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Large Amplitude Free Vibration of a Very Sag Marine Cable

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***Abstract*—**This paper focuses on a variational formulation of large amplitude free vibration behavior of a very sag marine cable. In the static equilibrium state, the marine cable has a very large sag configuration. In the motion state, the marine cable is assumed to vibrate in in-plane motion with large amplitude from the static equilibrium position. The total virtual work-energy of the marine cable at the dynamic state is formulated which involves the virtual strain energy due to axial deformation, the virtual work done by effective weight, and the inertia forces. The equations of motion for the large amplitude free vibration of marine cable are obtained by taking into account the difference between the Euler's equation in the static state and the displaced state. Based on the Galerkin finite element procedure, the linear and nonlinear stiffness matrices, and mass matrices of the marine cable are obtained and the eigenvalue problem is solved. The natural frequency spectrum and the large amplitude free vibration behavior of marine cable are presented.

***Keywords*—**Axial deformation, free vibration, Galerkin Finite Element Method, large amplitude, variational method.

I. Introduction

MARINE cables refer to the long, slender and flexible members used for connecting the anchor point at the sea floor and the floating platform at the sea surface. The static and dynamic analysis of the marine cable due to its self-weight and environmental load is an interesting topic in engineering in order to understand the behavior such as static equilibrium configuration, free and forced vibrations. In literature, research works related to the marine cables have been done extensively. The linear theory of a free vibration of a suspended cable with the support at the same elevation is developed by Irvine and Caughey [1]. The asymptotic equations for the natural frequencies and mode shapes of the inclined cable are derived by Triantafyllou and Grinfogel [2]. The effect of axial deformation on the natural frequencies for the marine cable was studied by Chucheepsakul and Huang [3]. The model formulation is developed base on the virtual work-energy functional of marine cables. Chucheepsakul and Srinil [4] developed the model formulation to analyses the three-dimensional vibration behaviors of an inclined extensible marine cable using virtual work-energy functional, the coupled equations of motions obtained from the difference between Euler's equation and equilibrium equation. Srinil et al. [5] investigated the nonlinear characteristics of the large amplitude-free vibrations of inclined sagged elastic cable, based on a three-dimensional model formulation and the axial deformation effect is taken into account. Recently, the natural frequencies and mode shape of a very large sag cable have been proposed by Phanyasahachart et al. [6], [7]. The model formulation developed based on the variational formulation involves with the axial deformation strain energy, the virtual work done due to self-weight and inertia force. The equation of motion was addressed; the finite element method was used to obtain the numerical solution. The purpose of this study is to extend the authors' research work on large amplitude free vibration. The model formulation is firstly developed and appeared in literature. The interesting features of nonlinear free vibration behaviors of marine cables are presented and highlighted.

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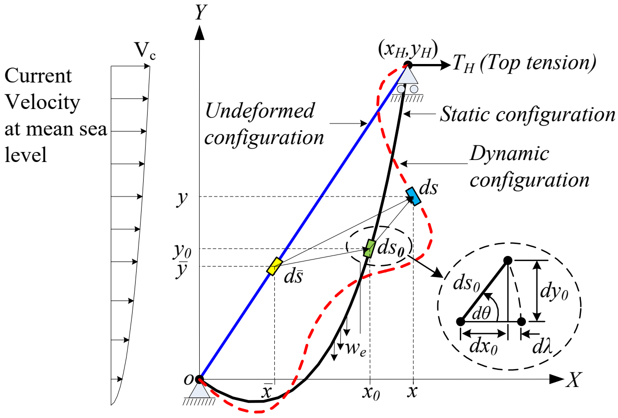


Fig. 1 Configuration of the marine cable in three states

II. Variational Model Formulation

The configuration of the very sag marine cable in three states is illustrated in Fig. 1, a variational formulation of the mechanical behavior of the marine cable is derived based on the work-energy principle in two-dimensional the Cartesian coordinate system. The marine cable is modeled between the hinged support at one end and the free-sliding support at the other end. In static analysis, the external virtual work done composed of the top horizontal tension force, effective weight, and the current drag force. For dynamic analysis, the axial deformations are taken into account for internal strain energy while the external virtual work done composed of the effective weight, and inertia force. The arc-length coordinate is used as an independent variable.

From the geometrical configuration of the marine cable illustrated in Fig. 1, the following relations can be obtained.

$$sin\theta = \frac{dy_0}{ds_0} = \frac{y_0'}{s_0'} \quad (1a)$$

$$cos\theta = \frac{dx_0}{ds_0} = \frac{x_0'}{s_0'} \quad (1b)$$

$$s_0'^2 = x_0'^2 + y_0'^2 \quad (1c)$$

where the prime symbol $(')$ is used to represent the derivative with respect to the unstrained marine cable arc-length $\bar{s}$, subscript (s_0) defined the condition in equilibrium state, the angle (θ) is measured between the horizontal and the marine cable arc length. Differentiating (1a) with respect to the arc-length parameter, s_0 gives the curvature (κ) of the marine cable element, one obtains

$$\kappa = \frac{d\theta}{ds_0} = \frac{y_0''}{\left(1-y_0'^2\right)^{\frac{1}{2}}} \quad (2)$$

For the extensible marine cable, the total axial strain (ε_0) at the equilibrium state can be expressed by

$$\varepsilon_0 = \frac{ds_0 - d\bar{s}}{d\bar{s}} \quad (3)$$

The arc length of the marine cable at static equilibrium state, ds_0 can be defined in terms of the Cartesian coordinate components (x_0, y_0) by

$$ds_0 = (1+\varepsilon_0)d\bar{s} = \sqrt{x_0'^2 + y_0'^2}\,d\bar{s} \quad (4)$$

where $d\bar{s}$ is the arc length at undeformed state, with (1c) and (4), one obtains:

$$x_0' = \sqrt{(1+\varepsilon_0)^2 - y_0'^2} \quad (5)$$

The infinitesimal arc-length ds_0 can be determined using the geometric relation of the marine cable in equilibrium state as shown in Fig 1. Thus, it is given as

$$ds_0^2 = (dx_0 - d\lambda)^2 + dy_0^2 \quad (6)$$

With (4) and (5), one can obtain:

$$d\lambda = \left\{(1+\varepsilon_0) - \sqrt{(1+\varepsilon_0)^2 - y_0'^2}\right\}d\bar{s} \quad (7)$$

