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Investigation of Undular Hydraulic Jump over Smooth Beds

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Abstract**—**Undular hydraulic jumps are illustrated by a smooth rise of the free surface followed by a train of stationary waves. They are sometimes experienced in natural waterways and rivers. The characteristics of undular hydraulic jumps are studied here. The height, amplitude and the main characteristics of undular jump is depended on the upstream Froude number and aspect ratio. The experiments were done on the smooth bed flume. These results compared with other researches and the main characteristics of the undular hydraulic jump were studied in this article.

Keywords**—**Undular Hydraulic Jump, low Froude Number, wave characteristics

I. INTRODUCTION

RAPID transition from super-critical to sub-critical flow is called Hydraulic jump. This phenomenon is commonly happening in rivers, natural waterway or canals. Hydraulic jump is one of the interesting happenings in the open channel hydraulic. The undular hydraulic jump is a special case of the hydraulic jump. For super critical flow with Froude number near to unity, an undular hydraulic jump is formed by a smooth rise of the surface and undulations that stretch out over a long length. In this type of hydraulic jump, there is not any significant energy dissipation and air entrainment. As the undular hydraulic jump does not have any considerable "roller", the energy-loss is retarded forward into a train of stationary waves [1-2].

The knowledge of the feasible flow circumstances of undular jumps is important for designing and managing channels, and hydraulic structures. In addition, information about the formation of undular jumps can be useful in planning water-sport facilities such as rafting chutes, which can also be an attractive feature of flowing water in a landscape.

The waves caused by undular hydraulic jump may also affect the downstream discharge measurement structures such as sharp-crested and broad-crested weir. Gibson [3] studied the effect of free surface waves on discharge measurement over weirs, and he found that for the wave heights of about $\Delta h/d_c \sim 0.375$, the discharge increase about 2 to 3%. The free-surface undulations might be taken place at the downstream of plunging jets, submerged out let or flip bucket [4].

Some researches [4-8] show that an undular jump is fundamentally two-dimensional but close to the sidewalls. The free-surface undulations are quasi-periodic, but the longitudinal profile is neither sinusoidal nor conoidal. Sidewall cross-waves were seen upstream of the first wave crest and sometimes immediately upstream of the downstream wave crests. Fig. 1 shows a typical free surface profile of undular hydraulic jump and illustrates the important parameters in this phenomenon.

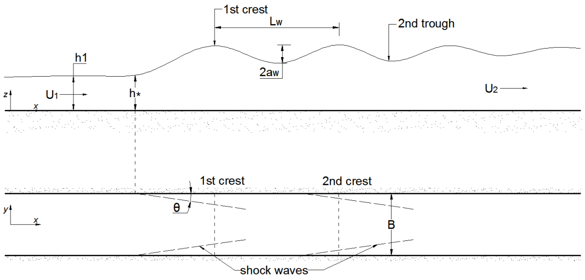

Fig. 1 Free-surface profile of an undular hydraulic jump

Some researchers [9-11] studied hydraulic jump included some undular jump cases, but the respects of undular hydraulic jumps was not considered in their researches. Some other authors have done some experiments on undular jumps [12-16]. Their analyses were based on similarity with undular surge in tranquil flow. Montes [2] and Ryabenko [17] disagree with the hypothesis of similarity between undular hydraulic jump and undular surge. Their results show that the undular hydraulic jump become weak jump at the upstream Froude numbers in the range 1.0 to 3.6 where this conversion is a function of upstream flow conditions (i.e. Froude number, aspect ratio and roughness). On contrary, an undular surge vanishes for surge Froude number higher than 1.5 to 1.8 [18-21].

Some of the recent studies about undular hydraulic jump are listed in Table I. The modern instrument like LDV, PIV, and ADV systems record just the velocity field. However, in the undular hydraulic jumps, the pressure field is not hydrostatic and the pressure data must be recorded together with velocity field. The only instrument that can measure velocity, pressure and total energy, and the bed shear stress after appropriate calibration is the Prandtl–Pitot tube [22, 24, 29].

