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# Kinematic Analysis of a Novel Complex DoF Parallel Manipulator

M.A. Hosseini, P. Ebrahimi Naghani

***Abstract*—**In this research work, a novel parallel manipulator with high positioning and orienting rate is introduced. This mechanism has two rotational and one translational degree of freedom. Kinematics and Jacobian analysis are investigated. Moreover, workspace analysis and optimization has been performed by using genetic algorithm toolbox in Matlab software. Because of decreasing moving elements, it is expected much more better dynamic performance with respect to other counterpart mechanisms with the same degrees of freedom. In addition, using couple of cylindrical and revolute joints increased mechanism ability to have more extended workspace.

***Keywords*—**Kinematics, Workspace, 3-CRS/PU, Parallel robot

## I. Introduction

PARALLEL manipulators have received extensive attention over the last two decades for their potential superior properties, such as low inertia, high stiffness, high precision and high load carrying capacity [1]-[2]. Performance indices such as manipulability, condition number, conditioning and dexterity indices are useful to comparison study for ability of different robot structures. Manipulability at first was introduced by Yoshikawa [3] which is determinant of the Jacobian matrix. The Jacobian matrix maps the sphere of unique joint space into an ellipsoid, whose volume is defined as manipulability. This mapping contains initial rotation, extension and a final rotation. The extension term magnifies the rotated spherical space in specified directions by the associate singular values [4]. Salisbury and Craig [5] introduced the ratio between maximum and minimum singular values as the condition number in Euclidean norm. The inverse of the Euclidean condition number is defined as condition index which varies from 0 to 1. Jacobian entries are inhomogeneous in complex degrees of freedom manipulators. Thus, making Jacobian homogeneous is studied by some researchers [6]-[13]. Rangbaran and Angeles [6] introduced carachteristic length to making Jacobian homogeneous. Thus, entries with length dimension will be divided by this factor. Gosselin [9] introduced a method for formulating dimensionally homogeneous Jacobian matrix for a planar mechanism with one rotational and two translational dof. This Jacobian matrix relates the actuator velocities to the velocities of the x -and y-coordinates of two points on the end-effector platform.

M.A.Hosseini is assistant professor in Department of Engineering, University of Mazandaran, Babolsar, Iran. (phone: 98-9112122170; fax: 98-1125254204; e-mail: am2hosseini@yahoo.com).

P.E. Naghani is lecturer at Sama Technical and Vocational College Islamic Azad University, Tehran Branch, Tehran-Iran. (phone: 98-9122861759; fax: 98-1125254204; e-mail: peyman587@ gmail.com).

Kim and Ryu [10] furthered this work by using the velocities of three points on the end-effector platform to develop a dimensionally homogeneous Jacobian matrix. Pond and Corretero [11] furthered this method again by using three independent coordinates of three points on an end-effector platform. Moreover, Angeles [12] introduced engineering characteristic length for a rigid body transformation matrix to make it homogeneous.

Here it is introduced a novel mechanism with high positioning and orienting rate. Kinematic is studied and Jacobian matrix is derived from these equations. Because of complexity of DoF, Jacobian matrix is homogenized by using weighted factor method [13]. Moreover, as a case study workspace volume and related indices has been studied through the workspace. Using GA method in Matlab software led to optimal mechanism structure with maximum workspace volume.

## II. 3-CRS/PU Parallel Manipulator

As depicted in Fig. 1, this mechanism consists of three legs. Each one includes the fixed length link which connects spherical joint attached to moving platform to the revolute joints attached to cylindrical joint slides by a ball screw on a slant link connected to the base platform. A middle links contains passive prismatic joint at bellow and an upper part which connected to the moving platform are adjoin by a passive universal joint. This universal joint permits the moving platform to have rotational displacement around x and y axis. Moreover lower prismatic joint enables moving platform to have translational motion along z direction.

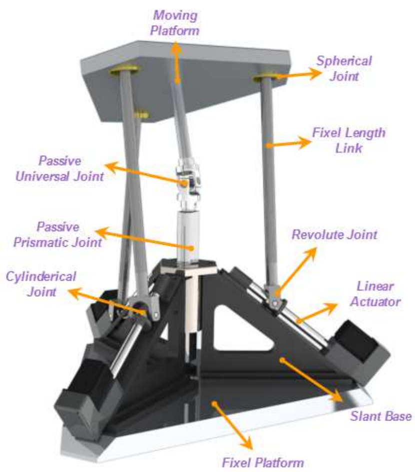


Fig. 1 CAD model of 3-CRS/PU parallel manipulator

## III. Kinematic Analysis

Geometrical model of mechanism is illustrated in Fig. 2. Two moving and global frames ({P (uvw)} and {O (xyz)}) are attached to the moving and base platforms, respectively. When the moving platform is parallel to the base, the two revolute axes of the passive leg universal joints (x' and y') are parallel with the base frame's axes (x and y), respectively.