A. Virtual Work Due to Top Horizontal Tension

The total displacement defined in (7) is used to formulate the virtual work done due to top horizontal tension applied to the free sliding roller support as

$$W_T = -\int_0^{s_t} T_H\left\{(1+\varepsilon_0) - \sqrt{(1+\varepsilon_0)^2 - y_0'^2}\right\}d\bar{s} \quad (8)$$

The virtual work of the top horizontal tension can be expressed by

$$\delta W_T = -\int_0^{s_t} T_H \frac{y_0'}{\sqrt{(1+\varepsilon_0)^2 - y_0'^2}}\delta y_0' d\bar{s} \quad (9)$$

B. Virtual Work Due to Effective Weight

The virtual work of effective weight for the marine cable is

$$\delta W_w = -\int_0^{s_t} w_e \delta y_0 d\bar{s} \quad (10)$$

while the effective weight of the marine cable (w_e) can be defined by

$$w_e = (\rho_c A_c - \rho_e A_e)g \quad (11)$$

where ρ_c and ρ_e are the densities of the cable and external fluid, respectively. A_c and A_e are the cross-sectional areas of the marine cable and outside diameter, respectively, and g is the gravitational acceleration.

C. Virtual Work Due to the Current Drag Force

The current drag force on the marine cable is composed of forces acting both in the normal and tangential directions with respect to the neutral axis, the virtual work of the current drag force can be expressed as

$$\delta W_H = -\int_0^{s_t}\left(f_{Hty} - f_{Hny}\right)\delta y_0 d\bar{s} \quad (12)$$

The current drag force in the normal and tangential directions are given by

$$f_{Hn} = \frac{1}{2}\rho_e D_e C_{Dn}|V_{Hn}|V_{Hn} \quad (13)$$

and

$$f_{Ht} = \frac{1}{2}\rho_e \pi D_e C_{Dt}|V_{Ht}|V_{Ht} \quad (14)$$

where D_e is the diameter of the external fluid, $C_{Dn,}$ and C_{Dt} are the normal and tangential drag coefficients, and V_{Hn} and V_{Ht} are the current velocities in normal and tangential directions, respectively.

The total virtual work in static equilibrium can be expressed as:

$$\delta\pi = \int_0^{s_t}\left\{\begin{matrix} T_H \frac{y_0'}{\sqrt{(1+\varepsilon_0)^2 - y_0'^2}}\delta y_0' \\ +w_e\delta y_0 + \left(f_{Hny} - f_{Hty}\right)\delta y_0 \end{matrix}\right\}d\bar{s} \quad (15)$$

Schematic of static equilibrium and dynamic configuration of the large-sag extensible marine cable and the dynamic displacement from static equilibrium position to dynamic displaced position in u and v of the Cartesian coordinate system is illustrated in Fig. 2.

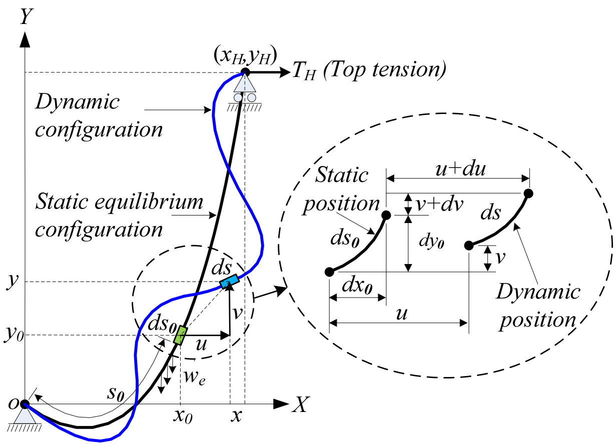


Fig. 2 Schematic of static and dynamic configuration of marine cable

The arc-length at the stretched state (ds) can be defined by

$$ds = \sqrt{(x_0' + u')^2 + (y_0' + v')^2}d\bar{s} \tag{16}$$

The total strain at displaced state (ε) described in total Lagrange descriptor can be expressed by

$$\varepsilon = \frac{ds - d\bar{s}}{d\bar{s}} = \frac{ds}{d\bar{s}} - 1 = \sqrt{(x_0' + u')^2 + (y_0' + v')^2} - 1 \tag{17}$$

and its derivative is

$$\delta\varepsilon = \frac{(x_0'+u')\delta u' + (y_0'+v')\delta v'}{\sqrt{(x_0'+u')^2 + (y_0'+v')^2}} \tag{18}$$

D. Virtual Strain Energy Due to Axial Deformation

The variation of the axial strain energy for the marine cable due to the stretching can be expressed by [10]

$$\delta U_a = \int_o^{S_t} EA\varepsilon(\delta\varepsilon)d\bar{s} \tag{19}$$

Substitution of (17) and (18) into (19) yields

$$\delta U_a = \int_o^{S_t} EA\left(\sqrt{(x_0' + u')^2 + (y_0' + v')^2} - 1\right)\frac{(x_0'+u')\delta u' + (y_0'+v')\delta v'}{\sqrt{(x_0'+u')^2 + (y_0'+v')^2}}d\bar{s} \tag{20}$$