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TABLE I
SUMMARY OF THE STUDIES ON UNDULAR HYDRAULIC JUMPS [22]

Author	Channel	Instrumentation
Chanson and Montes[6]	L = 20 m, B = 0.25 m, F/D inflow	Prandtl–Pitot tube (Ø3.3 mm)
Montes and Chanson [23]	L = 12 to 20 m, B = 0.2 to 0.3 m, F/D inflow	Prandtl–Pitot tubes
Chanson[24]	L = 20 m, B = 0.25 m, F/D inflow	Pitot–Preston tube (Ø 3.3 mm)
Ohtsu et al[25]	L = 5 to 20 m, B = 0.1 to 0.8 m, P/D & F/D inflow	Micro-propeller (Ø 3 mm), Prandtl–Pitot tube, 1D- LDV
Chanson [26]	L = 3.2 m, B = 0.5 m, P/D inflow	Prandtl–Pitot tube (Ø 3.3 mm)
Lennon and Hill [27]	L = 4.9 m, B = 0.3 m, F/D inflow	Particle Image Velocimetry (PIV)
Meftah et al. [28]	L = 15 m, B = 4 m, F/D inflow	Acoustic Doppler Velocimetry (ADV)
Chanson and Montes[6]	L = 20 m, B = 0.25 m, F/D inflow	Prandtl–Pitot tube (Ø3.3 mm)
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Notes. *B*: channel width; F/D: Fully developed; *L*: channel length, P/D: partially developed.

TABLE II
CLASSIFICATION OF THE UNDULAR HYDRAULIC JUMPS

Type	Range of Upstream Froude number	Description
Type A	$1 \le Fr_1 \le Fr^A$	Undulations of free surface have small amplitude and proportionately long wave length. There is a two-dimensional flow without any roller or shock wave in the surface of flow.
Type B	$Fr^A \le Fr_1 \le Fr^B$	Lateral shock waves generate at the upstream of the first wave crest. These shock waves cross slightly downstream of the first crest of wave. Lateral shock waves appear from both sidewalls next to the toe of the jump when the upstream Froude number is higher than 1.2
Type C	$Fr^B \le Fr_1 \le Fr^C$	The lateral shock waves intersect at the first crest, and a small roller appears at the first crest, directly after crossing of the shock waves. A "*cockscomb*" shape roller is places on the centerline. The roller has small size and does not occur at the later waves.
Type D	$Fr^C \le Fr_1 \le Fr^D$	For larger Froude numbers, the undular jump has a similar appearance with shock wave and air bubble enter the water at the top of the first wave. The bubbles trap at the intersection of the roller and lateral shock waves at the short distance (less than a single wave length). A small roller may appear at the second crest but no air bubble entrainment is observed here.
Type E	$Fr^D \le Fr_1 \le Fr^E$	In this type of undular jumps, the size and width of the roller (at the first crest of wave) raise and roller are limited by the lateral shock waves. Moreover, the air bubbles appear at the second crest.

II. CLASSIFICATION OF UNDULAR JUMPS

The undular hydraulic jump characteristics is related to inflow conditions: the upstream Froude number, the aspect ratio, the Reynolds number, the channel slope, the side wall and bed roughness and the boundary layer development at the toe of the jump, e.g., the turbulence level [8]. Chanson and Montes [6] classified the undular jumps base on the upstream Froude number (Table II).

For upstream Froude numbers Higher than Fr^E , the undulation of free surface will be disappeared and roller will be fully developed over the channel width and the jump become a weak jump.

Ohtsu et al. [8] divided the undular hydraulic jump into two cases. Case I and II include non-breaking and breaking undular jumps. A non-breaking undular jump has a continuous water surface without any breaking, although a breaking undular jump is referred to as an undular jump with a surface roller at the center part of the first wave.

III. CHARACTERISTICS OF UNDULAR HYDRAULIC JUMP

A. Experimental Apparatus

Experiments were performed in a 10-m long channel of uniform rectangular section made of glass (sidewalls) and steel(bottom), located in the Hydraulic Laboratory of the Universiti Sains Malaysia (Fig. 2).