The kinematic close loop equation can be written as follow for each leg:

$$\mathbf{c}+\mathbf{R}(\mathbf{a}_i+\mathbf{d})=b_i\mathbf{n}_{bi}+l_i\mathbf{n}_{li} \tag{1}$$

where **c** and **d** are the vectors from O to C and C to P, respectively. Moreover, **R** is rotation matrix carrying frame {P} into an orientation coincident with that of frame {O}; $\mathbf{a}_i$ is $i^{th}$ spherical joint position vector in moving frame. The connection radius of universal and spherical joint on the base and moving platform are illustrated by $b_i$ and $l_i$ respectively and associated unit vector by $\mathbf{n}_{bi}$ and $\mathbf{n}_{li}$, as well.


Fig. 2 Geometrical Model of 3-CRS/PU

### A. Inverse Kinematic

Considering following schematic of the mechanism, it is noteworthy that end of each fixed length link should be mounted on the sphere surface with center of $A_i$ and radius of l. Moreover, this point should slide on the slant base whose geometry is identified by the height (h). Position vector of $A_i$ can be defined by the following equation.

$$\mathbf{A_i}=\mathbf{R}(\mathbf{a}_i+\mathbf{d})+\mathbf{c} \tag{2}$$

Thus, intersection of slant base and associate sphere determines the revolute joint position and consequently actuator length.

Considering spherical and universal joints position vector as $\mathbf{A}_i=[x_{ai}\ \ y_{ai}\ \ z_{ai}]^T$ and $\mathbf{B}_i=[x_{bi}\ \ y_{bi}\ \ 0]^T$ the parametric equation of $GB_i$ can be written as follow.

$$\begin{aligned} x &= -x_{bi}\,t_i + x_{bi} \\ y &= -y_{bi}\,t_i + y_{bi} \\ z &= h\,t_i \end{aligned} \tag{3}$$

Substituting above equation in the parametric equation of sphere as the following:

$$(x-x_{ai})^2+(y-y_{ai})^2+(z-z_{ai})^2-l^2=0 \tag{4}$$

Led to following equation

$$m_i t_i^2 - 2n_i t_i + p_i = 0 \tag{5}$$

In which coefficients are as bellow

$$m_i=(x_{bi}{}^2+y_{bi}{}^2+h^2) \tag{6}$$

$$n_i=(x_{bi}{}^2+y_{bi}{}^2-x_{bi}x_{ai}-y_{bi}y_{ai}+hz_{ai}) \tag{7}$$

$$\begin{aligned} p_i=(&x_{bi}{}^2+x_{ai}{}^2-2x_{bi}x_{ai}+y_{bi}{}^2 \\ &+y_{ai}{}^2-2y_{bi}y_{ai}+z_{ai}{}^2-l^2) \end{aligned} \tag{8}$$

Substituting resultant $t_i$ from Eq. (5) into Eq. (3) determines the actuator length. However, it is noteworthy that $t_i$ limits should vary between 0 to 1.

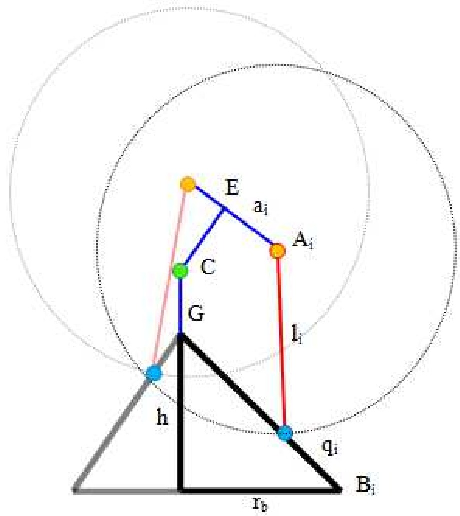


Fig. 3 Schematic configuration of 3-CRS kinematic

Accordingly, following cases may occur.

Case 1) The slant guide way does not intersect associate sphere: thus there is not any root for inverse kinematic equation. Correspondingly, assumed position will be out of reach by end-effector (EE).

Case 2) The slant guide way tangent to the associate sphere: thus there is one root for the assumed link and with respect to other links conditions there may be 1, 2 or 4 roots or any root for kinematic equation.