The dynamic updated Green's strain (γ_d) given by Chucheepsakul et al. [8] is

$$\gamma_d = \frac{x_0'u' + y_0'v'}{x_0'^2 + y_0'^2} + \frac{1}{2}\frac{\left(u'^2 + v'^2\right)}{\left(x_0'^2 + y_0'^2\right)} = \frac{1}{(1+\varepsilon_0)^2}\left(x_0'u' + y_0'v' + \frac{1}{2}u'^2 + \frac{1}{2}v'^2\right) \tag{21}$$

With the approximation using the binomial series, and the higher order term is neglected for linearization purpose,

$$\frac{1}{\sqrt{1+2\gamma_d}} \approx 1 - \frac{1}{2}(2\gamma_d) + \frac{1}{2!}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(2\gamma_d)^2 + \approx 1 - \gamma_d \tag{22}$$

The arc-length at the stretched state can be simplified using (22):

$$\sqrt{(x_0' + u')^2 + (y_0' + v')^2} = \sqrt{x_0'^2 + y_0'^2 + 2(x_0'u' + y_0'v') + u'^2 + v'^2} = \sqrt{1 + 2\gamma_d}(1 + \varepsilon_0) \tag{23}$$

The variation of the axial strain energy can be simplified by using (22) and (23):

$$\delta U_a = \int_o^{S_t} \frac{EA}{1+\varepsilon_0}(\varepsilon_0 + \gamma_d)[(x_0' + u')\delta u' + (y_0' + v')\delta v']d\bar{s} \tag{24}$$

Equation (24) can be rearranged by using (21) and the expression of tension in equilibrium state $T = EA\varepsilon_0$ becomes

$$\delta U_a = \int_o^{S_t}\left[\begin{array}{c}\frac{T}{1+\varepsilon_0} \\ + \frac{EA}{(1+\varepsilon_0)^3}\begin{pmatrix} x_0'u' + y_0'v' \\ +\frac{1}{2}u'^2 + \frac{1}{2}v'^2\end{pmatrix}\end{array}\right](x_0' + u')\delta u' d\bar{s} + \int_o^{S_t}\left[\begin{array}{c}\frac{T}{1+\varepsilon_0} \\ + \frac{EA}{(1+\varepsilon_0)^3}\begin{pmatrix} x_0'u' + y_0'v' \\ +\frac{1}{2}u'^2 + \frac{1}{2}v'^2\end{pmatrix}\end{array}\right](y_0' + v')\delta v' d\bar{s} \tag{25}$$

E. Virtual Work Due to Effective Weight and Inertia Force

The virtual work done due to effective weight and inertia forces for the marine cable is expressed by [10].

$$\delta W_a = -\int_0^{S_t} w_e \left\{\begin{array}{c}\frac{\sqrt{x_0'^2 + y_0'^2}}{g(1+\varepsilon_0)}\ddot{u}\delta u \\ + \left(\frac{\sqrt{x_0'^2 + y_0'^2}}{1+\varepsilon_0} + \frac{\sqrt{x_0'^2 + y_0'^2}}{g(1+\varepsilon_0)}\ddot{v}\right)\delta v\end{array}\right\} d\bar{s} \tag{26}$$

where w_e is the effective weight; $w_e = (\rho_c A_c + \rho_e A_e C_a)g$ and C_a is the added mass coefficient, $\ddot{u}$ and $\ddot{v}$ are the acceleration in x and y directions, respectively.

III. Equations of Motion

The total virtual work-energy is written as

$$\delta\Pi = \delta U_a - \delta W_a = 0 \tag{27}$$

Substitution of (24) and (26) to (27) yields the expression of total virtual work as follows:

$$\delta\Pi = \int_0^{S_t}\left\{\begin{array}{c}\left[\begin{array}{c}\frac{T}{1+\varepsilon_0} \\ + \frac{EA}{(1+\varepsilon_0)^3}\begin{pmatrix} x_0'u' + y_0'v' \\ +\frac{1}{2}u'^2 + \frac{1}{2}v'^2\end{pmatrix}\end{array}\right](x_0' + u')\delta u' \\ + w_e\frac{\sqrt{x_0'^2 + y_0'^2}}{g(1+\varepsilon_0)}\ddot{u}\delta u\end{array}\right\} d\bar{s} +$$

$$\int_0^{S_t}\left\{\begin{aligned}&\left[\begin{aligned}&\frac{T}{1+\varepsilon_0}\\&+\frac{EA}{(1+\varepsilon_0)^3}\begin{pmatrix}x_0'u'+y_0'v'\\+\frac{1}{2}u'^2+\frac{1}{2}v'^2\end{pmatrix}\end{aligned}\right](y_0'+v')\delta v'\\&+w_e\left(\frac{\sqrt{x_0'^2+y_0'^2}}{1+\varepsilon_0}+\frac{\sqrt{x_0'^2+y_0'^2}}{g(1+\varepsilon_0)}\ddot{v}\right)\delta v\end{aligned}\right\}d\bar{s} \quad (28)$$