The channel width is 0.30 m and the sidewall height is approximately 0.6 m. The channel slope can be adjusted using a geared lifting mechanism. Tailwater levels were controlled by a radial gate fitted at the downstream channel end. The upstream flow was controlled by a sluice gate. In this study, the water discharge ranges is 6.94 to 12.5 l/s and the upstream Froude number is more than 1.35.

Fig. 2 A photograph of an experiment on smooth bed

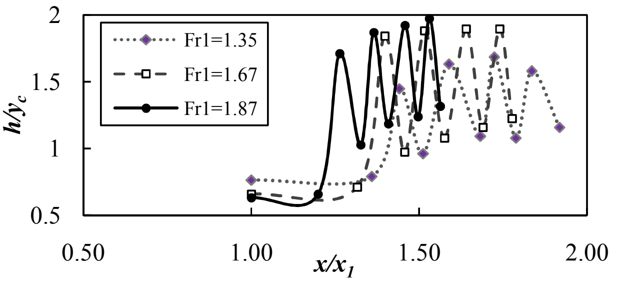

Fig. 3 Water surface profile in undular hydraulic jump (for *yc/B*=0.126)

B. Flow Pattern

In Type A of undular jumps, the flow is two-dimensional downstream of the top of the first wave. In type B, flow is symmetrical around the centerline of channel. For upstream Froude number higher than Fr^B, a various shape flow pattern is considered. After the first crest, the free surface is steady and symmetrical pattern and the crest of each wave is placed on the centerline (with similar phase shift and wavelength) but with smaller amplitudes [6, 30-32]. Fig. 1 shows the water surface profile in the undular hydraulic jump for *Q*=6.94 l/s until 4th undulation.

C. Lateral Shock Waves

The main properties of undular hydraulic jumps are lateral shock waves. Shock waves were generated in the positive pressure gradient domain. Configuration of vertical velocity and pressure distributions combine with lateral boundary layers, and the sidewall boundary layers caused to an immediate converse pressure gradient which cause a sudden decrease of velocity near the wall and maybe appear separation in this area [6].

The adverse pressure gradient takes place from being of subcritical flow depth in the jump, and the dimension of a sidewall-boundary-layer development is expanded from the toe of the jump. Then, the surface-flow velocity near the sidewalls is inclined to become critical, such that the lateral shock waves emerge from both sides of the toe section. Chanson[22] proposed a relation between Fr^*, θ and Fr_1 based on the experimental data:

$$Fr^* = Fr_1 - 0.119 \tag{1}$$

$$\theta = 28.1 \times Fr_1^{0.38} \tag{2}$$

Where, Fr^* is the Froude number at the start of the shock waves and θ is the angle between the sidewall and shockwaves. Properties of shock waves in the undular hydraulic jump were presented in Fig. *5*.

In Fig. *5*, the experimental data was compared with studies of undular hydraulic jump by Chanson [4]. From Fig. *5*(a), there are close comparisons between presented result and Chanson [4] and these differences is related to applied aspect ratios. The trend of rising the Fr^* with $Fr_1$is near to Chanson [4] experiments but the differences between present and Chanson [4] experiments for θ is further more (Fig. *5*(b)). Based on recent experiments the modified formulations for Fr_* and θ is:

$$Fr^* = 1.0591 Fr_1 - 0.0033,\ R^2 = 0.89 \tag{3}$$

$$\theta = 35.57 \times Fr_1^{0.0681}, R^2 = 0.909 \tag{4}$$

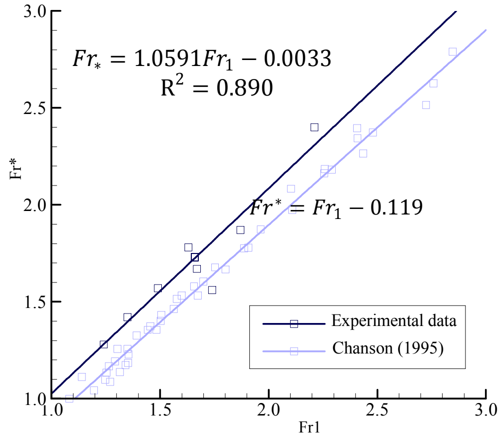

Fig. 4 Flow Froude number Fr* at the inception of the lateral shock waves with the sidewall