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Case 3) The slant guide way intersects associate sphere: thus there is two roots for the assumed link and with respect to other links conditions there may be 2 or 4 roots or any root for kinematic equation.

## IV. Jacobian Matrix and Velocity Analysis

Taking first time derivation from Eq. (1) led to following equation:

$$\dot{\mathbf{c}} + \boldsymbol{\omega}_p \times \mathbf{R}(\mathbf{a}_i + \mathbf{d}) = \dot{q}_i \mathbf{n}_{qi} + \boldsymbol{\omega}_l \times l\mathbf{n}_{li} \tag{9}$$

In which $\boldsymbol{\omega}_l$ and $\boldsymbol{\omega}_p$ are fixed length link and moving platform angular velocities, respectively. Inner product of the both sides of Eq.9 by $\mathbf{n}_{li}$, upon simplifications leads to:

$$\dot{\mathbf{c}}\mathbf{n}^T_{li} + \boldsymbol{\omega}_p \times \mathbf{R}(\mathbf{a}_i + \mathbf{d})\mathbf{n}^T_{li} = \dot{q}_i \mathbf{n}_{qi}\mathbf{n}^T_{li} \tag{10}$$

Equation (10) can be rewritten as bellow

$$\mathbf{n}^T_{li}\dot{\mathbf{c}} + (\mathbf{n}_{li} \times \mathbf{R}(\mathbf{a}_i + \mathbf{d}))^T \boldsymbol{\omega}_p = \dot{q}_i \mathbf{n}^T_{li}\mathbf{n}_{qi} \tag{11}$$

Above equation can be written as following general form

$$\mathbf{A}\dot{\mathbf{x}} = \mathbf{B}\dot{\mathbf{q}} \tag{12}$$

In which $\dot{\mathbf{x}}$ and $\dot{\mathbf{q}}$ are end-effector twist array and joint space velocity vector. Moreover, **A** and **B** are Two Jacobian matrices which are as Eq. 13 and 14.

$$\mathbf{A} = \begin{bmatrix} \mathbf{n}_{l1} & (\mathbf{n}_{l1} \times \mathbf{R}(\mathbf{a}_1 + \mathbf{d}))_x & (\mathbf{n}_{l1} \times \mathbf{R}(\mathbf{a}_1 + \mathbf{d}))_y \\ \mathbf{n}_{l2} & (\mathbf{n}_{l2} \times \mathbf{R}(\mathbf{a}_2 + \mathbf{d}))_x & (\mathbf{n}_{l2} \times \mathbf{R}(\mathbf{a}_2 + \mathbf{d}))_y \\ \mathbf{n}_{l3} & (\mathbf{n}_{l3} \times \mathbf{R}(\mathbf{a}_3 + \mathbf{d}))_x & (\mathbf{n}_{l3} \times \mathbf{R}(\mathbf{a}_3 + \mathbf{d}))_y \end{bmatrix} \tag{13}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{n}^T_{l1}\mathbf{n}_{q1} & 0 & 0 \\ 0 & \mathbf{n}^T_{l2}\mathbf{n}_{q2} & 0 \\ 0 & 0 & \mathbf{n}^T_{l3}\mathbf{n}_{q3} \end{bmatrix} \tag{14}$$

The Jacobian matrix can be determined by Eq. 15.

$$\mathbf{J} = \mathbf{B}^{-1}\mathbf{A} \tag{15}$$

## V. Singularity Analysis

Generally, singularity occurs in the condition in which there is not any root for Eq. 12 or there are infinity roots. In this condition, robot misses one or more own degrees of freedom or gains more degrees of freedom. This condition may occur with respect to determinant of two Jacobian matrices, i.e. **A** and **B**. Moreover, Zalatanov et. al. [14] illustrated some constraint singularity, for mechanisms, too.

For our mechanism, four cases may cause to singularity as follow:

Case 1) Det (**A**) = 0 or Direct singularity; this condition occurs when the z coordinates of fixed length links be zero. In this condition all three legs lie in moving platform plane which is parallel to the base one. Moreover, if any fixed length link locates along the line connect the middle passive universal joint to associate spherical joint ($CA_i$) by increasing the actuator length, there are two positions to locate, as depicted in Fig. 4.

Case 2) Det(**B**)=0 or Inverse singularity; In this condition one of fixed length link will perpendicular to the associate linear guide way direction. This case is the same condition which one root may exist.

Case 3) Constraint singularity; this case will occur when the moving platform rotates 90 degrees around x or y axis. In this condition platform will lose another rotation capability.