In order to perform the equation of motion, we applied integration by part twice and considering the marine cable for static equilibrium, $\delta\pi = 0$ and $u = v = u' = v' = u'' = v'' = 0$. The Euler equation in (28) in u and v directions is reduced to

$$-\frac{T}{1+\varepsilon_0}(x_0')' = 0 \quad (29)$$

and

$$-\frac{T}{1+\varepsilon_0}(y_0')' = 0 \quad (30)$$

For the marine cable in motion, $u \neq 0, v \neq 0, u' \neq 0, v' \neq 0, u'' \neq 0, v'' \neq 0$. The Euler equation in u and v directions becomes

$$\begin{gathered}-\frac{T}{1+\varepsilon_0}(x_0'+u')' -\\ \frac{EA}{(1+\varepsilon_0)^3}\begin{pmatrix}x_0'^2u'+x_0'y_0'v'+\frac{1}{2}x_0'u'^2+\frac{1}{2}x_0'v'^2\\+x_0'u'^2+y_0'u'v'+\frac{1}{2}u'^3+\frac{1}{2}u'v'^2\end{pmatrix}' +\\ w_e\frac{\sqrt{x_0'^2+y_0'^2}}{g(1+\varepsilon_0)}\ddot{u} = 0\end{gathered} \quad (31)$$

and

$$\begin{gathered}-\frac{T}{1+\varepsilon_0}(y_0'+v')' -\\ \frac{EA}{(1+\varepsilon_0)^3}\begin{pmatrix}x_0'y_0'u'+y_0'^2v'+\frac{1}{2}y_0'u'^2+\frac{1}{2}y_0'v'^2\\+x_0'u'v'+y_0'v'v'+\frac{1}{2}u'u'v'+\frac{1}{2}v'^3\end{pmatrix}' +\\ w_e\frac{\sqrt{x_0'^2+y_0'^2}}{1+\varepsilon_0}+w_e\frac{\sqrt{x_0'^2+y_0'^2}}{g(1+\varepsilon_0)}\ddot{v} = 0\end{gathered} \quad (32)$$

Subtracting (29) from (31) and subtracting (30) from (32), we obtain the equations of motion for large sag extensible marine cable in u and v directions, respectively. This can be written as

$$m_u\ddot{u} + f_1(u'', v'') = 0 \quad (33)$$

and

$$m_v\ddot{v} + g_1(u'', v'') = 0 \quad (34)$$

The linear and nonlinear stiffness matrix coefficients in (33) and (34) can be expressed as:

$$f_1(u'',v'') = \frac{Tu''}{1+\varepsilon_0}+\frac{EA}{(1+\varepsilon_0)^3}\begin{pmatrix}x_0'^2u''+x_0'y_0'v''\\+(3x_0'u'+y_0'v')u''\\+(x_0'v'+y_0'u')v''\\+\left(\frac{3}{2}u'^2+\frac{1}{2}v'^2\right)u''+u'v'v''\end{pmatrix} \quad (35)$$

$$g_1(u'',v'') = \frac{Tv''}{1+\varepsilon_0}+\frac{EA}{(1+\varepsilon_0)^3}\begin{pmatrix}x_0'y_0'u''+y_0'^2v''\\+(x_0'v'+y_0'u')u''\\+(3y_0'v'+x_0'u')v''\\+u'v'u''+\left(\frac{1}{2}u'^2+\frac{3}{2}v'^2\right)v''\end{pmatrix} \quad (36)$$

A. Linear Free Vibration

The linear stiffness matrix can be arranged in the matrix form as:

$$\begin{bmatrix}m_u & 0\\ 0 & m_v\end{bmatrix}\begin{Bmatrix}\ddot{u}\\ \ddot{v}\end{Bmatrix}+\begin{bmatrix}k_{uu} & k_{uv}\\ k_{vu} & k_{vv}\end{bmatrix}\begin{Bmatrix}u''\\ v''\end{Bmatrix}=\{\mathbf{0}\} \quad (37)$$

where the mass of the marine cable in u and v directions is defined by

$$m_u = m_v = w_e\frac{\sqrt{x_0'^2+y_0'^2}}{g(1+\varepsilon_0)} \quad (38)$$

The linear axial stiffness matrix of the second order derivative is

$$\begin{bmatrix}k_{uu} & k_{uv}\\ k_{vu} & k_{vv}\end{bmatrix}=\begin{bmatrix}\frac{T}{1+\varepsilon_0}+\frac{EAx_0'^2}{(1+\varepsilon_0)^3} & \frac{EAx_0'y_0'}{(1+\varepsilon_0)^3}\\ \frac{EAx_0'y_0'}{(1+\varepsilon_0)^3} & \frac{T}{1+\varepsilon_0}+\frac{EAy_0'^2}{(1+\varepsilon_0)^3}\end{bmatrix} \quad (39)$$

B. Nonlinear Free Vibration

The nonlinear free vibration of the very sag extensible marine cable can be expressed by

$$\begin{bmatrix}m_u & 0\\ 0 & m_v\end{bmatrix}\begin{Bmatrix}\ddot{u}\\ \ddot{v}\end{Bmatrix}+\left(\begin{bmatrix}k_{uu} & k_{uv}\\ k_{vu} & k_{vv}\end{bmatrix}+\begin{bmatrix}k_{uu} & k_{uv}\\ k_{vu} & k_{vv}\end{bmatrix}_{NL}\right)\begin{Bmatrix}u''\\ v''\end{Bmatrix}=\{\mathbf{0}\} \quad (40)$$

The first order nonlinear axial stiffness matrix is

$$\begin{bmatrix}k_{uu} & k_{uv}\\ k_{vu} & k_{vv}\end{bmatrix}_{NL1}=\frac{EA}{(1+\varepsilon_0)^3}\begin{bmatrix}3x_0'u'+y_0'v' & x_0'v'+y_0'u'\\ x_0'v'+y_0'u' & 3y_0'v'+x_0'u'\end{bmatrix} \quad (41)$$

The second order nonlinear axial stiffness matrix is

$$\begin{bmatrix}k_{uu} & k_{uv}\\ k_{vu} & k_{vv}\end{bmatrix}_{NL2}=\frac{EA}{(1+\varepsilon_0)^3}\begin{bmatrix}\frac{3}{2}u'^2+\frac{1}{2}v'^2 & u'v'\\ u'v' & \frac{3}{2}v'^2+\frac{1}{2}u'^2\end{bmatrix} \quad (42)$$