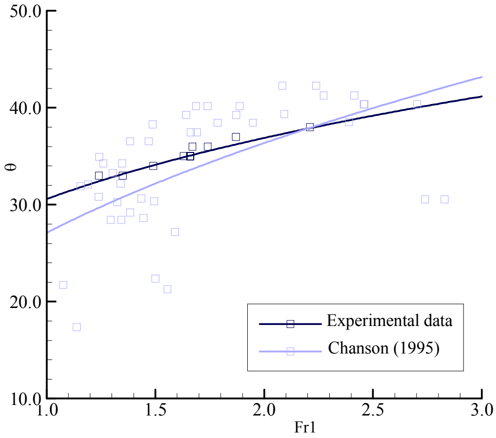

Fig. 5 Angle θ of the lateral shock waves with the sidewall

D. Wave Properties

Wave length (l_w): For a fixed Froude number, decrease of discharge reaches rising of dimensionless wave length and relative roughness and friction. Wave length decrease exponentially along the canal and the rate of this reduction is free from the upstream Froude number, the aspect ratio and the type of undular hydraulic jump [6].

Wave amplitude (a_w): For Froude number near the critical value, the wave amplitude close to the theoretical solution of Boussinesq equation [6, 12]. With increasing Froude number, the wave amplitude data separating from the solution of the motion equation and arrive in maximum value [1, 6, 10, 13].

For higher Froude numbers, with increasing Froude number, the wave amplitude decreases. Before the free-surface undulations disappear, the wave amplitude does not decrease with increasing Fr_1. When the undular jump become a weak jump, the amplitude of the first wave will aproximate to half of the roller height, and the amplitude of the second wave will be close to zero [6]. Fig. 6 and Fig. 7 show the dimensionless wave amplitude and wave length. The data show the maximum wave amplitude that takes place for Froude number 1.5 to 1.7.

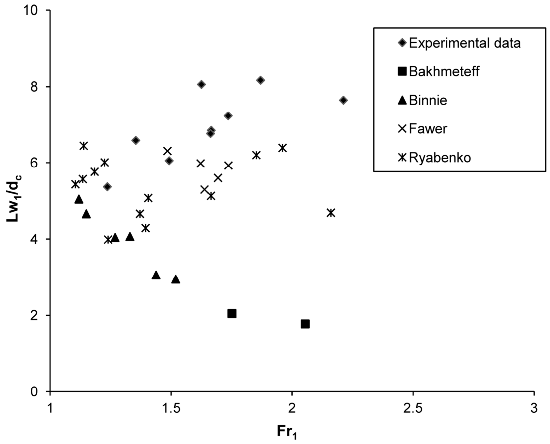

Fig. 6 Dimensionless first wave length

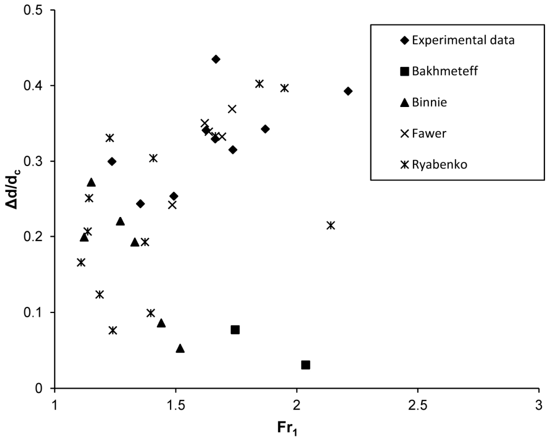

Fig. 7 Dimensionless first wave amplitude

E. Characteristics of the First Wave in Undular Jumps

For case I (non-breaking and breaking undular jumps), the flow parameters are only affected by Froude number [8]:

$$\frac{h_{wc}}{h_1} = -0.76\left(Fr_1 - 1\right)^2 + 2.3\left(Fr_1 - 1\right) + 1, \qquad 1.0 < Fr_1 \le Fr_{1u} \tag{5}$$

$$\frac{h_{wt}}{h_1} = 0.90\left(Fr_1 - 1\right)^{2.5} + 0.2\left(Fr_1 - 1\right) + 1, \qquad 1.0 < Fr_1 \le Fr_{1u} \tag{6}$$

