effect on average Nusselt numbers.

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Conduction heat transfer manner means the fluid heat transfer as same as those occur in solids.

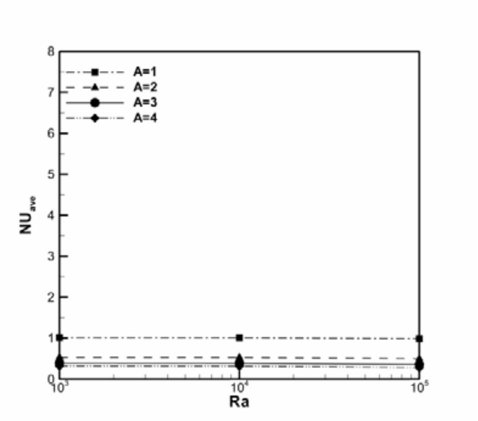

Fig.14 Average Nusselt for $\theta = -90$,Pr=0.71

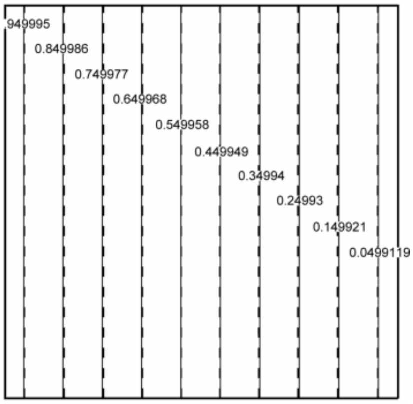

Fig.15 Average Nusselt for A=1, $\theta = -90$,Pr=0.71

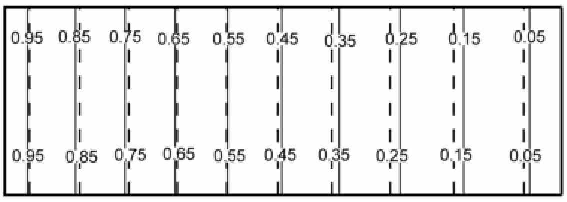

Fig.16 Average Nusselt for A=3, $\theta = -90$,Pr=0.71

In Figs.15 and 16 the conduction heat transfer manner can be seen clearly.

NUMENCLATURE

c_i	Discrete lattice velocity in direction i
c_s	Speed of sound in Lattice scale
c_p	specific heat at constant pressure (kJ/kg K)
F	External force in direction of lattice velocity
f_i^{eq}	Equilibrium distribution of flow field.
$\tilde{f}_\alpha$	Post-Collision Equilibrium distribution of flow field.
g	Acceleration due to gravity (m/s2)
g_i^{eq}	Equilibrium distribution of Temperature field
$\tilde{g}_\alpha$	Post-Collision Equilibrium distribution of flow field.
i	Discrete lattice direction
k	Thermal conductivity (W/m K)
L	Length of hot wall
NU	Nusselt number
NU_{ave}	Average Nusselt number
Pr	Prandtl number (ν / α)
Ra	Rayligh Number
w_i	Weighting factor in direction i

Greek symbols

β	Thermal expansion coefficient $(1/k)$
θ	Angle of rotation
Δt	Lattice time step
ρ	Density (kg/m3)
τ_v, τ_D	Relaxation time for fluid and Temperature field

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