IV. Finite Element Method

The static equilibrium configuration is obtained using finite element method and Newton-Raphson iterative procedure in previous work by Punjarat and Chucheepsakul, [9], [10]. The equation of motion is solved using the method of the Galerkin finite element method by Cook et al. [11]. The displacement components vector in Cartesian coordinate is written as.

$$\{\boldsymbol{u}\} = \{u \quad v\}^T \approx [\boldsymbol{N}]\{\boldsymbol{d}\} \tag{43}$$

where the cubic polynomial shape function matrix, $[\boldsymbol{N}]$ at the displaced state is

$$[\boldsymbol{N}] = \begin{bmatrix} N_1 & N_2 & 0 & 0 & N_3 & N_4 & 0 & 0 \\ 0 & 0 & N_1 & N_2 & 0 & 0 & N_3 & N_4 \end{bmatrix} \tag{44}$$

and the generalized coordinate of nodal displacement is

$$\{\boldsymbol{d}\} = \{u_1 \quad u'_1 \quad v_1 \quad v'_1 \quad u_2 \quad u'_2 \quad v_2 \quad v'_2\}^T \tag{45}$$

A. Linear Free Vibration Solution

Equation (37) can be written in the form of matrix following the Galerkin finite element method as:

$$\sum_{j=1}^{Nelem}\left(\int_0^l [N]^T \begin{bmatrix} m_u & 0 \\ 0 & m_v \end{bmatrix} [N] ds\{\ddot{d}\} + \int_0^l [N']^T \begin{bmatrix} k_{uu} & k_{uv} \\ k_{vu} & k_{vv} \end{bmatrix} [N'] ds\{d\}\right) = \{0\} \tag{46}$$

where j is the element number and $[\boldsymbol{N'}]$ and $[\boldsymbol{N''}]$ is the derivative of cubic polynomial shape function.

The finite element equation of the global system for free vibration can be expressed by

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{D}}\} + [\boldsymbol{K_L}]\{\boldsymbol{D}\} = \{\boldsymbol{0}\} \tag{47}$$

where $\{\ddot{\boldsymbol{D}}\}$ and $\{\boldsymbol{D}\}$ are the acceleration and displacement vectors, respectively can be obtained by assembling the element acceleration and displacements, therefore

$$\{\boldsymbol{D}\} = \sum_{j=1}^{nelem}\{\boldsymbol{d}\} \tag{48a}$$

and

$$\{\ddot{\boldsymbol{D}}\} = \sum_{j=1}^{nelem}\{\ddot{\boldsymbol{d}}\} \tag{48b}$$

The global mass matrices $[\boldsymbol{M}]$ is defined by

$$[\boldsymbol{M}] = \sum_{j=1}^{Nelem}[\boldsymbol{m}] \tag{49}$$

where $[\boldsymbol{m}]$ is the element mass matrix which given by

$$[\boldsymbol{m}] = \int_0^l (m_c + C_a^*)[\boldsymbol{N}]^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [\boldsymbol{N}] ds \tag{50}$$

where m_c and C_a^* are the mass of the marine cable and the external fluid including the added mass coefficient, respectively. The linear global stiffness matrices $[\boldsymbol{K_L}]$ is

$$[\boldsymbol{K_L}] = \sum_{j=1}^{Nelem}[\boldsymbol{k_L}] \tag{51}$$

where $[\boldsymbol{k_L}]$ is the element linear stiffness matrix

$$[\boldsymbol{k_L}] = \int_0^l \left\{ \begin{array}{c} \frac{1}{1+\varepsilon_0}[\boldsymbol{N'}]^T \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} [\boldsymbol{N'}] \\ + \frac{EA}{(1+\varepsilon_0)^3}[\boldsymbol{N'}]^T \begin{bmatrix} {x'_0}^2 & x'_0 y'_0 \\ x'_0 y'_0 & {y'_0}^2 \end{bmatrix} [\boldsymbol{N'}] \end{array} \right\} d\bar{s} \tag{52}$$

Using standard procedure of the Galerkin finite element method, (47) leads to the eigenvalue problem as:

$$([\boldsymbol{K_L}] - \omega_i^2[\boldsymbol{M}])\{\bar{\boldsymbol{D}}\} = \{\boldsymbol{0}\} \tag{53}$$

where ω_i represents the natural frequency of vibration and $\{\bar{D}\}$ is the corresponding mode shapes in the Cartesian coordinate.

B. Nonlinear Free Vibration Solution

For the nonlinear free vibration, the equation of motion can be written as

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{D}}\} + ([\boldsymbol{K_L}] + [\boldsymbol{K_{NL}}])\{\boldsymbol{D}\} = \{\boldsymbol{0}\} \tag{54}$$

where the nonlinear axial stiffness matrix is.

$$[\boldsymbol{K_{NL}}] = [\boldsymbol{k_{NL}^1}] + [\boldsymbol{k_{NL}^2}] \tag{55}$$

in which the first order nonlinear axial stiffness matrix is

$$[\boldsymbol{k_{NL}^1}] = \int_0^l \frac{EA}{(1+\varepsilon_0)^3}[\boldsymbol{N'}]^T \begin{bmatrix} 3x'_0u' + y'_0v' & x'_0v' + y'_0u' \\ x'_0v' + y'_0u' & 3y'_0v' + x'_0u' \end{bmatrix} [\boldsymbol{N'}] d\bar{s} \tag{56}$$

and the second order nonlinear axial stiffness matrix is

$$[\boldsymbol{k_{NL}^2}] = \int_0^l \frac{EA}{(1+\varepsilon_0)^3}[\boldsymbol{N'}]^T \begin{bmatrix} \frac{3}{2}{u'}^2 + \frac{1}{2}{v'}^2 & u'v' \\ u'v' & \frac{3}{2}{v'}^2 + \frac{1}{2}{u'}^2 \end{bmatrix} [\boldsymbol{N'}] d\bar{s} \tag{57}$$

Using standard procedure of the Galerkin finite element method, (54) leads to the eigenvalue problem as:

$$\Big(([\boldsymbol{K_L}] + [\boldsymbol{K_{NL}}]) - \omega_i^2[\boldsymbol{M}]\Big)\{\bar{\boldsymbol{D}}\} = \{\boldsymbol{0}\} \tag{58}$$

where ω_i represents the natural frequency of vibration and $\{\bar{D}\}$ is the corresponding mode shapes in the Cartesian coordinate.