Where, h_{wc} and h_{wt} are depth of flow in first crest and trough, respectively. Fig. 8 and Fig. 9 show the experimental data recognize the trend of h_{wc}/h_1 and h_{wt}/h_1 but some differences. The reason of this divergence is related to experimental condition. The undular hydraulic jumps are also affected by the aspect ratio while Ohtsu et al. [8] did not consider the effect of the aspect ratio in their formulas.

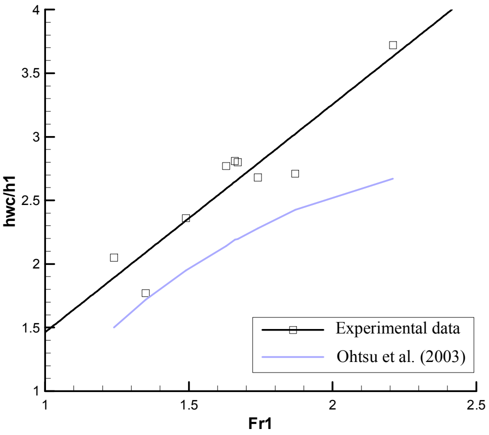

Fig. 8 Depth of first crest of undular hydraulic jump

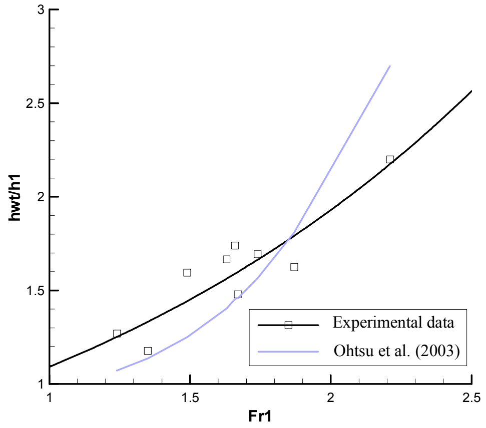

Fig. 9 Depth of first trough of undular hydraulic jump

F. Velocity and Pressure Distributions in Undular Jumps

In the undular hydraulic jump due to the existence of wavy surface, the velocity and pressure distributions are different in the crests and troughs. Velocity and pressure distributions are presented in Fig. 10 and Fig. 11. The velocities in Fig. 10 were normalized by the mean centerline velocity U_{cl} and shown as a function of y/h, where y is distance from bed measured normal to channel bottom and h is flow depth in centerline. Fig. 10 shows the main differences of velocity profile happened between upstream flow and first wave crest. In addition, the strong velocity decrease was observed in first and second crest, near the free surface for all cases of smooth bed but in the lowest Froude number, this decrease is small.