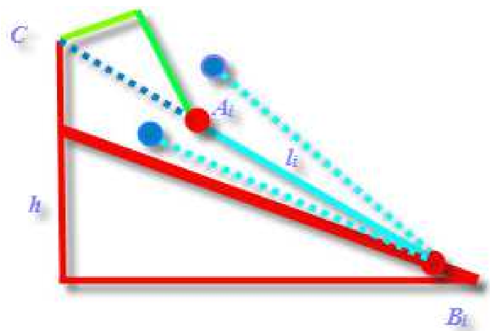


Fig. 4 Schematic for direct singularity

## VI. Workspace and Optimization

Applying inverse kinematic equation and a search algorithm in different height will determine bound of reachable points [15]. Using genetic algorithm leads to optimized structure cause to maximum workspace volume [3], [15]. So, design parameters and their limitation should be considered as the optional items which workspace volume is the function of. Respect to Fig. 1, design parameters and assumed limitations are cited in Table (1).

TABLE I
Geometrical Constraint for Mechanism

| Actuator (mm) | ζ (deg) | d (mm) | l (mm) |
|---|---|---|---|
| 400-750 | ±60 | 20-200 | 100-300 |
| λ (deg) | $r_b$ (mm) | | $r_a$ (mm) |
| 10-80 | 300-500 | | 100-300 |

As a case study, considering structure with parameters cited in Table (2) and constraints according to Table (1), the search algorithm lead to workspace volume as the Fig. 5.

TABLE II
The Case Study Design Parameters

| d (mm) | l (mm) | λ (deg) | $r_b$ (mm) | $r_a$ (mm) |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| 100 | 250 | 45 | 300 | 100 |

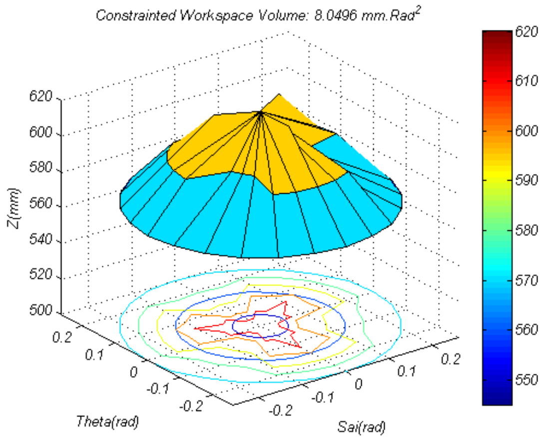


Fig. 5 Case study workspace

As illustrated in Fig. 5, the workspace volume is 8.05 (mm.rad$^2$).

Variation of local conditioning index, maximum and minimum singular values in altitude of z=500 mm is depicted in following figures. It is noteworthy that Jacobian matrix is homogenized by weighted factor as much as 200mm [13].

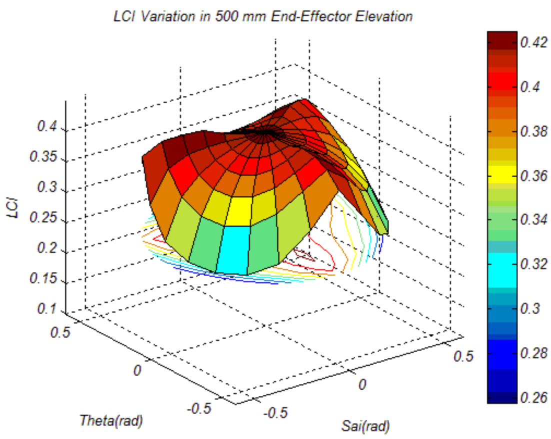


Fig. 6 Local Conditioning Index versus EE orientation in z=500mm
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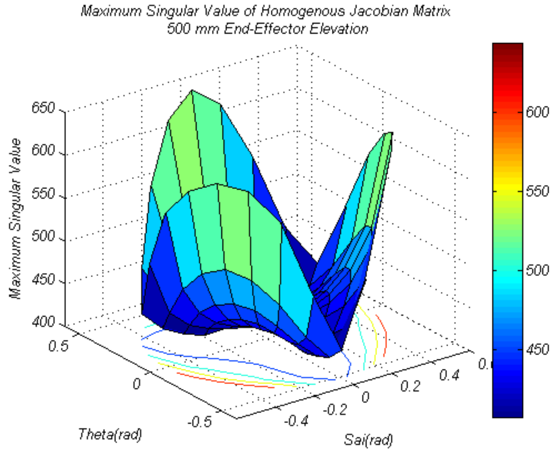