The nonlinear equation of motion shown in (58) is time-dependent; a time-independent nonlinear eigenvalue problem can be obtained by substituting a certain properties of the time function at the point of the maximum amplitude or at the reversal point of the motion. The eigenvalue problem is obtained by assuming the dynamic displacement value and substituting the characteristic of the time function as an instant with harmonic function [12].

$$\{\ddot{\boldsymbol{D}}\}_{max} = -\omega_i^2\{\boldsymbol{D}\}_{max} \tag{59}$$

where ω_i is the natural frequencies of the marine cable and $\{\boldsymbol{D}\}_{max}$ represents the dynamic displacement at the nodal point of maximum amplitude [10].

Substitution of (59) to (54) yields the equation of motion for large amplitude free vibration with time independent as:

$$\Big(([\boldsymbol{K_L}] + [\boldsymbol{K_{NL}}]) - \omega_i^2[\boldsymbol{M}]\Big)\{\boldsymbol{D}\}_{max} = \{\boldsymbol{0}\} \tag{60}$$

The relationship between the dynamic displacements of the marine cable at the point of maximum amplitude in (60) and the vibration mode shape can be expressed by

$$\{\boldsymbol{D}\}_{max} = a\{\boldsymbol{V_n}\} \tag{61}$$

where a represents the maximum amplitude of vibration and $\{\boldsymbol{V_n}\}$ for the normalized corresponding mode shapes.

V. Numerical Results

A. Natural Frequency Spectrum of Marine Cable

This section presents the study of the natural frequency spectrum for the cable, where the dimensionless frequencies (Ω) are plotted against the cable parameter (λ^2) which is proposed by Irvine and Caughey [1] for the cable with the support at the same elevation and sag value is relatively small, Triantafyllou and Grinfogel [2] proposed the cable parameter for the cable with the inclined support. The following dimensionless quantities are employed.

The cable parameter, λ^2

$$\lambda^2 = \left(\frac{wL_x}{T_\alpha}\right)^2 L_x \left(\frac{EA}{T_\alpha L_e}\right) cos^2\phi_\alpha \tag{62}$$

The dimensionless natural frequency, Ω

$$\Omega = \frac{\omega_i L_e}{\pi}\sqrt{\frac{M}{T_\alpha}} \tag{63}$$

where w is weight of cable (N/m), T_α is the static cable tension (N) at $\phi = \phi_\alpha$, ϕ_α is the angle of cable chord inclination (radian), E is the Elastic modulus of cable (N/m^2), A is the cross-sectional area of cable (m^2), L_x is the horizontal span length (m), L_e is the stretched cable length (m), ω_i is the natural frequency (radian/second), M is the total mass of cable per unit length (kg/m).

In this study, the cable with support at the same elevation is investigated for the natural frequency spectrum. The various top horizontal tensions from large to small value are applied to the free-sliding roller support. The cable length and the cable diameter are 869.42 m and 0.023 m, respectively. The cable unit weight is 9.48 N/m and the elastic modulus is varying from 1.794x10^6 kN/m^2 to 1.794x10^9 kN/m^2.

The natural frequency spectrum of the cable with the support at the same elevation is plotted between the cable parameter (λ/π) and the dimensionless frequency parameter (Ω/π) for the first eight mode shapes in Figs. 3-6. The small values of cable parameter correspond to a small sag and the large values of cable parameter correspond to a large-sag cable. The cable tends to change vibration behaviors for the top horizontal tension of 4 N, the plotted reverses back when top horizontal tension lower than 4 N which gives the very large sag cable, this illustrated the occurrence of two frequencies for the same cable parameters of very large-sag cable but the mode shapes are different.

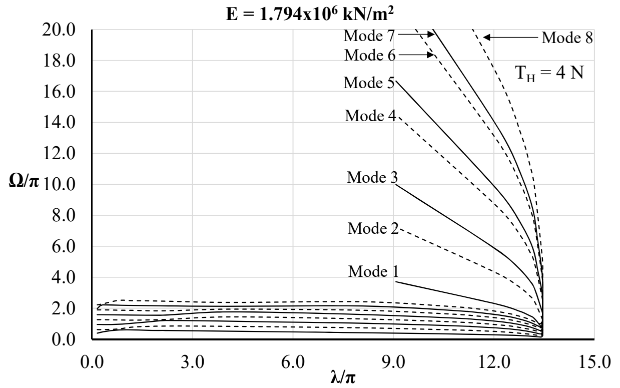


Fig. 3 Natural frequency spectrum of cable with elastic modulus, E = 1.794 x 10^6 kN/m^2

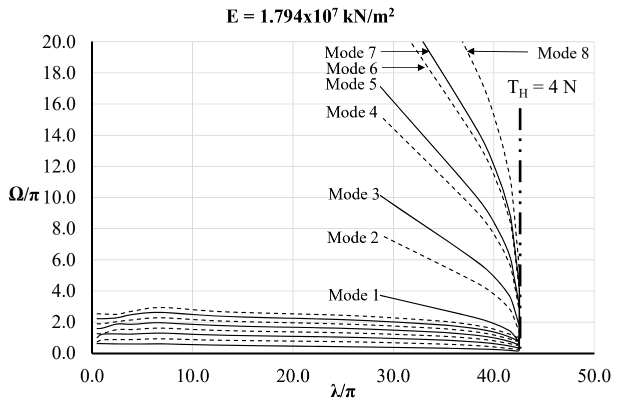


Fig. 4 Natural frequency spectrum of cable with elastic modulus, E = 1.794 x 10^7 kN/m^2