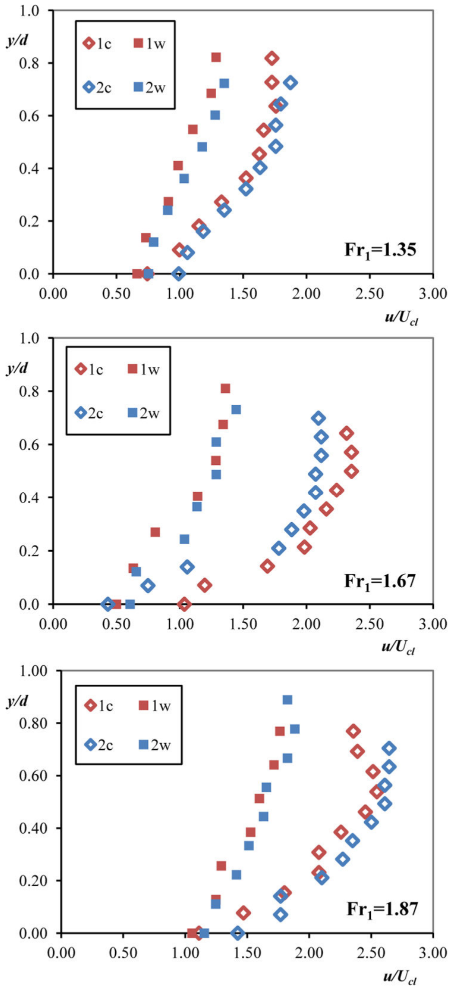

Fig. 10 Dimensionless velocity distributions in undular jumps

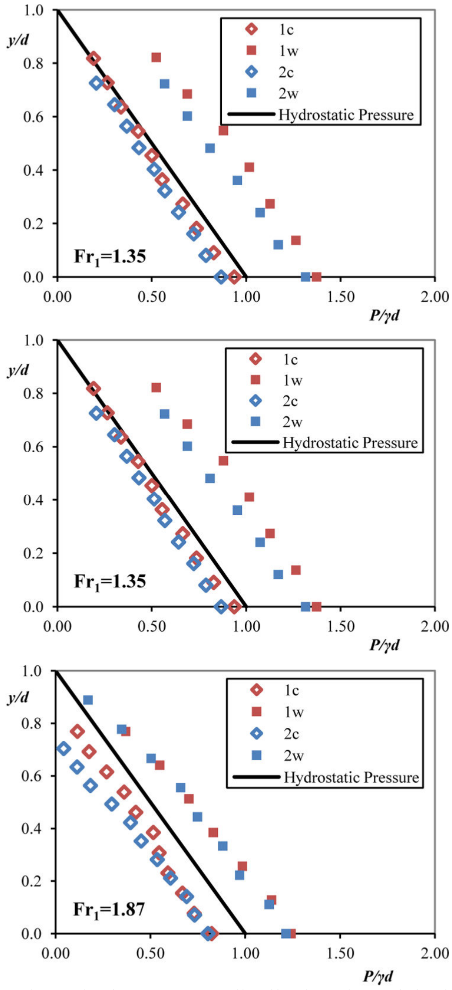

Fig. 11 Dimensionless pressure distributions in undular jumps

The flow pattern at the first crest of the undular hydraulic jump diverges considerably from those at the consequent wave crest. For later sections, the agreement between measured data and calculated data is improved. The results of velocity distribution show that the flow acceleration next to the bed (i.e. y/d_c<0.3) between crest and trough is higher than between crest and trough.

The dimensionless pressure in Fig. 11 is shown as a function of *y/h*. In the undular hydraulic jumps, due to the undulation of free surface, the pressure distribution is not hydrostatic. Where the free surface has concaved shape, the real pressure is larger than hydrostatic and in the convex surface the pressure is less than hydrostatics. This case can be defined by the irrotational flow motion theory [21, 33]. This concept is perceptible in Fig. 11.

IV. CONCLUSION

For super critical flow with Froude number near to unity, an undular hydraulic jump is formed by a smooth rise of the surface and undulations. An undular hydraulic jump may happen in irrigation and water supply channels, in narrow or shallow straights and downstream of short drop structures or in the transitional region from steep to mildly sloping channels. The knowledge of the feasible flow circumstances of undular jumps is important for designing and managing channels, and hydraulic structures. When undular jump forms in a channel, large amplitude waves were produced and extended downstream. These phenomena may damage the channel banks and the rising of water level due to these undulations must be considered in design of channels. Furthermore, the generation of free-surface waves might enforce additional impact loads and vibrations on downstream structures. Hence finding some ways to reduce the length of distribution of waves, wavelength and amplitude is very important.

The experimental measurements indicate some metrical and kinematical properties of undular hydraulic jumps over the smooth beds. The water surface profile shows a traveling train of conical waves. These waves propagate along the length of channel. The velocity profile of crests and troughs are different. From hydraulic point of view, due to the curve shape of free surface, the pressure distribution is non-hydrostatic and this concept is obvious on the pressure measurement.

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