Fig. 7 Maximum singular value of homogenous Jacobian matrix versus EE orientation in z=500mm

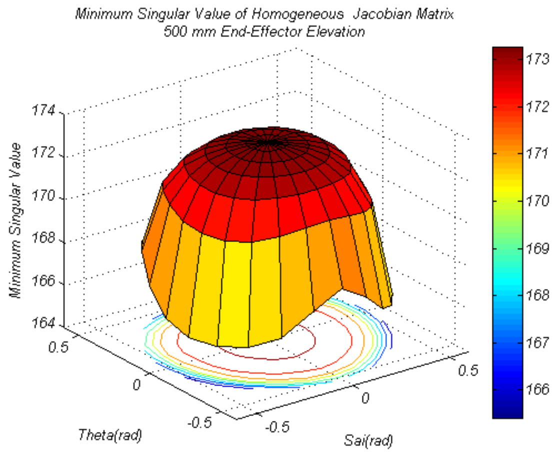


Fig. 8 Minimum singular value of homogenous Jacobian matrix versus EE orientation in z=500mm

Figure 6, shows that how far is the mechanism from the singularity. Moreover, it shows the mechanism isotropic behavior around zero rotations. The amplification factor via unique displacement in actuators is shown by singular values of homogenous Jacobian matrix. Figures 7 and 8, illustrates that these factors are too much. This is very important which identifies compatibility of mechanism in high positioning and orienting tasks.

Using GA in Matlab software, according to following objective function and constraints, it is lead to structure whose parameter is cited in Table III.

$$V^{*}=Max(V(d,r_a, r_b, \lambda, l)) \quad (16)$$

Subject to:

1- $100<r_a<300$, $300<r_b<500$, $20<d<200$,
2- $10<\lambda<80$, $100<l<300$,
3- $300<$Actuator Length $(L)<650$
4- $-60<\zeta_i<60$ deg.

Optimization operation leads to design parameters according to Table III. It shows 19.28 times increasing in workspace volume. Moreover, optimal workspace is illustrated in Fig. 9.

TABLE III
THE CASE STUDY DESIGN PARAMETERS

| $V^*$ (mm.Rad$^2$) | l (mm) | $\lambda$ (deg) |
|---|---|---|
| 155.18 | 451.292 | 70.308 |
| d (mm) | $r_b$ (mm) | $r_a$ (mm) |
| 21.547 | 357.76 | 211.28 |

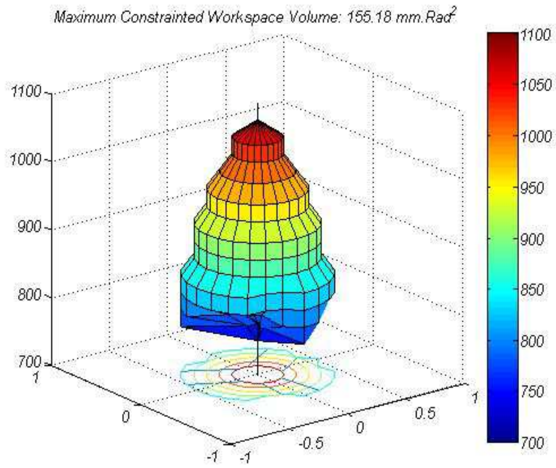


Fig. 9 Optimal workspace for the case study mechanism

## VII. CONCLUSION

In this research work a novel parallel manipulator with 3-CRS/PU structure is introduced. The mechanism has two rotational and one rotational degrees of freedom. Inverse kinematic equations with a geometrical approach have been solved and used to workspace evaluation. Jacobian matrix is derived by taking the first time derivation respect to time. Jacobian entries inhomogeneity has resolved by weighted factor approach. Variation of associate singular values through the workspace illustrates the capability of mechanism in high positioning and orienting tasks. Moreover, using genetic algorithm in Matlab software led to optimal structure with maximal workspace volume.

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**Mir Amin Hosseini** is currently Assistant Professor in the Department of Engineering, at Mazandaran University, Babolsar, Iran. He received his B.Sc. in Manufacturing Engineering at Tehran Polytechnic University and M.Sc. in Manufacturing Engineering at Tarbiat Modarres University. He received his PhD in Manufacturing Engineering too, at University of Mazandaran. His research interests focus on parallel machine tools.

**Peyman Ebrahimi Naghani** is currently lecturer at Tehran Sama College, Tehran, Iran. He received his B.Sc and M.Sc. in Mechanical Engineering at Islamic Azad University, Karaj branch and Tehran Center branch.