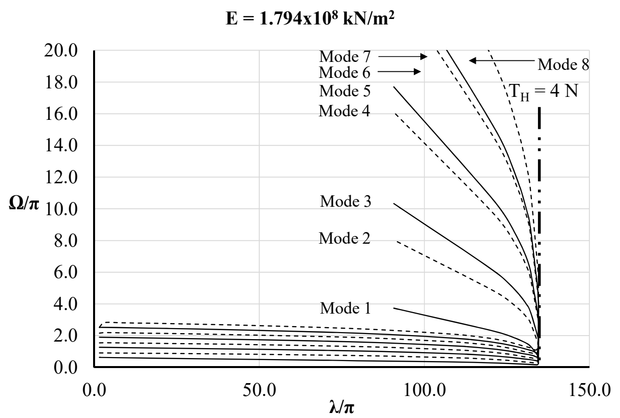


Fig. 5 Natural frequency spectrum of cable with elastic modulus, E = 1.794x10^8 kN/m^2

B. Large Amplitude Free Vibration Behavior

In order to demonstrate the effect of the extensibility on the nonlinear free vibration of the marine cable, the numerical investigation is carried out with the input parameters remain the same as natural frequency spectrum; except the current velocity of 1.0 m/s and the added mass coefficient of 1.0 are added.

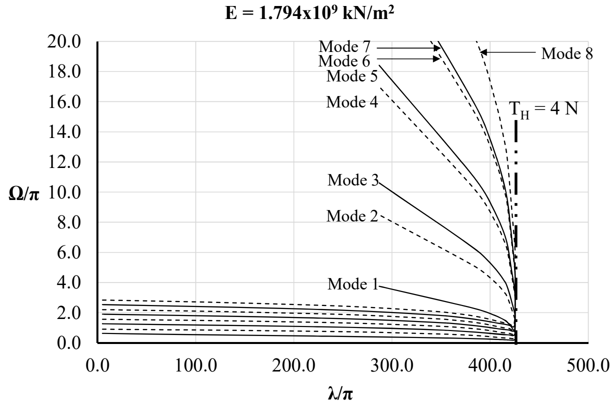
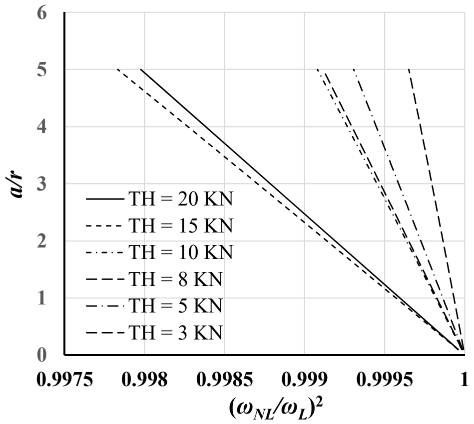


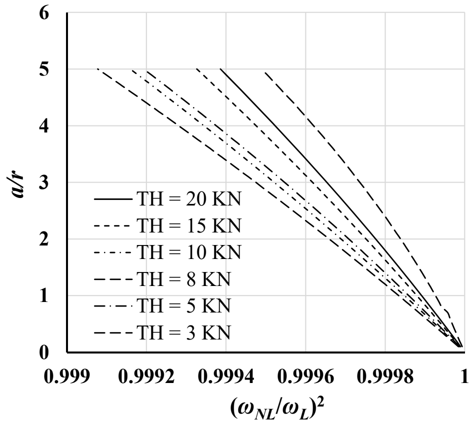
Fig. 6 Natural frequency spectrum of cable with elastic modulus, E = 1.794 x 10^9 kN/m^2

The effect of top horizontal tension on the relation between nonlinear frequency ratios $(\omega_{NL}/\omega_L)^2$ and amplitude of vibration (a/r) for cable supports at the same level and specified elastic modulus, E = 1.794x10^7 kN/m^2, 1.794x10^8 kN/m^2, 1.794x10^9 kN/m^2, and 1.794x10^{10} kN/m^2 are plotted in Figs. 7 (a)-(d), respectively. The nonlinear frequency in these figures showed a softening type for all value of elastic modulus and the degree of softening increased as the top horizontal tension value decreased.

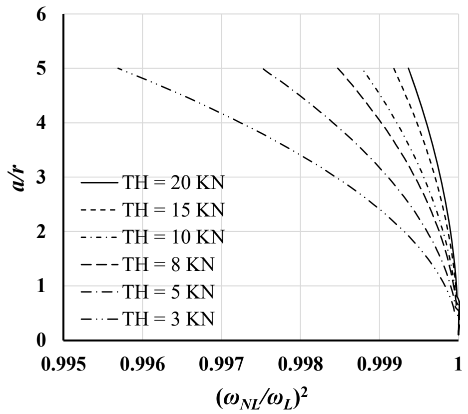
The nonlinear behavior of the marine cable with support at different level of 300 m exhibits the hardening type for lower elastic modulus value of 1.794x10^7 kN/m^2 and 1.794x10^8 kN/m^2, and a softening type for the elastic modulus value of 1.794x10^9 kN/m^2 and 1.794x10^{10} kN/m^2.



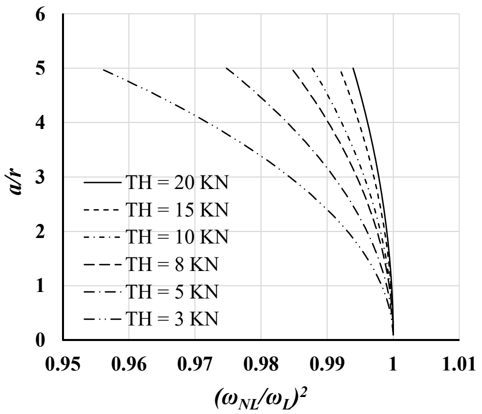
(a) Elastic modulus, E = 1.794 x 10^7 kN/m^2



(b) Elastic modulus, E = 1.794 x 10^8 kN/m^2

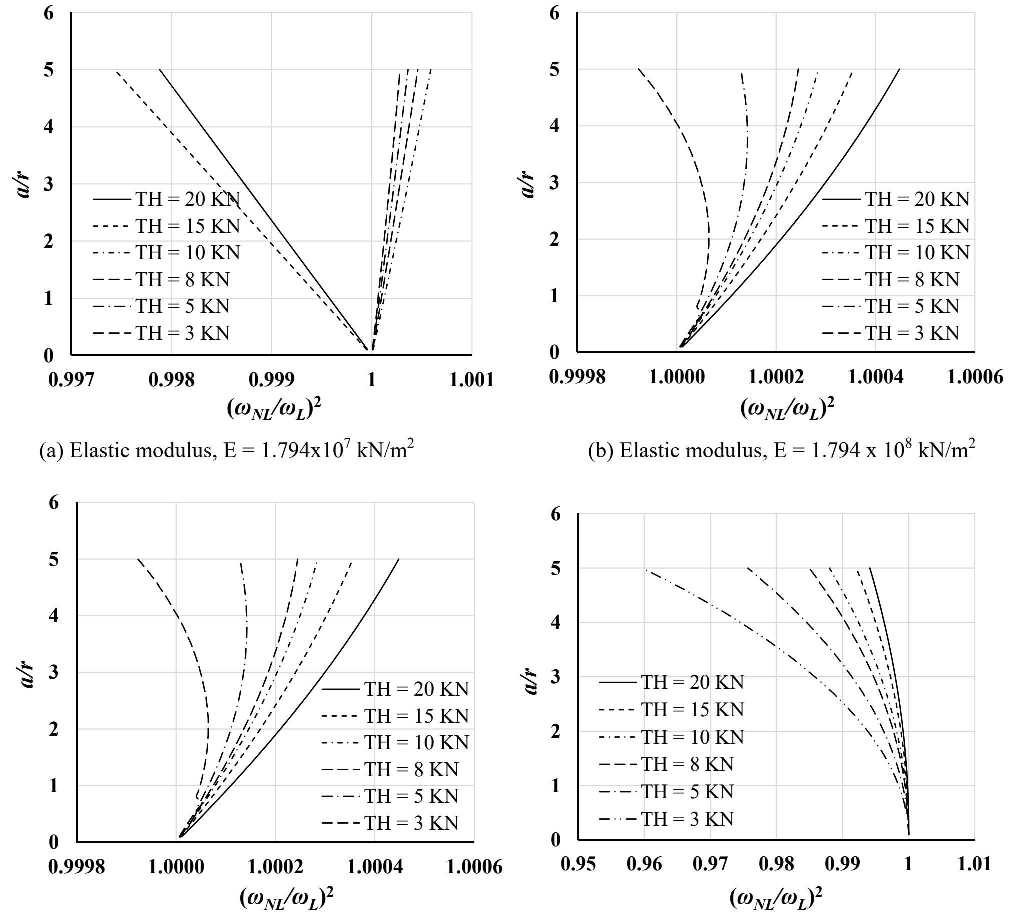


(c) Elastic modulus, E = 1.794 x10^9 kN/m^2



(d) Elastic modulus, E = 1.794 x 10^{10} kN/m^2

Fig. 7 Effect of top horizontal tension on the relation between nonlinear frequency ratios $(\omega_{NL}/\omega_L)^2$ and amplitude of vibration (a/r) for cable supports at the same level and specified elastic modulus, E = 1.794 x 10^7 kN/m^2 to E = 1.794 x 10^{10} kN/m^2



(a) Elastic modulus, E = 1.794x10^7 kN/m^2

(b) Elastic modulus, E = 1.794 x 10^8 kN/m^2

(c) Elastic modulus, E = 1.794 x 10^9 kN/m^2

(d) Elastic modulus, E = 1.794 x 10^{10} kN/m^2

Fig. 8 Effect of top horizontal tension on the relation between nonlinear frequency ratios $(\omega_{NL}/\omega_L)^2$ and amplitude of vibration (*a*/*r*) for cable supports at different level, 300 m and specified elastic modulus, E = 1.794x10^7 kN/m^2 to E = 1.794x10^{10} kN/m^2

VI. Conclusion

The model formulation based on the variational approach for large amplitude free vibration of a very sag extensible marine cable had been proposed. In the formulation, the arc-length coordinate adopted from the Lagrangian description was used as the independent variable. The total virtual work for the extensible marine cable in two dimensions was formulated, the linear and nonlinear stiffness matrices, and mass matrices were obtained. The eigenvalue problem of the linear and nonlinear free vibration analysis was solved by the inverse iteration method and the direct iteration method, respectively.

The natural frequency spectrum of the cable with the support at the same elevation was plotted between the cable parameter (λ/π) and the dimensionless frequency parameter (Ω/π) for the first eight mode shapes and illustrated that the small cable parameter corresponded to a small sag cable, while the large cable parameter corresponded to a very-large-sag cable. The cable had a tendency to change vibration behaviors for the low value of top horizontal tension, the natural frequencies spectrum reverses back and illustrated the occurrence of two frequencies parameter for the same cable parameters. The simple case of the large amplitude free vibration of the very sag extensible marine cable were presented and shown the hardening type for lower elastic modulus value and a softening type for the higher value of the elastic modulus.

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